Canad. Math. Bull. Vol. **41** (1), 1998 pp. 118–124

ON PERMANENTAL IDENTITIES OF SYMMETRIC AND SKEW-SYMMETRIC MATRICES IN CHARACTERISTIC p

ANGELA VALENTI

ABSTRACT. Let $M_n(F)$ be the algebra of $n\times n$ matrices over a field F of characteristic $p > 2$ and let $*$ be an involution on $M_n(F)$. If $s_1,\dots,s_r$ are symmetric variables we determine the smallest r such that the polynomial

$$P_r(s_1,\dots,s_r)=\sum_{\sigma\in S_r} s_{\sigma(1)}\cdots s_{\sigma(r)}$$

is a $*$-polynomial identity of $M_n(F)$ under either the symplectic or the transpose involution. We also prove an analogous result for the polynomial

$$C_r(k_1,\dots,k_r,k'_1,\dots,k'_r)=\sum_{\sigma,\tau\in S_r} k_{\sigma(1)}k'_{\tau(1)}\cdots k_{\sigma(r)}k'_{\tau(r)}$$

where $k_1,\dots,k_r,k'_1,\dots,k'_r$ are skew variables under the transpose involution.

1. **Introduction.** Let $x_1,\dots,x_m$ be non commuting indeterminates and S_m the symmetric group on $1,2,\dots,m$. The standard polynomial

$$S_m(x_1,\dots,x_m)=\sum_{\sigma\in S_m}(\operatorname{sgn}\sigma)x_{\sigma(1)}\cdots x_{\sigma(m)}$$

plays an important role in the study of the polynomial identities of $M_n(F)$, the algebra of $n\times n$ matrices over a field F. The Amitsur-Levitzki theorem ([6, Theorem 1.4.1]) states that $S_{2n}(x_1,\dots,x_{2n})$ is a polynomial identity of minimal degree for $M_n(F)$ and also, if f is a polynomial identity of degree $2n$ then $f=\alpha S_{2n}$ for some $\alpha\in F$.

A permanental version of the standard polynomial is given by the polynomial

$$P_m(x_1,\dots,x_m)=\sum_{\sigma\in S_m} x_{\sigma(1)}\cdots x_{\sigma(m)}.$$

Clearly P_m is obtained from the polynomial x^m by complete linearization. Hence, if $\operatorname{char}F=0$ (or $\operatorname{char}F=p>0$ and $p>m!$) it easily follows that if $m\geq 1$, $P_m(x_1,\dots,x_m)$ is not a polynomial identity for $M_n(F)$, for all $n\geq 1$. Nevertheless in [8] Zalesskii proved that if $\operatorname{char}F=p>0$ then $P_{np}(x_1,\dots,x_{np})$ is a polynomial identity for $M_n(F)$; moreover P_{np} is of minimal degree in the sense that, if for some $k\geq 1, P_k(x_1,\dots,x_k)$ is a polynomial identity for $M_n(F)$ then $k\geq np$.

Received by the editors June 26, 1996.
Research supported in part by MURST of Italy.
AMS subject classification: 16R50.

Let F be a field, $X = \{x_1, x_2, \ldots\}$ a countable set and $F\{X, *\}$ the free algebra with involution $*$ over F. We denote by x_i^* the image of the variable x_i under $*$. If R is an F-algebra with involution $*$ we shall consider only involutions such that $(\alpha a)^* = \alpha a^*$ for all $a \in R$ and $\alpha \in F$. Recall that a non-zero polynomial $f(x_1, x_1^*, \ldots, x_m, x_m^*)$ in $F\{X, *\}$ is a $*$-polynomial identity ($*$-PI) for R if $f(r_1, r_1^*, \ldots, r_m, r_m^*) = 0$ for all $r_1, \ldots, r_m \in R$.

Now, if $*$ is an involution on $M_n(F)$, it is well known that if F is an infinite field of characteristic not 2, then only two involutions play a role in the study of the $*$-polynomial identities of $M_n(F)$: the transpose involution, denoted $* = t$, and the canonical symplectic involution, denoted $* = s$.

Recall that s is defined only in case $n = 2m$ is even and it is given by the rule: if $A \in M_n(F)$, write $A = \begin{pmatrix} B & C \\ D & E \end{pmatrix}$ where $B, C, D, E \in M_m(F)$ and set

$$A^s = \begin{pmatrix} E^t & -C^t \\ -D^t & B^t \end{pmatrix}$$

where t is the usual transpose.

