INTERNAL REFRACTION IN MERIDIAN CIRCLES

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ABSTRACT

Tube refraction of the order 0".3 inside a meridian circle is a source of systematic errors that has not been noticed before. Natural convection due to cooling at night or heating during day will create primarily vertical temperature gradients in the air. This is shown by experiments at a collimator where typically 1 K/m was found inside the tube even when the gradient outside the tube was much smaller. A method of tangential ventilation of the tube was used to eliminate the gradients. It is shown that the resulting effect on seeing or image motion will be to the better when the ventilation system is properly designed, and the design for a conventional meridian circle and for a Glass Meridian Circle is discussed.

1. INTRODUCTION

The air inside the tube of an ordinary meridian circle can cause significant systematic errors in absolute observations of declination. The reason is that small and approximately vertical temperature gradients in the air cause an *Internal Refraction* or *Tube Refraction*.

Internal refraction has been studied and removed in some other astrometric instruments. Astrolabes and Photo Zenith Telescopes with vacuum are in successful operation in China as described by Hu Ningsheng **(1986)**. Kirijan et al. (1982) have studied refraction anomalies in the horizontal tube of the Pulkovo Horizontal Mirror Meridian Circle. They have diminished the internal refraction by spiralling air in a double wall tube.

The effect of internal refraction in an ordinary meridian circle does not seem to have been studied or even noticed by anyone before. The reason for the present study was the development of the Glass Meridian Circle (GMC) (Høg,**1986)**, hereafter called Paper I, where refraction in a horizontal tube was anticipated and a new method for

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its removal was to be tested. These tests to be reported below, were done on the North Collimator of the Carlsberg Automatic Meridian Circle (CAMC) while still in Brorfelde in early 1983. The vertical gradient was removed by blowing air tangentially into the tube at one end, so that all air would rotate around the optical axis. The line of sight was then observed to shift about 0".2 in the vertical coordinate as if a vertical gradient about 1 K/m, positive upwards, had been removed in the 1.6m long collimator tube.

This *Tangential Ventilation* was found to cause only negligible seeing or image motion effects, if it were used in a GMC or a conventional meridian circle. Provisions for tangential ventilation were made on the CAMC where an effect about 0".3 would be expected if it is proportional to the length of the tube. The time schedule for moving the instrument to La Palma did not allow the system to be tested in Brorfelde.

2. REFRACTION IN TUBES AND NATURAL CONVECTION

If a parallel beam of light travels perpendicular to a temperature gradient it will be curved convex towards the warm side, since air of higher temperature has a lower index of refraction and therefore a higher velocity of light. The angular deflection is given by

$$\Delta\Theta = 0''.20 T'L \qquad (1)$$

for atmospheric air at 20°C. T' (in K/m) is the gradient and L (in m) is the length of light travel.

For a converging beam of light between the lens and focus the image will shift

$$\Delta y = \frac{1}{2}\Delta\Theta = 0''.10 T'L \qquad (2)$$

It thus follows that the shift $\Delta y = 0''.2$ on L = 1.6m mentioned in Section 1 corresponds to T' = 1.25K/m. Since the diameter of the collimator tube is 0.2m the temperature difference across the tube must be $\Delta T = 0.25$K which could not be safely measured with our electronic thermometer.

The natural convection at so small a gradient is very slow so that the removal of the gradient by making the walls of the tube isothermal does not seem very efficient. The Pulkovo spiralling air method mentioned above acts by keeping the inner wall isothermal and is reported to have a good effect - but the language barrier to Russian prevents me from further understanding of the results.

I have observed the natural convection in a horizontal tube by means of a Ronchi grid at the centre of curvature of a concave mirror. Motion of turbulence elements of e.g. 1 cm/s could be seen. The motion was also frequently studied by watching tobacco smoke blown into the tube through a straw. This smoke method led to an understanding of some phenomena which would otherwise have escaped attention.

