

THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

ALONZO CHURCH    S. C. KLEENE    ALICE A. LAZEROWITZ

*Managing Editor:* ALFONS BORGERS

*Consulting Editors:*

W. ACKERMANN  
C. A. BAYLIS  
PAUL BERNAYS  
G. D. W. BERRY  
MARTIN DAVIS

ROBERT FEYS  
FREDERIC B. FITCH  
CARL G. HEMPEL  
LEON HENKIN  
JOHN G. KEMENY

ANDRZEJ MOSTOWSKI  
RÓZSA PÉTER  
BARKLEY ROSSER  
THORALF SKOLEM  
A. R. TURQUETTE

VOLUME 21

1956

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC  
WITH THE AID OF SUBVENTIONS FROM  
EDWARD C. HEGELER TRUST FUND

UNIVERSITY OF CHICAGO  
DARTMOUTH COLLEGE  
DUKE UNIVERSITY  
HARVARD UNIVERSITY  
INSTITUTE FOR ADVANCED STUDY  
UNIVERSITY OF ILLINOIS  
LEHIGH UNIVERSITY

UNIVERSITY OF MICHIGAN  
UNIVERSITY OF NEBRASKA  
UNIVERSITY OF PENNSYLVANIA  
PENNSYLVANIA STATE UNIVERSITY  
PRINCETON UNIVERSITY  
RUTGERS UNIVERSITY  
SMITH COLLEGE

STANFORD UNIVERSITY





THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

ALONZO CHURCH    S. C. KLEENE    ALICE A. LAZEROWITZ

*Managing Editor:* ALFONS BORGERS

*Consulting Editors:*

W. ACKERMANN  
C. A. BAYLIS  
PAUL BERNAYS  
G. D. W. BERRY  
MARTIN DAVIS

ROBERT FEYS  
FREDERIC B. FITCH  
CARL G. HEMPEL  
LEON HENKIN  
JOHN G. KEMENY

ANDRZEJ MOSTOWSKI  
RÓZSA PÉTER  
BARKLEY ROSSER  
THORALF SKOLEM  
A. R. TURQUETTE

VOLUME 21

1956

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC

WITH THE AID OF SUBVENTIONS FROM

EDWARD C. HEGELER TRUST FUND

UNIVERSITY OF CHICAGO  
DARTMOUTH COLLEGE  
DUKE UNIVERSITY  
HARVARD UNIVERSITY  
INSTITUTE FOR ADVANCED STUDY  
UNIVERSITY OF ILLINOIS  
LEHIGH UNIVERSITY

UNIVERSITY OF MICHIGAN  
UNIVERSITY OF NEBRASKA  
UNIVERSITY OF PENNSYLVANIA  
PENNSYLVANIA STATE UNIVERSITY  
PRINCETON UNIVERSITY  
RUTGERS UNIVERSITY  
SMITH COLLEGE

STANFORD UNIVERSITY

The four numbers of Volume 21 were issued at the following dates:

Number 1, pages 1–112, March 20, 1956.

Number 2, pages 113–224, July 2, 1956

Number 3, pages 225–336, September 28, 1956

Number 4, pages 337–426, April, 1957

All numbers of this volume are copyrighted by the Association for Symbolic Logic, Inc. Reproduction of copyrighted numbers of the Journal by photostat, photo-print, micro-film, or like process is forbidden, except by written permission of the Managing Editor.

## TABLE OF CONTENTS

A new approach to semantics – Part I. By JOHN G. KEMENY . . . . .	1
Two concepts from the theory of models. By LEON HENKIN . . . . .	28
Note on a problem of L. Henkin. By ABRAHAM ROBINSON . . . . .	33
On the axiom of extensionality – Part I. By R. O. GANDY . . . . .	36
Solution of a problem of Tarski. By JOHN MYHILL . . . . .	49
Another variant of natural deduction. By IRVING M. COPI . . . . .	52
Reduction to a symmetric predicate. By ALAN COBHAM . . . . .	56
Modality and quantification in S5. By A. N. PRIOR . . . . .	60
Formal systems of constructive mathematics. By M. H. LÖB . . . . .	63
Reviews . . . . .	76
Nineteenth meeting of the Association for Symbolic Logic. By IRVING M. COPI and R. L. WILDER . . . . .	109
Abstract of papers . . . . .	109
Begründung einer strengen Implikation. By WILHELM ACKERMAN . . . . .	113
On the definition of ‘formal deduction’. By RICHARD MONTAGUE and LEON HENKIN . . . . .	129
On the interpretation of Aristotelian syllogistic. By J. C. SHEPHERDSON . . . . .	137
On formulas with valid cases. By W. V. QUINE . . . . .	148
A new approach to semantics – Part II. By JOHN G. KEMENY . . . . .	149
Arithmetical representation of recursively enumerable sets. By RAPHAEL M. ROBINSON . . . . .	162
Reviews . . . . .	187
Problems in symbolic logic . . . . .	215
Twentieth meeting of the Association for Symbolic Logic. By G. D. W. BERRY . . . . .	216
Abstract of papers . . . . .	216
Elections . . . . .	223
Notice of a meeting of the Association for Symbolic Logic . . . . .	224
Identity, variables, and impredicative definitions. By K. JAAKKO J. HINTIKKA . . . . .	225