In what follows we shall write $\big(M_n(F), s\big)$ and $\big(M_n(F), t\big)$ to indicate that $M_n(F)$ is endowed with an involution of transpose or symplectic type respectively.

If $\operatorname{char} F \neq 2$, for $i = 1, 2, \ldots$, we define $s_i = x_i + x_i^*$ and $k_i = x_i - x_i^*$ so that $F\{X, *\} = F\{s_1, k_1, s_2, k_2, \ldots\}$ and s_i and k_i are symmetric and skew-symmetric variables respectively.

In a paper relating the Amitsur-Levitzki theorem to the cohomology of the unitary group [3], Kostant proved that if n is even $S_{2n-2}(k_1, \ldots, k_{2n-2})$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$. This theorem was later extended by Rowen in [5] who showed that $S_{2n-2}(k_1, \ldots, k_{2n-2})$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$ for all n. Concerning the symplectic involution Rowen in [7] proved that $S_{2n-2}(s_1, \ldots, s_{2n-2})$ is a $*$-polynomial identity for $\big(M_n(F), s\big)$. It is not known for generic n if the above polynomials are $*$-polynomials identities of minimal degree for $\big(M_n(F), *\big)$.

In the present paper, by generalizing Zalesskii's theorem, we shall prove that if $\operatorname{char} F = p > 2$ then $P_{mp}(s_1, \ldots, s_{mp})$ is a $*$-polynomial identity for $\big(M_n(F), s\big)$ where $m = n/2$. Also $P_{mp}(s_1, \ldots, s_{mp})$ is of minimal degree in the sense that if $P_r(s_1, \ldots, s_r)$ is a $*$-polynomial identity for $\big(M_n(F), s\big)$, then $r \geq mp$. In case of transpose involution we prove that $P_r(s_1, \ldots, s_r)$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$ if and only if $r \geq np$.

Another result we shall prove concerns a permanental version of the Capelli polynomial, *i.e.*, the polynomial

$$C_r(x_1, \ldots, x_r, y_1, \ldots, y_r) = \sum_{\sigma, \tau \in S_r} x_{\sigma(1)} y_{\tau(1)} \cdots x_{\sigma(r)} y_{\tau(r)}$$

where $x_1, \ldots, x_r, y_1, \ldots, y_r$ are distinct non commuting variables.

In [2] it was proved that $C_r(x_1, \ldots, x_r, y_1, \ldots, y_r)$ is a polynomial identity for $M_n(F)$ if and only if $r \geq np$ where $\operatorname{char} F = p > 0$.

https://doi.org/10.4153/CMB-1998-018-9 Published online by Cambridge University Press

Let $k_1, k_1', \ldots, k_r, k_r'$ be distinct skew variables. By extending the above result, in [4] Revesz and Szigeti proved that, if $\operatorname{char} F = p > 2$, then $C_r(k_1, \ldots, k_r, k_1', \ldots, k_r')$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$ provided that $r \geq [\frac{n+1}{2}]p$ and $p > \sqrt{[\frac{n+1}{2}]}$.

We shall improve this theorem by showing that the conclusion still holds if we remove the hypothesis $p > \sqrt{[\frac{n+1}{2}]}$; we shall also prove that, if n is even, this polynomial is of minimal degree in the sense that if $C_r(k_1, \ldots, k_r, k_1', \ldots, k_r')$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$ then $r \geq [\frac{n+1}{2}]p$.

One final remark is in order. All the results proved in this note extend to matrix rings over an arbitrary commutative ring with 1 such that $p \cdot 1 = 0$.

Throughout F will be a field of characteristic $p > 2$.

2**. Permanental standard polynomials.** Let A be the algebra generated over F by the countable set $\{a_1, a_2, \ldots\}$ with relations $a_i^p = 0$ and $a_i a_j = a_j a_i$ for all i, j. If $*$ is an involution on $M_n(F)$, then the algebra $M_n(A) \cong M_n(F) \otimes_F A$ has a natural induced involution, that we shall still denote $*$, defined by requiring that $(b \otimes a)^* = b^* \otimes a$, for $b \in M_n(F)$ and $a \in A$.