If all surroundings, i.e. tube, lens and micrometer could be kept isothermal *and* at constant temperature any gradients in the air would, of course, disappear after some time, but this time would be quite long since the thermal conductivity of air is very low. Since the instrument is, however, to be used in an open building where the temperature drops by a few degrees at night it is impossible to keep the instrument at constant temperature. Since tube, lens and micrometer have different and non-negligible time constants, it is not even possible to keep them isothermal at ambient air temperature. (But this should be an ideal for design of meridian instruments.) Let us imagine that the tube is kept at ambient decreasing temperature (e.g. by spiralling air) and that the lens follows at a slightly higher temperature. The air in the tube will follow at a slightly higher temperature than the wall and a vertical positive gradient will be established. The air close to the inside of the lens may be heated by the lens and will drift upwards and into the upper part of the tube if this is horizontal.

If the tube is not horizontal the isothermal surfaces will again tend to be horizontal, i.e. the gradient be vertical upwards, but modified by a slow convective streaming. The deflection will be zero when the telescope is pointed to zenith distance $z = 0$, and it will be maximum at $z = 90°$, thus approximately $\Delta\Theta = \text{const.} \cdot \sin z$ - the same form as the classical flexure term. But a term due to internal refraction would not necessarily be constant throughout the night as a purely mechanical flexure should be. In fact Helmer (1982, priv.comm.) has sometimes obtained variable flexure terms at the CAMC.

These local gradients and convective motions of the air would probably be proportional to the *rate of change of temperature*. They could be avoided by some kind of forced ventilation. A rotation of the air about the optical axis seems the most direct way to avoid systematic refraction. Hu Ningsheng (1986) has proposed to rotate the air by rotation of an inside tube with blades. I have proposed (Høg,1973) and subsequently studied the method of tangential ventilation.

Such a study ought to tell more precisely how tangential ventilation is to be realized in order that the gradients are removed and no new problems are introduced, e.g. seeing or image motion. Image motion can be quantitatively estimated at best if a photoelectric micrometer is used, preferably the same micrometer that is to be used in the real instrument later on. This was the case in Brorfelde. Otherwise the power spectra of image motion would have to be estimated in order to make quantitative predictions. It is not so difficult to scale the results to another optical system with another focal length. The variation with size and form of the optical aperture, with amount of air and size of injecting nozzle can, however, only be found by experiment.

A vertical gradient in the free air between the two collimators of a conventional meridian circle wil *not* introduce a systematic error in the determination of horizontal flexure if the same gradient exists inside the meridian circle telescope. Since this is not generally to be expected it is worth noting some typical gradients found in the Brorfelde dome, cf. Høg (1978,p.250). With open dome: no measurable gradient was found, i.e. $\leqq 0.1$ K/m, at day or night. With closed dome: 1.5K/m was typical at full sunshine in the winter and already a little sunshine would give 1.0 K/m, but in strong wind (10 m/s) the gradient would be only half as large. On a cloudy day with closed dome 0.2 K/m was still typical, but even then a gradient about 1 K/m was found inside the collimator and the protection tube as derived from the optical measurements.

It can be concluded that flexure determinations in day time must be affected by large systematic errors since they are traditionally carried out with closed dome. This is probably one of the reasons why different values of horizontal flexure are found in night and in day time, see e.g. Miyamoto and Yoshizawa (1986,Fig.9)

3. THE COLLIMATOR EXPERIMENTS

3.1 Equipment and tangential ventilation

The CAMC in Fig.1 is pointed at the North collimator. The protection tube of 4 mm thick plastic can be easily turned away on a hinge. The collimator tube has 5 cm thick insulation that can be removed.

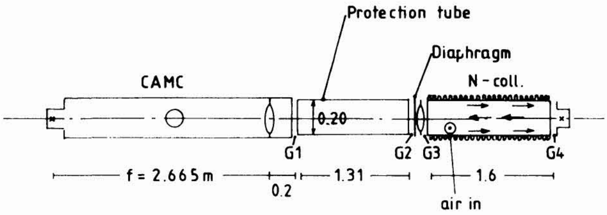

Fig.1. The CAMC pointed at the North collimator. The vertical dimension of the drawing is magnified twice relatively to the horizontal.

Clean air can be injected tangentially into the tube at "air in", thus rotating the air and removing systematic transversal temperature gradients.
The arrows show only the axial components of air motion.

