STABILITY OF A CIRCULAR CYLINDRICAL HOLE IN A GLACIER

By J. F. NYE

(Hammond Laboratory, Yale University, New Haven, Connecticut, U.S.A.*)

ABSTRACT. If a bore hole of circular cross-section is made in an ice sheet or glacier, will it remain circular, or will it perhaps by some process of instability tend to depart from the circular shape? An argument is given to show that the circular shape is stable.

RÉSUMÉ. *Stabilité d'un trou cylindrique dans un glacier.* Si un forage de section circulaire est effectué dans une calotte glaciaire ou dans un glacier, conservera-t-il sa forme primitive ou alors tendra-t-il par quelques processus d'instabilité à perdre sa forme circulaire? On expose un raisonnement montrant que la forme circulaire est stable.

ZUSAMMENFASSUNG. *Stabilität eines kreiszylindrischen Loches in einem Gletscher.* Bleibt ein Bohrloch mit kreisförmigem Querschnitt—niedergebracht in eine Eiskappe oder einen Gletscher—kreisförmig oder strebt es vielleicht durch einen Vorgang der Instabilität darnach, von der Kreisform abzuweichen? Die folgende Beweisführung zeigt, dass die Kreisform stabil ist.

WHEN an unlined tunnel or bore hole is made in a glacier it gradually closes up under the weight of the ice that lies above it. If the cross-section is originally circular the question has been raised as to whether it will remain circular in the course of time, or whether it might, for example, become elliptical. The question is of special interest in connection with the vertical holes now being made in the Greenland and Antarctic Ice Sheets.

If one assumes that the cross-section does remain circular, the rate of closure can be calculated (Nye, 1953) from the flow law of ice measured in the laboratory. One way of investigating the stability of the circular section would be to impose an arbitrary small perturbation on the shape of the section, and then to study the flow equations to see whether the perturbation grew or diminished. Although the resulting equations would be linear, such an analysis is not simple, and it might be necessary to consider various different types of initial perturbation before reaching a firm conclusion.

The following argument shows in a semi-intuitive way, without recourse to detailed analysis, that the circular hole is indeed stable.

Consider first, as in the original paper (Nye, 1953), a cylindrical cavity of circular cross-section, radius a, in an infinite weightless material, with a surface traction σ_a per unit area applied inside the cavity (Fig. 1). The material obeys the flow theory applicable to ice. The cavity is then contracting at a rate given by the original analysis; the radial stress
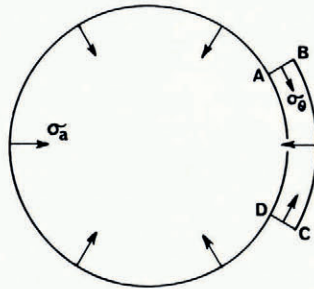

Fig. 1. Diagram to illustrate stresses acting on a circular cylindrical hole in a glacier if the section ABCD of its wall is removed

505

component σ_r falls off from the value σ_a on the cavity wall, $r = a$, to zero at infinity according to the relation

$$\frac{\sigma_r}{\sigma_a} = \left(\frac{a}{r}\right)^{2/n};$$

(1)

while the circumferential stress component σ_θ obeys

$$\frac{\sigma_\theta}{\sigma_a} = \frac{n-2}{n}\left(\frac{a}{r}\right)^{2/n},$$

(2)

n being the exponent in Glen's flow law for ice: strain-rate $\propto (\text{stress})^n$. Experimentally n is between 2 and 4.

Now remove the piece of material ABCD in Figure 1, bounded by radial planes AB, CD and the circular cylindrical surface BC. Apply the surface traction σ_a to the newly exposed surfaces, and consider whether the perturbed section thus formed tends to return to a circular form.

We first examine the effect of the radial stress component. Before the material was removed the radial tension σ_r across BC was less than σ_a, by equation (1). If, after removal of the material, a traction σ_r of this same amount were imposed on BC the flow would be maintained unchanged. In fact a greater traction σ_a is applied. Hence the material is pulled in faster than before and the circular shape tends to be restored.

The effect of the circumferential stress σ_θ is similar. σ_θ on the wall of the unperturbed cavity is, by equation (2), $\{(n-2)/n\}\sigma_a$, which is less than σ_a. A stress slightly smaller than this (because of the fall-off with r), say σ'_θ, acts across AB and DC before ABCD is removed. If a surface traction equal to σ'_θ were imposed on AB and DC after removal of ABCD the flow would be maintained (so far as the circumferential stress is concerned). In fact a greater traction σ_a is applied. Hence the sides AB and DC are pulled together, and so once again the effect is to annul the disturbance.

We see then that the combined effects of σ_r and σ_θ are to tend to restore the circular shape. (A similar argument applied to the axial component σ_z shows that if the disturbance ABCD extends only for a finite distance along the hole it still diminishes.) Now, any disturbance from the circular shape can be regarded as a linear superposition of square-wave perturbations like ABCD. Although the flow law governing the main flow is non-linear, the equations governing the behaviour of a small perturbation will be linear. The effects of the component perturbations may therefore be superposed. We therefore conclude that any small perturbation from the circular shape will tend to disappear, and therefore that the circular shape is stable.

Now consider a cylindrical hole of circular section, radius a, lying at a depth d $(d \gg a)$ below the free horizontal surface of a block of ice, and let the ice now have weight. As in the original paper we take the weightless solution for an infinite medium, put $\sigma_a = \rho g d$ where ρ is the density, and add to the stresses everywhere a hydrostatic pressure equal to ρg times depth. This gives (almost) zero traction on the hole wall and on the upper surface, as required. The addition of a hydrostatic pressure makes no difference to the arguments about stability. A hole or tunnel of circular section lying sufficiently far below a horizontal surface in a glacier or ice sheet will thus tend to remain circular. Of course, if the upper surface is sloping, the glacier will itself be deforming, and any excavation will be deformed by the main glacier flow as well as by the closure mechanism discussed here.

A corollary to these arguments is that if a deep tunnel is made with an oval section the proportional decrease of the larger diameter should be greater than that of the smaller diameter. But this conclusion is only valid if the tunnel is sufficiently deep $(d \gg a)$, for otherwise the proximity of the surface could cause anisotropic contraction on its own account. It is also necessary that the ice be fully consolidated; if it is not, horizontal stratification

could cause anisotropic contraction. For these reasons, and also because of possible distortion by the main glacier flow, it is perhaps unsafe to test the conclusion by looking at the behaviour of horizontal tunnels in glaciers. But none of the provisos applies to a deep *vertical* hole in a slowly deforming ice mass such as the Antarctic Ice Sheet. There, if the arguments in this note are sound, they predict that a slightly oval vertical hole should, at depth, gradually return to a circular shape. It would be interesting if the experimenters could test this prediction.

MS. received 11 June 1964

REFERENCE

Nye, J. F. 1953. The flow law of ice from measurements in glacier tunnels, laboratory experiments and the Jungfraufirn borehole experiment. *Proceedings of the Royal Society*, Ser. A, Vol. 219, No. 1139, p. 477–89.