Recall that a $*$-polynomial $f(x_1, x_1^* \ldots, x_m, x_m^*)$ is multilinear if in every monomial of f, x_i or x_i^*, $i = 1, \ldots, m$, appears exactly once. We have the following

LEMMA 1. *Let $f(x_1, x_1^* \ldots, x_m, x_m^*)$ be a multilinear $*$-polynomial. Then f is a $*$-polynomial identity for $M_n(F) \otimes_F A$ if and only if f is a $*$-polynomial identity for $M_n(F)$.*

PROOF. Since f is multilinear it is enough to check f on elements of the type $b \otimes a$ with $b \in M_n(F)$ and $a \in A$. Let $b_1, \ldots, b_m \in M_n(F)$ and $a_1, \ldots, a_m \in A$ then

$$f(b_1 \otimes a_1, b_1^* \otimes a_1, \ldots, b_m \otimes a_m, b_m^* \otimes a_m) = f(b_1, b_1^*, \ldots, b_m, b_m^*) \otimes a_1 a_2 \cdots a_m.$$

It is now clear that f is a $*$-PI for $M_n(F)$ if and only if f is a $*$-PI for $M_n(F) \otimes_F A$. ∎

Let C be a commutative ring and suppose that $M_{2m}(C)$ is endowed with the symplectic involution. Let e_{ij}, $(i, j = 1, \ldots, n)$ be the usual matrix units of $M_n(C)$. Let us denote by $Pf(c)$ the Pfaffian of the matrix c. If $b = b^* \in M_{2m}(C)$, it is know [6, Theorem 2.5.10] that every eigenvalue of b has even multiplicity; also if $u = \sum_{i=1}^m (e_{ii+m} - e_{i+mi})$ and $p(x) = Pf\big((x - b)u\big)$ then $p(b) = 0$ and $p(x)^2$ is the characteristic polynomial of b.

The polynomial P_k satisfies the following obvious relation

REMARK. For every $h \leq k$,

$$P_k(x_1, \ldots, x_k) = \sum_{i_1, \ldots, i_h} x_{i_1} \cdots x_{i_h} P_{k-h}(x_1, \ldots, \hat{x}_{i_1} \cdots \hat{x}_{i_h} \ldots x_k)$$

where $\hat{x}_j$ means that the variable x_j is omitted.

The following theorem shows that the bound of Zalesskii's theorem can be considerably lowered if one evaluates the polynomial P_k on symmetric matrices under the symplectic involution.

THEOREM 2. *$P_r(s_1, \ldots, s_r)$ is a $*$-polynomial identity for $\big(M_{2m}(F), s\big)$ if and only if $r \geq mp$.*

PROOF. Let A be the algebra defined above and consider $M_{2m}(A)$ with canonical symplectic involution. For $b = b^* \in M_{2m}(A)$ let $p(x) = Pf\big((x-b)u\big)$, and write

$$p(x) = x^m - \mu_1 x^{m-1} + \ldots + (-1)^m \mu_m.$$

Since $\mu_i \in A$ it follows that $\mu_i^p = 0$, for all $i = 1, \ldots, m$; hence, since $p(b) = 0$,

$$0 = p(b)^p = b^{mp} - \mu_1^p b^{(m-1)p} \ldots + (-1)^m \mu_m^p = b^{mp}$$

and x^{mp} is a $*$-PI for $M_{2m}(A)$ in one symmetric variable x. By completely linearizing the polynomial x^{mp} we get that $P_{mp}(s_1, \ldots, s_{mp})$ is a $*$-polynomial identity for $M_{2m}(A) \cong M_{2m}(F) \otimes_F A$; since P_{mp} is multilinear, by Lemma 1, it follows that P_{mp} is a $*$-PI for $M_{2m}(F)$.