Clean air blown through a "microfilter" can be injected tangentially into the collimator tube by a fan unit (not shown). The air may be heated a few degrees before it enters the collimator tube, as is needed for some experimental purposes cf. Section 3. The air in the tube will rotate around the optical axis and it will escape at the gaps G3 and G4, but mostly at G4 since G3 should be kept small, < 1mm. Injection velocities of 5 to 10 m/s through tubes 1 or 2 cm diameter, i.e. flows of q = 0.5 to 2 l/s were found to be adequate. Originally q = 2 to 10 l/s injected through a 70 mm diameter tube was used, but this is not necessary if the gap G3 is small.

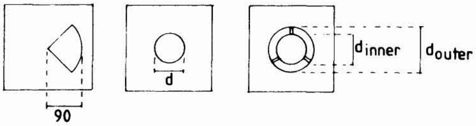

Fig.2. Diaphragms for reducing the aperture of the collimator -

The quadrant aperture is used for determination of the amount of de-focus from observation of, say, Δx with the diaphragm as shown above, and rotated 180°.

The small aperture and the annular aperture are similar to those needed for autocollimation measurements in the GMC.

Rotation of the air and sufficient radial mixing must be obtained along the tube. As this was checked by watching tobacco smoke a disturbing effect was found. Air at the axis returned slowly and with only little rotation from the G4-end back towards where it had been injected, see arrows in Fig.1. Motion at the periphery was both spiralling, axial towards G4, and turbulent. Rotation slowed down as the air approached G4.

For instance injection of 0.4 l/s at 10 m/s through a tube 7 mm diameter gave the air at the collimator objective a rotation of 1 second per revolution. At 1 m from the injection the rotation had slowed to 2s/rev and at 1.4m distance to 3s/rev. It is important for maintaining a rotation along the tube that it is smooth inside - on a meridian circle telescope enforced by inside ribs an inner tube must be inserted. This is available on the CAMC.

The axial return flow was not expected, but is easily explained by the underpressure at the axis at places where the peripheral rotation is fastest. It is an undesired flow and it produced quite large effects when warm air was injected, since the air gradually cooled along the wall of the tube and then returned cooler along the axis. A radial temperature gradient up to 10 percent of the difference to outside air was observed on a tube without insulation. This radial gradient produced a large change of focus which was found rather early in the experiment and long remained a mystery until the explanation was finally revealed.

Another undesired effect of the cooler axial air must be a larger image motion. The observed values of image motion, Eq.(3), when heated air was blown in, must therefore be upper limits which can be reduced if the axial return flow can be eliminated.

3.2 Observations with slit micrometer

The CAMC double slit micrometer, see Helmer (1986), was used to observe the point light source in the collimator focus.

An observation with the slit micrometer takes 16s during which 8 traverses of the light source with the inclined double slit is obtained. The mean errors of the horizontal x and vertical y components are computed as $8^{-0.5}$ times the mean error of one traverse. Repeated observations have shown that this is a realistic estimate of the mean error of a 16s observation and the following SD_y refers to the mean error (or standard deviation) in the vertical component of a 16s observation.

The protection tube is normally used at the experiments since it decreases the mean error of an observation with full 180mm diameter aperture from typically 0".020 to only 0".015, cf. SD_y in Fig.3.

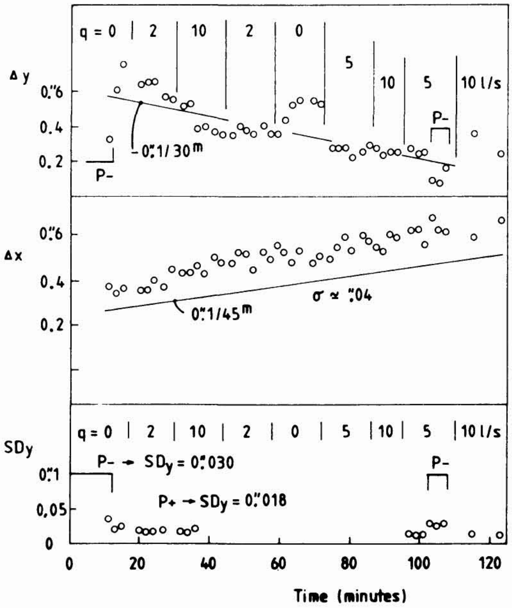

Fig.3. Systematic vertical shift (Δy) depending on ventilation of collimator tube and on use of the protection tube which is removed at intervals marked P-.

