Students wishing to start in this area would be well advised to stick to established texts, or, at the expense of a few postcards to selected authors, obtain pre-prints on their chosen subject.

R. J. HENERY

CSASZAR, A., *General topology* (Adam Hilger, 1978), 488 pp., £20·50.

This comprehensive treatise on general topology is heavily influenced by the book "Topological spaces", by E. Čech, assisted by Z. Frolíc and M. Katětov (Interscience, 1966), and so it is natural to compare the two works. The scope of the present work is clear from the chapter headings: 1. Introduction, 2. Topological spaces, 3. Proximity and uniform spaces, 4. Completely regular spaces, 5. Complete and compact spaces, 6. Extensions of spaces, 7. Product and quotient spaces, 8. Paracompact spaces, 9. Baire spaces, 10. Connected spaces, 11. Topological groups. A novel feature of the book is the early introduction of proximities and uniformities. In contrast to Čech's book, the language is less formal and consequently the present work is much more readable. There are many examples in the text and numerous exercises at the end of each section. This is not an introductory text on topology. However the wealth of material presented should prove invaluable to the research worker in topology and related disciplines.

H. R. DOWSON

ANDRÁSFAI, BÉLA, *Introductory Graph Theory* (Adam Hilger, 1977), 268 pp., £8·00.

This is a translation by András Recski of a book which first appeared in Hungary in 1969. It was another Hungarian author, Dénes König (written in German), who gave the world the first book on graph theory in 1936, and for many years *Theorie der endlichen und unendlichen Graphen* had no competitors. However, in recent years there has been no shortage of publications, about a dozen of them in English, so it is natural to ask if this translation is really necessary. I consider the answer to be: perhaps not necessary, but certainly valuable.

The justification of this book lies in its method of communication. The author passionately believes that graph theory is an excellent means of developing problem-solving ability, where advanced knowledge is not necessary, but where ingenuity and deep consideration are often called for. The book is therefore written around problems: "The results are presented *in statu nascendi*, following the procedure of discovery, solution of sub-statements, definition of new concepts which prove to be useful, and determination of the possibilities of generalisation for the solution of practical problems. Exercises, problems and their solutions are given throughout, with suggestions of new problems, simplification of complicated statements, and, above all, stimulation of readers." The result is a book which is different; it reads more like a mathematical detective story than a book of theorems, and it is accordingly more suited to private reading than to prescription as a text for a course of lectures.

The choice of material, too, differs from the norm for an introductory text. There is no mention at all of "topological" graph theory, so Euler's formula and the four colour theorem are not dealt with. The seven chapters deal with: (1) basic ideas, (2) trees and forests (including spanning trees and the circuit space of a graph), (3) routes following the edges of a graph (Eulerian graphs), (4) routes covering the vertices of a graph (Hamiltonian circuits, including the theorems of Dirac and Pósa), (5) matchings (of bipartite graphs, using alternative paths, and the König max-min theorem), (6) extremal graph theory (including some Ramsey theory), (7) solutions to exercises.

Although the ideas develop from simple problems, this is by no means an easy book. Some of the arguments, particularly in chapters 5 and 6, are quite involved, and the reader who perseveres will undoubtedly emerge with wits sharpened and a greater respect for proof by contradiction. It is in chapter 6 in particular that the Hungarian school of graph theory, carefully nurtured by Erdös, is most evident. The "extremal" graph theory here can be illustrated by the following simple example: if a graph with n vertices and e edges has no triangles, then $e \leqslant [n^2/4]$, equality occurring only for specified extremal graphs. A generalisation of this result has a nice application due to Erdös: if we have $3s$ points in the plane ($s \geqslant 2$), such that the distance between any two is at most 1, then at most $3s^2$ of the distances between points are greater than $1/\sqrt{2}$.

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The standard of translation throughout is excellent. A second volume is promised, which will include the relation of graphs to surfaces, matrices and probability. It is eagerly awaited.

IAN ANDERSON

JOHNSTONE, P. T., *Topos Theory* (L.M.S. Monograph No. 10, Academic Press, London, 1977), xxiii + 367 pp., £17·50.

The subject has evolved rapidly since the work of Lawvere and Tierney in 1969–70, many of its ideas being common knowledge to specialists but not easily available to others. Peter Johnstone's book is the first comprehensive publication of both the elements of the theory and their development and application; it is intended as an "introduction . . . for the research student with some experience in category theory" and as "a comprehensive reference work for the specialist". It succeeds admirably in these intentions, and with its wit and style should also interest many other mathematicians: from logic, topology, algebraic geometry, universal algebra, set theory. . . .

A topos is a cartesian closed category with finite limits and a "subobject classifier", which allows the construction of "classifying maps" of subobjects just as the two-element set allows that of characteristic functions of subsets in the topos $\mathcal{S}$ of sets. For instance, the category of set-valued functors on a small category, the category of presheaves or of sheaves over a space, the category of finite sets, or of sets acted on by a group—all are toposes; and, more significantly, each of these examples, and many others, can be imitated "internally" inside (almost) any topos, to create a new topos. From this elementary standpoint one may conveniently study the Grothendieck toposes of modern algebraic geometry, or universal algebra, which may be done in any topos with a natural number object $\mathbb{N}$ (for the construction of free algebras). Johnstone, however, does not regard topos theory as a machine for solving "major outstanding problems of mathematics", but rather as an idea which will "inevitably lead to the deeper understanding of the real features of a problem which is an essential prelude to its correct solution". And the essence of this idea is the "rejection of the idea that there is a fixed universe of 'constant' sets within which mathematics can and should be developed, and the recognition that the notion of 'variable structure' may be more conveniently handled within a universe of *continuously variable* sets than by the method, traditional since the rise of abstract set theory, of considering separately a domain of variation (i.e. a topological space) and a succession of constant structures attached to the points of this domain".

The book begins with a brief sketch of the categorical background, including Giraud's theorem characterising Grothendieck toposes as categories satisfying certain exactness and size conditions. The basic elements of the theory then follow: axioms, representability of partial maps, Pare's theorem that the opposite category of a topos E is equivalent to the category of algebras for the power-object monad on E, pull-back functors, image factorisations, internal categories and limits, commutation of finite limits with filtered colimits, topologies in a topos, sheaves and sheafification, geometric morphisms (between toposes) and their factorisation, Wraith's glueing construction, finite lax colimits in the 2-category of toposes, Diaconescu's theorem characterising geometric morphisms from a fixed topos to the topos of internal functors on a category inside another topos, the Mitchell-Diaconescu generalisation of Giraud's theorem, Boolean toposes and the double-negation topology, the axiom of choice, toposes generated by the subobjects of 1, and, perhaps most interesting for the general reader, the Mitchell-Benabou 'internal language of a topos', carrying further the idea that a topos should be considered as a universe of discourse rather than just yet another sort of category.

Chapters 6–9 independently cover the aspects of the subject most relevant to applications: first, in a topos with a natural number object, we can talk about finite cardinals, algebraic and geometric theories, and real numbers; second, we have Deligne's theorem (every "coherent" topos has enough points), one version of which is the completeness theorem for finitary geometric theories, and Barr's theorem that every Grothendieck topos is the image under "surjective" geometric morphism of a topos in which the axiom of choice is valid (example: the classical relationship between the toposes of sheaves over a topological space and over the same set discretely topologised). Third, cohomology with Abelian coefficients is studied in a Grothendieck topos, with the beginnings of Duskin's non-abelian version; Chapter nine discusses the various notions of finiteness in a topos, the equivalence of various extensions of topos theory to Zermelo-Fraenkel set theory, and Tierney's