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Orthogonal expansions and transforms of vector-valued measures and functions

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In the first section of this thesis we give necessary and sufficient conditions for a sequence $\{a_n\}$ of elements of a vector space X to be the coefficients of an X-valued measure or function; that is, to be of the form

$$a_n = \int_a^b \overline{\theta}_n(t)\mu(dt)$$

or

$$a_n = \int_a^b \overline{\theta}_n(t)f(t)dt ,$$

where $\{\theta_n(t)\}$ is an orthonormal sequence of complex-valued functions on a compact interval $[a, b]$ and μ is an X-valued measure and f an X-valued function on $[a, b]$. In Chapter II we briefly discuss the case when X is the complex number field and the results concern complex-valued measures and functions in $L_p((a, b))$, $1 \le p \le \infty$. Chapter III deals with the coefficients of measures with values in a quasi-complete locally convex space and investigates when the measure has relatively compact range and finite variation. In Chapter IV necessary and sufficient conditions are given for a sequence of elements of a Banach space X to be the coefficients of a Bochner integrable function, a strongly measurable, essentially bounded function or, when X is a reflexive Banach space, a function in $B_p([a, b], X)$, $1 < p < \infty$. The chapter includes the case of

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elements of a locally convex space being the coefficients of a given Pettis integrable function and also a theorem on the coefficients of elements in the completion of the space of Pettis integrable functions with values in a separable Banach space.

Section Two deals with the continuous analogues of the theorems of Section One. We let (A, B) and (C, D) denote intervals of the real line and let K be a complex-valued function on $(A, B) \times (C, D)$ for which certain assumptions are satisfied. We give necessary and sufficient conditions for a function with values in a vector space X to be the transform of an X-valued measure or function with respect to the kernel K. The theorems are analogous to those of Section One. Transforms of complex-valued measures and functions are considered in Chapter VI while Chapters VII and VIII cover transforms of vector-valued measures and functions. In Chapter IX the theorems are proved under another set of assumptions on the kernel K.

In Section Three we briefly discuss how the theorems of Section One may be used to define solutions (in a generalized sense) of partial differential equations.

Some of the theorems of the first two sections appear in [1] and [2].

References

[1] Stephen K. McKee, "Orthogonal expansion of vector-valued functions and measures", *Mat. Časopis, Slovensk. Akad. Vied* 22 (1972), 71-80.

[2] Stephen K. McKee, "Transforms of vector-valued functions and measures", *Mat. Časopis, Slovensk. Akad. Vied* 23 (1973), 5-13.

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