No horizontal shift (Δx) is found, except a slow drifting.

The standard deviation (SD_y) of a 16s observation is smallest (0".018) when the protection tube is used. This improvement is more pronounced in an open dome.

In this figure each circle is the average of 5 observations of 16s each.

The contribution from photon noise is about 0".010 so that the image motion alone is even less. This is with the dome closed. The protection tube is, however, not recommended for flexure determinations due to the systematic effect mentioned below and due to the expected instability of the air in this tube, it being open at one end.

The mean error for an observation with 40 mm aperture is 0".07 and the same for an annular aperture of 60mm inner and 80mm outer diameter, cf. Fig.2.

When the collimator is ventilated tangentially by ambient air no change of these mean errors is seen since most of the image motion originates inside the protection tube and the meridian circle telescope where the longer paths are.

When the collimator is ventilated with air heated about 5 or 10 degrees K the mean error is increased by an amount to be added quadratically and being proportional to the increase of temperature ΔT above ambient. The increase of mean error is

$$\Delta SD_y = \begin{cases} 0''.009\Delta T & (3a) \\ 0''.004\Delta T & (3b) \end{cases}$$

where (3a) is for the small circular and (3b) for the full 180mm aperture. These figures were found to be independent of the air flow between 2 and 10 l/s injected through a 70mm diameter tube and also independent of insulation of the collimator tube with 5cm Rockwool or not. The reason for this independence is believed to be that most of the image motion originates through radial mixing of the peripheral warmer air with the cooler axial return flow, see Fig.1. A smaller part of the image motion comes from the boundary layer of a few centimeters of air close to the collimator lens which was 1 or 2 K from ambient temperature.

It is worth noting that the mean errors varied much less with aperture size during heated ventilation than without ventilation. A part of the larger variation without ventilation is due to photon noise, but this contribution is eliminated in the evaluation of image motion during heated ventilation.

The slower components of image motion including the systematic parts are illustrated by the two upper plots of Fig.3. These observations are typical for several other days during the winter of 1983. Each circle is the average of 5 observations of 16s each, and with full 180mm aperture.

The horizontal component Δx shows a slow drift of 0".1 per 45 minutes, presumably due to mechanical drifting of azimuth of the CAMC and for the collimator. The scatter around this line is $\sigma = 0''.04$ although the average of 5 observations should have a standard deviation of only $\sigma = SD_y \cdot 5^{-5} = 0''.008$. This increased scatter is presumably due to slow natural convection inside the whole optical path of Fig.1.

Tangential ventilation with a certain flow q(l/s) through a 70mm injection tube is used for about 10 minutes before changing to another flow as indicated in the Figure. It appears that the horizontal component is not affected neither systematically nor in its scatter by the amount of ventilation.

The vertical component Δy shows, however, a pronounced systematic shift about 0".2 when ventilation is stopped at time = 60 minutes. It appears to take about 5 minutes before the full effect is seen, and somewhat less when the ventilation is started again. It is known from watching tobacco smoke that the rotation takes about half a minute to start or stop. It is thus concluded that the thermal gradient takes several minutes to build up.

Systematic shifts of Δy are also seen to depend on the use of the protection tube in which a positive temperature gradient thus also builds up.

The vertical component shows a scatter and a systematic drift about the same size as the horizontal component.

4. CONCLUSIONS

4.1 Conventional meridian circle

A systematic flexure term about $0''.3 \sin z$ due to tube refraction is expected in the CAMC. An experiment is planned to measure the effect by tangential ventilation of the CAMC, and, hopefully, to eliminate this error source.

