The Infrared Period - 2.2μm Magnitude Relation for RR Lyrae Stars

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Abstract

The observational and theoretical basis for the $\log P$ vs. K-mag relation is reviewed. The observed gradient in all well observed globular clusters agrees well with the theoretical prediction. An indistinguishable result is found for field RR Lyrae stars whose absolute magnitudes have been determined from infrared Baade-Wesselink analyses. A full reference list is given for the source of these magnitudes. Application of the results to find the distance to globular clusters and the Galactic Centre is discussed.

1. Introduction

At the last pulsation meeting, in Bologna, I concluded my presentation by saying 'If one can even begin to realise the full potential of the contribution of IR photometry to the subject matter of this conference, it will not be possible to summarise it in 20 minutes at the next Pulsation Meeting.' This meeting's organisers clearly remembered these words for two years and have consequently allowed 25 minutes for this review! To meet the time requirement, I will concentrate on the use of K (2.2μm) magnitude observations, with special reference to the log(Period) vs K-mag relation. Although there are several variations on the IR photometric system (see Bessell & Brett 1988 for one excellent comparison) there is very little difference between K filters. Typical systematic corrections between systems are less than 0.02 mag, with negligible colour terms for stars hotter than 4500 K. Therefore all K magnitudes here are taken as published. Another bonus for 2.2μm observations is the reduced effect of dust extinction: $A_K{\sim}0.1A_V$.

Although my first 2.2μm observations of RR Lyrae stars were made in about 1980 the full picture of their usefulness did not emerge until more extensive observations in 1983/84. These were published in 1985 (Longmore *et al.*, the first Baade-Wesselink (BW) analysis using infrared photometry, which with other BW analysis of dwarf Cepheids constituted the subject of J.Fernley's thesis) and 1986 (Longmore, Fernley & Jameson [LFJ], demonstrating the $\log P$ vs K-mag relation). The 1985 BW analysis of VY Ser had a fortuitous timing coincidence. Despite their use of an excellent radial velocity curve, Carney & Latham (1984) were unable to find a solution for the radius of VY Ser with optical photometry alone. Finding a solution by combining optical and IR photometry and using the Carney & Latham velocities immediately

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demonstrated the 'infrared advantage'. This review traces the development of the $\log P$ vs K-mag relation (section 2), summarizes the latest BW results (section 3) and discusses additional applications of them (section 4).

2. The log(Period) versus K-mag relation

A simple way to think of the $\log P$ vs K-mag relation is as a K-mag, radius relation. Period is then introduced via the period, mean-density relation, which is strongly radius dependent. From the definition of bolometric luminosity $\Delta M_{bol} = 10\Delta\log T_e - 5\Delta\log R$ (all symbols have their usual meanings). Empirical (eg. Carney 1983) and model atmosphere (eg. R.Kurucz, 1992 private communication) calculations indicate a tight T_e-$(V-K)$ relation of the form

$$\log T_e = -0.112(M_V - M_K) + 3.934 \tag{1}$$

Therefore, noting that for RR Lyrae stars $M_{bol} \sim M_V$, it follows that $\Delta M_K \sim 0.10$ ΔM_{bol} - 4.5 $\Delta\log R$. The $\log P$ vs K-mag relation itself is easily derived from the pulsation equation (van Albada & Baker 1971, Cox 1988):