By the previous remark, if $r \geq mp$ then $P_r(s_1, \ldots, s_r)$ is still a $*$-PI for $M_{2m}(F)$. Also, in order to finish the proof we only have to show that $P_{mp-1}(s_1, \ldots, s_{mp-1})$ is not a $*$-PI for $M_{2m}(F)$. Let us consider the following symmetric elements of $M_{2m}(F)$:

$$t_1 = e_{12} + e_{m+2m+1}, \quad t_2 = e_{23} + e_{m+3m+2}, \ldots, t_{m-1} = e_{m-1m} + e_{2m2m-1},$$
$$t_m = e_{m1} + e_{m+12m}, \quad v_1 = e_{22} + e_{m+2m+2}, \ldots, v_{m-1} = e_{mm} + e_{2m2m}.$$

Then we get

$$e_{11} P_{mp-1}(t_1, \ldots, t_m, t_1, \ldots, t_m, \ldots, t_1, \ldots, t_m, v_1, \ldots, v_{m-1}) e_{11} = (p-1)!^m (p-1)^{m-1} e_{11} \neq 0.$$

We should remark that $(p-1)^{m-1}$ counts the monomials of the form

$$t_1 a_2 t_2 a_3 \cdots a_m t_m t_1 b_2 t_2 b_3 \cdots b_m t_m \cdots t_1 c_2 t_2 c_3 \cdots c_m t_m,$$

where, for all i, $\{a_i, b_i, \ldots, c_i\} = \{v_i, 1, \ldots, 1\}$.

Then $P_{mp-1}(s_1, \ldots, s_{mp-1})$ is not a $*$-PI for $M_{2m}(F)$. ■

Unfortunately the analogue of Theorem 2 for transpose involution doesn't give a bound smaller than np on the degree of the polynomial P_k.

THEOREM 3. *$P_r(s_1, \ldots, s_r)$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$ if and only if $r \geq np$.*

PROOF. By Zalesskii's theorem we know that $P_{np}(s_1, \ldots, s_{np})$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$. Therefore it is enough to show that $P_{np-1}(s_1, \ldots, s_{np-1})$ is not a $*$-PI for $M_n(F)$. Take

$$t_1 = e_{12} + e_{21}, \quad t_2 = e_{23} + e_{32}, \ldots, t_{n-1} = e_{n-1n} + e_{nn-1}, \quad t_n = e_{1n} + e_{n1}$$

and

$$v_1 = e_{22}, \ldots, v_{n-1} = e_{nn}.$$

https://doi.org/10.4153/CMB-1998-018-9 Published online by Cambridge University Press

Then

$$e_{11}P_{np-1}(t_1,\ldots,t_n,\ldots,t_1,\ldots,t_n,v_1,\ldots,v_{n-1})e_{11} = 2^{p-1}(p-1)!^n(p-1)^{n-1}e_{11} \neq 0$$

where the last inequality holds since $\operatorname{char} F = p \neq 2$ and the only monomials giving a nonzero contribution are those written as products of terms of the form $t_1a_2t_2a_3\cdots a_nt_n$ and $t_na_n\cdots t_2a_2t_1$, where $a_i = 1$ or v_i. ∎

3. **Permanental Capelli polynomials.** In [4] Revesz and Szigeti proved that, if $r \geq p[\frac{n+1}{2}]$ and $p > \sqrt{[\frac{n+1}{2}]}$ then

$$C_r(k,k') = C_r(k_1,\ldots,k_r,k'_1,\ldots,k'_r) = \sum_{\sigma,\tau\in S_r} k_{\sigma(1)}k'_{\tau(1)}\cdots k_{\sigma(r)}k'_{\tau(r)}$$

is a $*$-polynomial identity for $\big(M_n(F),t\big)$ in the skew variables k_is and k'_is. In this section we shall remove the condition $p > \sqrt{[\frac{n+1}{2}]}$ from this theorem and moreover we shall prove that, if $n = 2m$, $C_m(k,k')$ is of minimal degree in the sense that, if for some $r \geq 1$, $C_r(k,k')$ is a $*$-polynomial identity for $\big(M_n(F),t\big)$ then $r \geq mp$.

We also consider the polynomial

$$\begin{aligned} D_r(s,k) &= D_r(s_1,\ldots,s_{2r},k_1,\ldots,k_r) \\ &= \sum_{\substack{\sigma\in S_{2r}\\ \tau\in S_r}} s_{\sigma(1)}(s_{\sigma(2)}\circ k_{\tau(1)})s_{\sigma(3)}(s_{\sigma(4)}\circ k_{\tau(2)})\cdots s_{\sigma(2r-1)}(s_{\sigma(2r)}\circ k_{\tau(r)}) \end{aligned}$$

where $s_1,\ldots,s_{2r}$ are symmetric variables and $s\circ k = sk + ks$. Here we shall improve the result in [4] concerning $D_r(s,k)$ by removing as above the hypothesis on $\operatorname{char} F$.

