AN EXPLORATION OF VOLUME TEST V/Vm UNDER VARIOUS VALUES OF DECELERATION PARAMETER q_o

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ABSTRACT. The problem of volume test,V/Vm,in zero pressure and matter dominated Friedmann universe model is explored under various values of deceleration parameter q_o. The following conclusions are drawn. (i) For different values of q_o, the change of V/Vm is sensitive at small values of z. Since values of V/Vm are very small at small values of z, this change exerts little effect on the average values of V/Vm. (ii)

when values of z and z_m are fixed for each member of a sample, large values of q_o yield larger values of V/Vm,especially in the case of large z. (iii) For $q_o \leq 1$,the method of volume test is reliable. When $q_o>2$,especially $q_o>3$,this method has to be used with caution at large z.

1. FORMULAE AND RESULTS

Let H_o,q_o and z be the Hubble constant,deceleration parameter and the redshift of a source respectively. The value q_o has to be not smaller than zero in the matter dominated Friedmann model,then the correct formula of the luminisity distance is[1],

$$d_L=(c/H_o q_o^2)[zq_o+(q_o-1)(\sqrt{1+2q_o z}-1)]. \quad (1)$$

Defining

$$A=d_L H_o/c=zq_o^{-1}+q_o^{-2}(q_o-1)(\sqrt{1+2q_o z}-1) \quad (2)$$

After some substitution and deduction,we obtain,

$$V(z)=\frac{2\pi}{QH_o^3}\begin{cases}(1-2q_o)^{-\frac{3}{2}}(P\sqrt{1+P^2}-\sinh^{-1}P), & K=-1(q_o<\frac{1}{2})\\ (2q_o-1)^{-\frac{3}{2}}(\sin^{-1}P-P\sqrt{1-P^2}), & K=+1(q_o>\frac{1}{2})\\ (2/3)A^3[\frac{1}{2}(1+A+\sqrt{1+2A})]^{-3}, & K=0(q_o=\frac{1}{2})\end{cases} \quad (3)$$

where $$P=A\sqrt{K(2q_o-1)}/(1+z). \quad (4)$$

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Q=41254°,is the number of square degrees in the whole sky.

The ratios, $V(z)/V(z_m)$, under various values of q_o are computed. The results of the computation are stated as (i) in the abstract.

By taking $q_o=0$, 0.2,...3.0,the values of V/V_m and mean values, $\langle V/V_m \rangle$ are computed for 3 sampleswhich have been studied by the predecessors only for $q_o=0$ or 1. The variations of values of $\langle V/V_m \rangle$ with q_o, as the result of our computation,are shown in Fig.1 to-gether with the results of predecessors for comparison. The samples include 33 radio sources of 3CR,30 of 4C in the selected region and 60 of ±4° PKS[2],[3] [4].

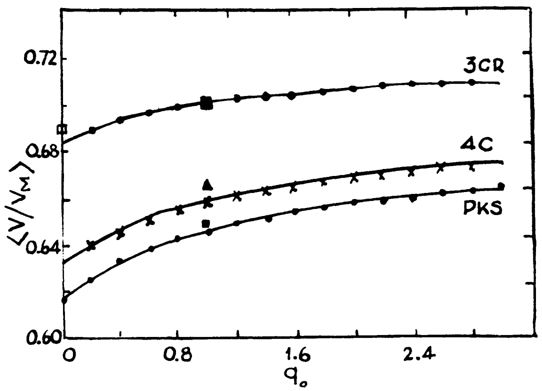

Fig. 1. Plot of $\langle V/V_m \rangle$ against q_o for three samples.

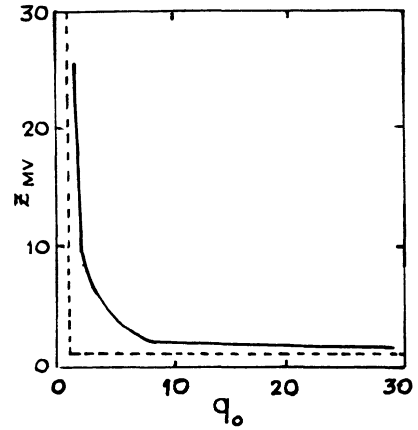

Fig. 2. Plot of q_o against z_{mv}.

