

EDWARD NELSON (1932–2014)

MIKHAIL G. KATZ

Department of Mathematics, Bar Ilan University

SEMEN S. KUTATELADZE

Novosibirsk State University

On September 10, 2014 we lost Edward Nelson, one of the most original and controversial mathematical thinkers of our time.

Nelson was born on May 4, 1932 in Decatur, Georgia. His father spoke fluent Russian, having spent some time in St. Petersburg in connection with issues related to prisoners of war (Nelson, 1995). Nelson was educated at the University of Chicago. In 1953 he was awarded a master's degree, and in 1955 he earned a doctorate under the supervision of Irving Segal, an expert in mathematical methods of quantum mechanics and functional analysis and one of the founders of the theory of C^* -algebras. Not surprisingly, Nelson's early papers were devoted to quantum mechanics. In 1995, Nelson received the Steele prize awarded by the American Mathematical Society, for his 1963 and 1977 papers on applications of probability theory to quantum fluctuations.

After graduating, Nelson spent three years at Princeton's Institute for Advanced Study, and from 1959 until his retirement in 2013, he worked at Princeton University where he became a Professor in 1964. In his memory the university flag was raised at half-mast for three days. In 1975 Nelson was elected to the American Academy of Arts and Sciences, and in 1997, to the National Academy of Sciences of the USA. Princeton University Press published the following six monographs by Nelson:

- (1) Dynamical theories of Brownian motion. Princeton University Press, Princeton, N.J. 1967.
- (2) Tensor analysis. Princeton University Press, Princeton, N.J. 1967.
- (3) Topics in dynamics. I: Flows. Mathematical Notes. Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo 1969.
- (4) Quantum fluctuations. Princeton Series in Physics. Princeton University Press, Princeton, N.J., 1985.
- (5) Predicative arithmetic. Mathematical Notes, 32. Princeton University Press, Princeton, N.J. 1986.
- (6) Radically elementary probability theory. Annals of Mathematics Studies, 117. Princeton University Press, Princeton, N.J. 1987.

All of these monographs are freely available online at
<https://web.math.princeton.edu/~nelson/books.html>

Already the short list above indicates the wide range of his interests and results related to probability theory, quantum mechanics, the theory of dynamical systems, and

Received: January 11, 2015.

mathematical logic. The volume (Faris, 2006) edited by Faris contains helpful surveys of Nelson's mathematical work in varied areas as well as a full bibliography.

Exceptional fame came to Nelson in the wake of his developing a fundamentally new, syntactic approach to Abraham Robinson's nonstandard analysis (Robinson, 1966), based on the internal set theory (IST) Nelson proposed in Nelson (1977). A related framework was developed independently by Hrbáček (1978). Detailed presentations of IST may be found in Gordon *et al.* (2002) and Kanovei & Reeken (2004).

Nelson rejected the prevailing views to the effect that nonstandard analysis operates with some fictional elements that extend the standard world of mathematical entities. In his approach, nonstandard objects inhabit the realm of the most ordinary mathematical objects. Nelson emphasized the creative syntactic contribution of the new approach in the following terms: "Really new in nonstandard analysis are not theorems but the notions, i.e., external predicates." (Nelson, 1988, p. 134)

A major event of the mathematical scene was Nelson's book *Radically elementary probability theory* (Nelson, 1987) which contains a modern exposition of von Mises frequency approach based on internal set theory IST. In the preface to this book, which was translated into French and Russian, Nelson writes:

Let me take this occasion to express a strong hope. This book is a bare beginning, and it will not succeed unless others carry on the job. There is a tendency in each profession to veil the mysteries in a language inaccessible to outsiders. Let us oppose this. Probability theory is used by many who are not mathematicians, and it is essential to develop it in the plainest and most understandable way possible. My hope is that you, readers of this book, will build a pavilion of probability open to all who search for understanding. (From the preface to the Russian edition.)

To elaborate on Nelson's IST approach to infinitesimals in a non-technical way, note that the general mathematical public often takes the Zermelo-Fraenkel theory with the Axiom of Choice (ZFC) to be *the* foundation of mathematics. How much ontological reality one assigns to this varies from person to person. Some mathematicians distance themselves from any kind of ontological endorsement, which is a formalist position in line with Robinson's. On the other hand, many do assume that the ultimate test of truth and/or verifiability is in the context of ZFC, so that in this sense ZFC still is *the foundation*, though it is unclear whether Robinson himself would have subscribed to such a view.