On $\omega$ -consistency and related properties. By STEVEN OREY . . . . .	246
On simplifying truth-functional formulas. By KURT BING . . . . .	253
Independent axiom schemata for S5. By ALAN ROSS ANDERSON . . . . .	255
The representations of S5. By GUSTAV BERGMANN . . . . .	257
Note on a system of Myhill. By J. C. SHEPHERDSON . . . . .	261
A note on transfinite iteration. By GIDEON SCHWARZ . . . . .	265
Unification of universes in set theory. By W. V. QUINE . . . . .	267
On the relative consistency of set theory. By STEVEN OREY . . . . .	280
Some proofs of independence in axiomatic set theory. By ELLIOTT MENDELSON . . . . .	291
On completely recursively enumerable classes and their key arrays By H. G. RICE . . . . .	304
Reviews . . . . .	309
Recursive functions in basic logic. By FREDERIC B. FITCH . . . . .	337
Alternative postulate sets for Lewis's S5. By E. J. LEMMON . . . . .	347
The independence of a weak axiom of choice. By ELLIOTT MENDELSON	350
Reviews . . . . .	367
Institutional contributing subscribers to the Journal . . . . .	414
List of officers and members of the Association for Symbolic Logic . . . . .	415



## ERRATA

### VOLUME 21

- Page 36, line 14. For «variagles», read «variables».
- Page 49, after the last displayed formula, insert a «I» at the right margin.
- Page 51, line 5 from below. For «PRESSBURGER», read «PRESBURGER».
- Page 69, line 15 from below. Insert a left-hand paranthesis immediately after the left-hand bracket.
- Page 70, line 14, For «ca», read «ca».
- Page 73, line 8. For « $a = b$ », read « $\alpha = \beta$ ».
- Page 74, line 19. For « $\alpha = \beta$ », read « $\alpha \equiv \beta$ ».
- Page 77, line 14. Insert an additional parenthesis after «XXI 79(4)».
- Page 98, line 10 from below. For «suchthat», read «such that».
- Page 98, line 12 from below. For «generalre cursive», read «general recursive».
- Page 100. line 11. For «contra productive», read «contraproductive».
- Page 102, Line 8, For «as equence», read «a sequence».
- Page 102, line 12. For «hyper-simple», read «hypersimple».
- Page 105, line 6 from below. For «112», read «109».
- Page 107, line 1. For «clasica», read «clásica».
- Page 107, line 11. Insert a comma after 0.
- Page 107, line 17. For «1951», read «1952».
- Page 110, Line 11. For «principal», read «principle».
- Page 206, line 3 from below. For  $(x\exists)$ , read  $(\exists x)$ .
- Page 211, line 21. For «ceruits», read «circuits».
- Page 213, line 22. For «Re- printed», read «Reprinted».
- Page 219, line 5 from below. For «““yellow””», read «“ “yellow” ”».
- Page 315, line 16 from below. For «intitation», read «initiation».
- Page 319, line 2. For  $(\exists R)/(S)$ , read  $(\exists R)(S)$ .
- Page 323, line 21. Before «1949», insert «Paris».
- Page 329, line 14 from below. For «precedure», read «procedure».
- Page 330. At the end of line 2 from the bottom. For «clauses» , read «clause».
- Page 331, line 2. For «latter», read «letter».
- Page 335, line 3 from the bottom. For «is», read «in».
- Page 336, line 16 from below. Insert a period after “Taxonomy”.