2. ANALYTICAL EXPLORATION OF $V(q_o,z)$

Is the volume-test method equally valid for different spaces? First,we have to examine whether the maximum volume $V_{mv}(q_o,z)$ exists;if it does exist,then the value of z_{mv} corresponding to $V_{mv}(q_o,z)$ should be found. In order to get V_{mv}, as usual we perform the differentiation, $\partial V(q_o,z)/\partial z=0$, for 3 cases: $q_o<\frac{1}{2}$, $q_o=0$, $q_o>\frac{1}{2}$. The result is only for the case $q_o>\frac{1}{2}$, more precisely only for $q_o>1$, there exist values of z_{mv}[5],

$$z_{mv}=(1/2q_o)\left[\frac{(2q_o-1)(\sqrt{2q_o-1}+q_o)^2}{(q_o-1)^2}-1\right]. \qquad (5)$$

Also,it can be shown, $\partial z_{mv}/\partial q_o<0$, i.e.,the values of z_{mv} decrease monotonously with the increase of the values of q_o; $z_{mv}\rightarrow 1$ when $q_o\rightarrow\infty$ and $z_{mv}\rightarrow\infty$ when $q_o\rightarrow 1$. The relation between z_{mv} and q_o is shown in Fig.2.

In order to observe clearly the variation of $V(q_o,z)$ with z under various values of q_o, we compute the values of $V(q_o,z)$ under $q_o=0$,0.1, 0.25,0.5,...15,20 and z=1,2,3,...9,10. The results are shown in Fig.3.

The purpose of volume test is to see the deviation between the mean values $\langle V/V_m \rangle$ of a sample and the expectation value ½ in the uniform distribution. For this aim, we put

$$V_h(q_o,z)=0.5V_m(q_o,z). \qquad (6)$$

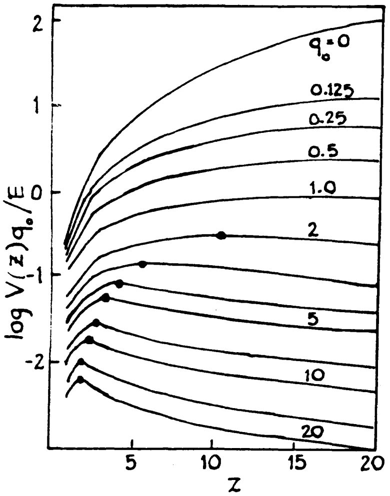

Fig. 3. Plot log $V(z)q_o^3/E$ against z ($E = 2\pi/QH_o^3$).

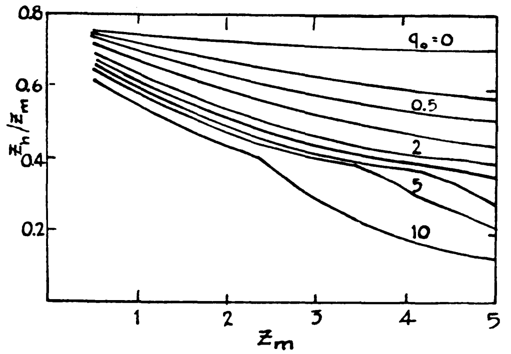

Fig. 4. Plot of z_h/z_m against z_m for different values of q_o.

We calculate a series of (z_h/z_m)-z_m curves which are illustrated in Fig.4.

3. DISCUSSION AND CONCLUSIONS

When z_m in Fig.3 is larger than z_{mv}, the redshift corresponding to the maximum volume $V_{mv}(z_{mv})$, and the value of z is $\lesssim z_{mv}$, it will lead to the absurd result, i.e., $V(z)/V(z_m)>1$. Whenever the values of q_o and z are large enough to cause $V(q_o,z)\cong V_{mv}(q_o,z_{mv})$, the method is invalid. From Fig.4, one can easily see that using the volume-test method, with respect to a definite value of z_m, smaller values of z_h result when larger values of q_o are adopted , which lead to larger values of $V(z)/V_m(z_m)$ and $\langle V/V_m \rangle$; and vice versa.

Conculsions are stated in the abstract.

References

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