Much of the mathematical public attaches more significance to ZFC than merely a formalist acceptance of it as the ultimate test of provability, and tends to accept the existence of sets (wherever they are to be found exactly but that is a separate question), including infinite sets, and particularly the existence of the set of *real* numbers which are often assumed to be built into the nature of *reality* itself, as it were. When such views are challenged what one often hears in response is an indignant monologue concerning the categoricity of the real numbers, etc. As G. F. Lawler put it in his "Comments on Edward Nelson's 'Internal Set Theory: a new approach to nonstandard analysis'":

Clearly, the real numbers exist and have these properties. Indeed, many courses in elementary analysis choose not to construct the reals but rather to take the existence of an ordered field as given. This is reasonable: we are implicitly assuming such an object exists, otherwise, why are we studying it? (Lawler, 2011)

In such a situation infinitesimals may not thrive: the real numbers are thought truly to *exist*, infinitesimals *not*. This is where Nelson comes in with his syntactic novelty and declares: Guess what, we can find infinitesimals *within the real numbers themselves* if we are only willing to enrich the language we speak! The perceived ontological differences between the real numbers and infinitesimals are therefore seen to be merely a function of the technical choices, including syntactic limitations, imposed by Cantor, Dedekind, Weierstrass, and their followers in establishing the foundations of analysis purged of infinitesimals. This and related issues are explored in Bair *et al.* (2013) and Bascelli *et al.* (2015).

Nelson was a unique mathematician, whose creative interests shifted from the classical areas of mathematical physics and functional analysis toward mathematical logic and computer science under the influence of technological progress of modern computation. Nelson was the first webmaster of the Mathematics Department at Princeton University. Free in his *Ars Inveniendi*, he advocated freedom of knowledge and information. His motto was the paradoxical thesis “Intellectual property is an oxymoron.”

His memory will endure in our hearts.

BIBLIOGRAPHY

- Bair, J., Błaszczyk, P., Ely, R., Henry, V., Kanovei, V., Katz, K., Katz, M., Kutateladze, S., McGaffey, T., Schaps, D., Sherry, D., & Shnider, S. (2013). Is mathematical history written by the victors? *Notices of the American Mathematical Society*, **60**(7), 886–904. Available from: <http://www.ams.org/notices/201307/rnoti-p886.pdf> and <http://arxiv.org/abs/1306.5973>.
- Bascelli, T., Błaszczyk, P., Kanovei, V., Katz, K., Katz, M., Schaps, D., & Sherry, D. (2015). Leibniz vs Ishiguro: Closing a quarter-century of syncategoremania. *HOPOS: The Journal of the International Society for the History of Philosophy of Science*. To appear.
- Faris, W. (2006). Diffusion, quantum theory, and radically elementary mathematics. In Faris, W. G., editor. *Mathematical Notes*, Vol. 47. Princeton, NJ: Princeton University Press.
- Gordon, E., Kusraev, A., & Kutateladze, S. (2002). *Infinitesimal Analysis. Mathematics and its Applications*, Vol. 544. Updated and revised translation of the 2001 Russian original. Translated by Kutateladze. Dordrecht: Kluwer Academic Publishers.
- Hrbáček, K. (1978). Axiomatic foundations for nonstandard analysis. *Fundamenta Mathematicae*, **98**(1), 1–19.
- Kanovei, V., & Reeken, M. (2004). *Nonstandard Analysis, Axiomatically*. Springer Monographs in Mathematics. Berlin: Springer.
- Lawler, G. (2011). Comments on Edward Nelson’s “Internal set theory: A new approach to nonstandard analysis”. *Bulletin of the American Mathematical Society (N.S.)*, **48**(4), 503–506.
- Nelson, E. (1977). Internal set theory: A new approach to nonstandard analysis. *Bulletin of the American Mathematical Society*, **83**(6), 1165–1198.
- Nelson, E. (1987). *Radically Elementary Probability Theory. Issue 117 of Annals of Mathematics Studies*. Princeton, NJ: Princeton University Press.
- Nelson, E. (1988). The syntax of nonstandard analysis. *Annals of Pure and Applied Logic*, **38**(2), 123–134.
- Nelson, E. (1995). Private communication. January 16, 1995.
- Robinson, A. (1966). *Non-standard Analysis*. Amsterdam: North-Holland Publishing.

DEPARTMENT OF MATHEMATICS
BAR ILAN UNIVERSITY
RAMAT GAN 52900 ISRAEL
E-mail: katzmik@macs.biu.ac.il

SOBOLEV INSTITUTE OF MATHEMATICS
NOVOSIBIRSK STATE UNIVERSITY
RUSSIA
E-mail: sskut@math.nsc.ru