rattachant; la quatrième: La lettre à A. Chevalier, le troisième mémoire (très fragmentaire) "Théorie des intégrales dont les différentielles sont des fonctions algébriques", et les "derniers vestiges" c'est à dire "tout un lot de calculs restés jusqu'à ce jour inédits" sans commentaires, mais ou l'on a "découpé et isolé chaque calcul, numéroté dans l'ordre d'édition, I à CXXIX"; "il se trouvera, j'espère, des gens qui trouveront profit à déchiffrer tout ce gâchis" (mots de Galois, cités par l'éditeur). La cinquième partie comprend tout ce qui concerne les travaux particuliers et les premiers essais écrits par Galois comme élève au collège Louis-le-Grand et à l'Ecole Normale. Ensuite vient la correspondance, c'est à dire les neuf lettres actuellement connus de Galois. Dans deux appendices on trouve une bibliographie des "Oeuvres", l'inventaire des manuscrits, leur description avec quelques commentaires et des remarques sur le drame précédant la mort de Galois, en particulier les lettres de Stéphanie D.... Le volume contient aussi une reproduction fac-similée de douze pages de manuscrits et de lettres de Galois et les deux portraits bien connus.

H. Schwerdtfeger, McGill University

Representation Theory of Finite Groups and Associative Algebras, by C. W. Curtis and Irving Reiner. Interscience, New York, 1962.

Contents: I Background from Group Theory, II Representations and Modules, III Algebraic Number Theory, IV Semi-simple Rings and Group Algebras, V Group Characters, VI Induced Characters, VII Induced Representations, VIII Non-semi-simple Rings, IX Frobenius Algebras, X Splitting Fields and Separable Algebras, XI Integral Representations, XII Modular Representations.

An examination of the above Table of Contents makes it clear that to write an adequate review of this comprehensive volume is a formidable task. Though the authors do not claim to be encyclopaedic, they have succeeded in correlating two large and important fields of modern mathematics, showing how many ideas which arise in the study of the representation theory of finite groups have a more general interpretation for associative algebras. One thing which strikes one immediately and with increasing force as the development proceeds is the correctness of the initial approach of Frobenius, Burnside, Schur and Dickson, since few of the ideas which have been introduced in the last fifty years are fundamentally new. Though the notion of a finite group is almost trivial it has proved to be so important in many different contexts that an enormous amount of time and energy has been devoted to exploring its representation theory and other properties. Foremost in these explorations of recent years has been Richard Brauer, a pupil of Schur, to whom many of the deep results recorded here are due.

How should a review of a book like this be written? In order to break down the problem let us divide the chapters as follows I-II, III, IV-V, VI-VII, VIII-IX, X-XII. Clearly chapter III is in a separate category, - it could almost have been an Appendix - but where to place it is unimportant; what is important is that the authors have devoted much pains to make it adequate for their needs and modern in treatment. It is not easy reading without some background in the classical theory of algebraic numbers, but its study is rewarding and many of the results obtained are fundamental in subsequent applications.

Chapters I, II, IV and V cover the representation theory of Frobenius, Burnside and Schur set in the algebraic mould of Emmy Noether. The following sentence from the introduction to chapter IV describes what the authors are trying to do. "Some of the main results in this theory depend not so much on special properties of group algebras as on properties which group algebras have because they belong to the large class of rings with minimum condition." Put in this way, the development cannot become a race to reach the famous formulae of character theory before the reader loses interest, - it must rather be an elaboration, and deepening of Schur's 'Neue Begründung' of 1905. For this the reader should be grateful since here are the rolling hills and wide flowing streams which lead us to the crags and precipices to be encountered later.

In § 28 the authors give a brief account of Young's approach to the group algebra and the representation theory of the symmetric group $S_n$. It remains a tantalizing fact that in this case the group algebra yields the actual matrices of the irreducible representations with all their interest and significance, while character theory plays a secondary role. Clearly, there is still much to be learned and the fact that these ideas have such wide application in physics and chemistry acts as a spur to the mathematicians to continue their study, since the rainbow may lead to the pot of gold 'the other side of the mountain'.

Chapters VI and VII describe the upward trail, first marked by Frobenius' Reciprocity Theorem, and recently followed by Brauer, Mackey, Clifford and Shoda. The notion of an induced representation ties together two fundamental aspects of group theory, namely that of a linear representation of a subgroup $H$ of $G$ and the abstract relationship between $H$ and $G$ described by the permutation representation of $G$ induced by $H$. That every finite group is isomorphic to a subgroup of $S_n$ has been known for a long time, but it has not contributed greatly to our knowledge of group theory up to the present. Is it a rash conjecture that this relationship, properly expressed, is the rainbow which could lead to the pot of gold?

Chapters VIII-IX develop the properties of semi-simple rings and algebras. Much of this work is due to Nakayama and the levels into which it naturally falls can best be indicated by the chain of inclusion relations (p. 440).

$$\{\text{group algebras}\} \subset \{\text{symmetric algebras}\} \subset \{\text{Frobenius algebras}\} \subset \{\text{quasi Frobenius algebras}\}$$

over a field K. If the characteristic p of K divides the order of the group G then the group algebra KG is no longer semi-simple and we are involved in the modular representation theory of G treated in chapter XII.

In the last three chapters of the book we are in the midst of the mountains, trying to follow the streams to their sources whose directions were so clear on the plains. In chapter X the authors give a lucid account of the effect on reducibility and decomposability of extending the field K. This aspect of representation theory goes back to Schur but the significance of the 'Schur index' (§ 70) has only been appreciated lately, largely as a result of Brauer's work. Though the ideas are difficult they hold promise for the future.

Chapter XI on integral (Z-) representations is the longest in the book (75 pages). Perhaps this is not surprising since every effort has been made to explain the difficulties inherent in a study of the group ring. To clarify ideas the authors give some simple examples which show how Z- equivalence differs from Q- equivalence and in § 74 discuss the Z- representation theory of a cyclic group of order p. The extension of these results to more general groups is one of the outstanding problems of the theory. This work goes back to Zassenhaus and Maranda, while many recent results are due to Reiner and Swan.

The last chapter on modular representations tells the story of the work of Brauer and his students carried out in Toronto, Michigan and Harvard over the past 30 years. It is an exciting story for those who have been connected with it. The road has become increasingly difficult, but analogy has played a strong rôle and character theory, appropriately modified and extended to include the Cartan invariants, leads to a very beautiful system of formulae. As in the case of the ordinary theory, it is possible to use the modular theory to deduce deep properties of abstract groups as Feit and Thompson have recently shown.

- And what of the 'pot of gold'? Will we ever really understand the implications of the simple axioms which define a group? Are the difficulties inherently similar to those which have made the theory of numbers so fascinating and so obscure? Whatever the future holds in store, we are deeply indebted to Professor Curtis and Professor Reiner for completing a tremendous task and producing a book which fills a long felt need. It is a trivial comment to say that few books are free of typographical errors, and few mathematical books are free of mathematical errors. Though some of both kinds have been detected, one may be sure that they will be corrected in future editions of this important work.

G. de B. Robinson, University of Toronto

* * *

Correction to Book Reviews, Vol. 6, No. 3

On page 438, in the review of the book "Introduction to Differentiable Manifolds" the name of the author, S. LANG, has been omitted.