If $f(x_1,x_2,\ldots,x_n)$ is a homogeneous polynomial then we can apply to f the well known process of multilinearization [6, p. 126] that will produce a multilinear polynomial. At each stage of this process we still get a homogeneous polynomial g that we shall call a proper linearization of f provided $g \neq f$.

The following lemma is stated only for polynomials in skew variables since this is the setting in which we shall apply it. Anyway it is obvious that it can be generalized to polynomials in any number of symmetric and skew variables.

LEMMA 4. *Let R be an F-algebra with involution $*$ and $f(k_1,k_2,\ldots,k_n) \in F\{X,*\}$ a homogeneous polynomial which is not multilinear. Suppose that all proper linearizations of f are $*$-polynomial identities for R. If B is a basis for the space of skew elements of R and $f(b_1,b_2,\ldots,b_n) = 0$, for all $b_1,\ldots,b_n \in B$, then f is a $*$-polynomial identity for R.*

PROOF. Let $a_1,\ldots,a_n$ be skew elements of R and write $a_i = \sum\alpha_{ij}b_j$ where $b_j \in B$ and $\alpha_{ij} \in F$. Notice that $f(\sum\alpha_{1j}b_j,\ldots,\sum\alpha_{nj}b_j)$ can be written as a linear combination of valuations of the proper linearizations of f and terms of the type $f(b_{i_1},\ldots,b_{i_n})$ with $b_{i_j} \in B$. Then, by applying the hypothesis, we get that f is a $*$-PI for R. ∎

THEOREM 5. *$C_{mp}(k,k')$ is a $*$-polynomial identity for $\big(M_n(F),t\big)$ where $m = [\frac{n+1}{2}]$. Moreover, if $n = 2m$ and $C_r(k,k')$ is a $*$-polynomial identity for $\big(M_n(F),t\big)$ then $r \geq mp$.*

https://doi.org/10.4153/CMB-1998-018-9 Published online by Cambridge University Press

PROOF. We may assume that $n = 2m$ is even. Let A be the algebra defined at the beginning of Section 2. Let K, K' be two skew matrices in $M_n(A)$ under the transpose involution and suppose that K is invertible. Then the map $\phi\colon D \longrightarrow KD^tK^{-1}$ defines an involution of symplectic type on $M_n(A)$ and under this involution KK' is a symmetric element. As in the proof of Theorem 2 it follows that KK' satisfies a polynomial of degree m; let

$$(KK')^m - \mu_1(KK')^{m-1} + \cdots + (-1)^m\mu_m = 0.$$

By a Zariski density argument we can remove the assumption that K is invertible and by taking p-th power, as in the proof of Theorem 2, it follows that $(KK')^{mp} = 0$. At this point it s enough to notice that $C_{mp}(k, k')$ is the multilinearization of the polynomial $(k_1k_1')^{mp}$; hence we get that $C_{mp}(k, k')$ is a $*$-PI for $\big(M_n(A), t\big)$ and, so, by Lemma 1, for $\big(M_n(F), t\big)$.

Suppose now that $C_r(k, k')$ is a $*$-PI for $\big(M_n(F), t\big)$ and $r < mp$. Recall that, since by Lemma 1, $M_n(F)$ and $M_n(A)$ satisfy the same multilinear $*$-PIs then $C_r(k, k')$ is also a $*$-PI for $M_n(A)$. Define now the two matrices

$$K = \begin{pmatrix} O & B \\ -B^t & O \end{pmatrix} \quad \text{and} \quad K' = \begin{pmatrix} O & -D^t \\ D & O \end{pmatrix}$$

where $B = (b_{ij}) \in M_m(A)$ is such that $b_{ij} = 0$ if $j \not\equiv i+1 \pmod m$, $b_{ii+1} = a_i$ and $b_{m1} = a_m$ and $D = (d_{ij}) \in M_m(A)$ is such that $d_{ij} = 0$ if $j \neq i$ and $d_{ii} = a_{m+i}$.