An inner smooth tube, cf. Section 3.1, will be inserted in the telescope. Tangential injection will be made close to both objective and micrometer through tubes of cross section $4\text{x}20=80\text{mm}^2$ at the end. The velocity at the end should be 10 m/s. The air will leave the telescope at its middle through a tube going to a fan and a coarse filter and it will return through tubes on the outside of the telescope, so that the same clean air is recirculated. It is expected that the air will be slightly heated by the fan and be about $\Delta T = 0.5$ K warmer than ambient at the place of injection. The resulting increase of image motion as given by Eq.(3b), but with a larger coefficient due to the longer focal length f= 2.665 m of the CAMC, would be for a 16s observation

$$\Delta SD_y = (2.665/1.6)^{0.5} 0.004\Delta T = 0''.005\Delta T \qquad (4)$$

i.e. 0".0025. This is an upper limit, valid if the axial return flow is the same as in the collimator experiment, and this image motion is entirely negligible. It will not contain the slow component of image motion discussed in Section 3.2 at the Δx component.

4.2 Glass meridian circle

Effects of the tube refraction are much more critical in the GMC than in an conventional MC because the light passes the tube two or more times and because a smaller aperture is used for the autocollimations.

Particularly the autocollimation on S2 is critical since the light passes four times and two of these as parallel light which gives twice as large deflection as a convergent beam, cf. Eqs. (1) and (2). It can be shown that scaling of Eq. (3a) for the standard deviation of a 16s observation taking the multiple path and the ratio og 2.665/1.6 between the focal length of GMC and collimator into account gives

$$SD_y = (2.665/1.6)^{0.5} \cdot 6 \cdot 0''.009\Delta T = 0''.068\Delta T \qquad (5)$$

This is an upper limit if tangential ventilation is used and if the axial return flow were the same as in the collimator experiment.

It is expected that $\Delta T = 0.5$ K can be obtained for the injected air. In this case a 4s autocollimation on S2 would have $\sigma_2 = 0''.068$ which is acceptable for the GMC, cf. Paper I.

The real mean error σ_2 is, however, expected to be much smaller because no axial return flow will be found in the GMC. This is achieved because the air will escape axially through the central hole of the 45° mirror. In addition, a central hole of 28 mm diameter was introduced in the GMC concave mirror where air will escape. This will prevent axial return flow from this end of the GMC and the radial flow will, furthermore, remove the boundary layer close to the concave mirror.

Since autocollimation on S2 is the most critical measurement in the GMC the standard deviations of all other measurements are negligible and need no further discussion.

In case of no ventilation of the GMC and with the same internal conditions giving $SD_y = 0''.07$ in the collimator experiment for small apertures, cf. Section 3.2, we would expect $SD_y = 0''.2$ for autocollimation on S2. This would be quite inconvenient. Without any ventilation systematic errors over 1 arcsec would be expected for the S2 autocollimation.

Thus, both for reducing systematic errors and image motion the tangential ventilation is required in the GMC. No reason has been found to doubt that it will also be sufficient for the purpose. A proof will be obtained by the coming experiments with a 24cm GMC, cf. Paper I.

Acknowledgement

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Discussion:

CURRIE: We have considered the use of spiral vanes in the arms of the optical interferometer. The vanes may give a mixing with less turbulence and thus less enlargment of the image and reduced back flow.

HØG I think you need turbulence for a good random mixing.

SMITH: I have studied (with the assistance of R. Miller) the flexure of the U.S.N.O. 6" and 7" transit circles and found in the case of the 7" T.C., a coefficient of the sine term in flexure of the order + 0.010"/°C ± 0.002"/°C based on 1,000 horizontal collimaton determinations. All of the foregoing determinations were made at night. In the case of the 6" T.C., no significant dependence of the flexure on temperature could be found, even though day-time flexures as well as night-time flexures were included. We used approximately 200 days and 400 nights observations in the study.

HØG: The variable flexure I was speaking about is not directly a function of the temperature, but a function of the temperature gradient inside the tube, and this may be a function of the rate of change of temperature.

TELEKI: If I correctly understood your explanations, I wonder how you eliminated the influence of flexure from your results?

HØG: I observed the direction and then started the air flow. Then I see a vertical shift which is canceled when the air flow stops. The method is thus completely differential.