BULL. AUSTRAL. MATH. SOC. 05B05, 51E10, 05B10, 05B15
VOL. 23 (1981), 319-320.

CONSTRUCTION AND RESOLUTION OF QUADRUPLE SYSTEMS

ALAN HARTMAN

A quadruple system (X, Q) or order V, denoted $QS(V)$, is a set X of cardinality V, and a set Q of 4-subsets of X, with the property that each 3-subset of X is contained in a unique member of Q.

A quadruple system (X, Q) is resolvable if Q can be partitioned so that each part is itself a partition of X.

A necessary condition for the existence of a $QS(V)$ is that $V \equiv 2$ or $4 \pmod 6$ or, trivially, $V = 0$ or 1. In 1960 Hanani proved that these conditions are also sufficient. He used six recursive constructions and direct constructions of a $QS(14)$ and a $QS(38)$ to prove this result. The first part of this thesis contains some new recursive constructions for quadruple systems, and a unified treatment of all the existing constructions. These results enable us to give a new proof of Hanani's result using only two recursive constructions, at the expense of increasing the number of initial designs necessary for the induction. New results on the problem of embedding quadruple systems in larger quadruple systems are also obtained in this part of the thesis.

The second part of the thesis is an attack on the existence problem for resolvable quadruple systems. A necessary condition for the existence of a resolvable $QS(V)$ is that $V \equiv 4$ or $8 \pmod{12}$ or, trivially, $V = 0, 1$ or 2. The sufficiency of these conditions remains an open question. We adapt the constructions given in the first part of the thesis so that they preserve resolvability. This enables us to reduce the

Received 23 January 1981. Thesis submitted to the University of Newcastle, August 1980. Degree approved, December 1980. Supervisor: Dr R.B. Eggleton.

existence problem for resolvable $QS(V)$ to a finite set of existence problems for resolvable $QS(V)$ with certain specified subsystem properties.

The second part of the thesis also contains many direct constructions of resolvable $QS(V)$, and direct constructions of $QS(V)$ with even more stringent resolvability properties than classical resolvability.

Department of Combinatorics and Optimization,
University of Waterloo,
Waterloo,
Ontario N2L 3G1,
Canada.