By direct computation we get that

$$(KK')^m = \begin{pmatrix} BD & O \\ O & B^tD^t \end{pmatrix}^m = a_1a_2\cdots a_ma_{m+1}\cdots a_{2m}I$$

where I is the $2m \times 2m$ identity matrix. From this equality, it follows that, since $a_i^p = 0$, mp is the smallest exponent such that $(KK')^{mp} = 0$. Thus $(KK')^r \neq 0$.

We claim that all the proper linearizations of the polynomial $(k_1k_1')^r$ are $*$-PIs for $\big(M_n(A), t\big)$. Let f be a proper linearization of $(k_1k_1')^r$. We use induction on the number of distinct variables appearing in f. Let f be of degree $d_i \geq 1$ in k_i and suppose first that $p \nmid d_i$. Then let f' be the linearization of f which is of degree $d_i - 1$ in k_i and of degree 1 in one new skew variable z. By inductive hypothesis f' is a $*$-PI for $M_n(A)$. If we identify $k_i = z$ in f' we obtain that d_if and, so, f is a $*$-PI for $M_n(A)$.

On the other hand suppose that $p \mid d_i$ and let $M_n(A)^-$ be the space of skew elements of $M_n(A)$. If B is a basis of $M_n(A)^-$, then the elements of B can be written in the form $b_i \otimes c_i$ where $b_i = -b_i^* \in M_n(F)$. Thus we compute

$$f(b_1 \otimes c_1, \ldots, b_u \otimes c_u) = f(b_1, \ldots, b_u) \otimes c_1^{d_1} \cdots c_u^{d_u}.$$

Since $p \mid d_i$ then $c_i^{d_i} = 0$ and, so, f vanishes on a basis of $M_n(A)^-$. By Lemma 4 then f is a $*$-PI for $M_n(A)$. This procedure also implies, again by Lemma 4, that $(k_1k_1')^r$ is a $*$-PI for $M_n(A)$, a contradiction. ∎

We now prove the following result that has been proved in [4] with the additional hypothesis $p > \sqrt{[\frac{n+1}{2}]}$.

THEOREM 6. *If $m = [\frac{n+1}{2}]$ then $D_{mp}(k, s)$ is a $*$-polynomial identity for $\big(M_n(F), t\big)$.*

https://doi.org/10.4153/CMB-1998-018-9 Published online by Cambridge University Press

PROOF. Let K, K' be two skew matrices in $M_n(A)$ under the transpose involution. As in the proof of the previous theorem we have that $(KK')^{mp} = 0$. Now take $K' = KS+SK$ where S is a symmetric elements of $\big(M_n(A), t\big)$; then $\big(K(KS + SK)\big)^{mp} = 0$. By multilinearizing this polynomial we get that $D_{mp}(k, s)$ is a $*$-PI for $\big(M_n(A), t\big)$ and, so, for $\big(M_n(F), t\big)$. ∎

REFERENCES

1. A. Giambruno and A. Valenti, *On minimal $*$-identities of matrices*. Linear and Multilinear Algebra, **39**(1995), 309–323.

2. P. Kortesi and J. Szigeti, *On permanental identities over matrix rings*. Comm. Algebra **22**(1994), 159–171.

3. B. Kostant, *A theorem of Frobenius, a theorem of Amitsur-Levitzki and cohomology theory*. Indiana J. Math. (and Mech.) **7**(1958), 237–264.

4. G. Revesz and J. Szigeti, *Identities of symmetric and skew-symmetric matrices in characteristic p*. Rend. Circ. Mat. Palermo (2) **44**(1995), 94–106.

5. L. H. Rowen, *Standard polynomials in matrix algebras*. Trans. Amer. Math. Soc. **190**(1974), 253–284.

6. ———, *Polynomial Identities in Ring Theory*. Academic Press, New York, 1980.

7. ———, *A simple proof of Kostant's theorem, and an analogue for the symplectic involution*. Contemp. Math. **13**, Amer. Math. Soc., Providence, Rhode Island, 1982, 207–215.

8. A. E. Zalesskii, *A symmetric analogue of the Amitsur-Levitzki Theorem*. Vestsī Akad. Navuk BSSR Ser. Fīz.-Mat. Navuk (1985), 108–110 (Russian).

Dipartimento di Matematica
Università di Palermo
Via Archirafi 34
90123 Palermo
Italy

https://doi.org/10.4153/CMB-1998-018-9 Published online by Cambridge University Press