$$\log P = -0.68\log(M/M_\odot) - 0.336M_{bol} - 3.48\log T_e + 13.09, \tag{2}$$

where, from the R. Kurucz models

$$M_{bol} = M_K - 7.560\log T_e + 29.846. \tag{3}$$

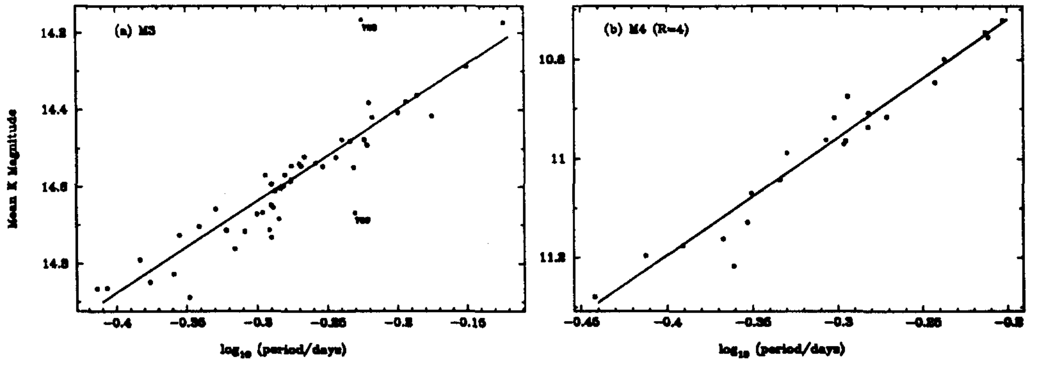
From (1), (2) & (3)

$$\log P = -0.441M_K + 0.105M_V - 0.68\log(M/M_\odot) - 0.635, \tag{4}$$

$$M_K = -2.27\log P + 0.24M_V - 1.54\log(M/M_\odot). \tag{5}$$

Equations (4) and (5) show clearly the relatively small scatter likely to be introduced by differences in the visual magnitudes of the stars. The small mass range exhibited by RR Lyrae stars implies ±0.02 mag effects on M_K within clusters and ±0.07 mag between clusters from the $\log(M/M_\odot)$ term, although the absolute value of RR Lyrae masses is still uncertain (Cox 1991, Kovacs *et al.* 1992, Simon & Clement 1993). Fig. 1 in Fernley *et al.* (1987) illustrates particularly the relevance of the $\log P$ vs K-mag relation to horizontal branch (HB) morphology.

On a sound basis theoretically, the $\log P$ vs K-mag relation is also well established observationally. It was found in all three clusters observed by LFJ, who pointed out that the relation could be used as a distance indicator if 'normalised' to a fixed period, chosen to be 0.5 days ($\log P = -0.3$). Results for eight clusters (M3, M4, M5, M15, M107, NGC 3201, NGC 5466 and ω Cen) were published by Longmore *et al.* (1990). Distances to these clusters were derived, all eight demonstrating a $\log P$ vs K-mag relation with the same gradients within the observational errors. Figures 1(a) and 1(b) show two of the five best established results, including some unpublished data for M3 and M4. Using these data for M3, Buckley *et al.* (1991) examined the K residuals. Despite the size of the residual being close to that expected simply from the scatter in the mean of approximately six random phase observations, they found a correlation between the residual and the V mag height ΔV above the zero age horizontal branch (ZAHB) as determined by Sandage (1990). The reason for this correlation is still being explored. Proving that the relation is not a function of observer, T.Liu (1992 private communication) has recently observed M5 and M15 in detail. Table 1 lists the gradients found for the five best observed clusters (six or more observations per star, $\geq$25 stars per cluster). The mean K residual is about 0.03 mag for all five clusters.



Figs. 1 (left) and **2** (right). The $\log P$ vs K-mag relation for M3 (left) & M4 (right). A fixed slope of −2.38, the mean value from Table 1, is drawn on each plot. M3-V23 is a cluster non-member, and M3-V59 suffers from contamination by a nearby star(s).

Table 1. Gradients of the five best determined $\log P$ vs K-mag relations in globular clusters.

Cluster	Gradient	Reference
ω Cen	-2.28	Longmore et al 1990
M3	-2.34	Buckley et al 1991
M4	-2.33	" "
M5	-2.42	T.Liu, private communication, 1992
M15	-2.46	" "

Fig. 3 (from Buckley *et al.* 1991) shows theoretical evolution away from the ZAHB tracks in the $\log P$ vs K-mag plane for stars of 0.68, 0.76 and $0.80 M_\odot$. For simplicity all pulsations are assumed to be in the fundamental mode. They are derived using

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the Lee & Demarque (1990) HB models and Equations (2) and (5) and indicate well the small effect that evolution has in this plane.

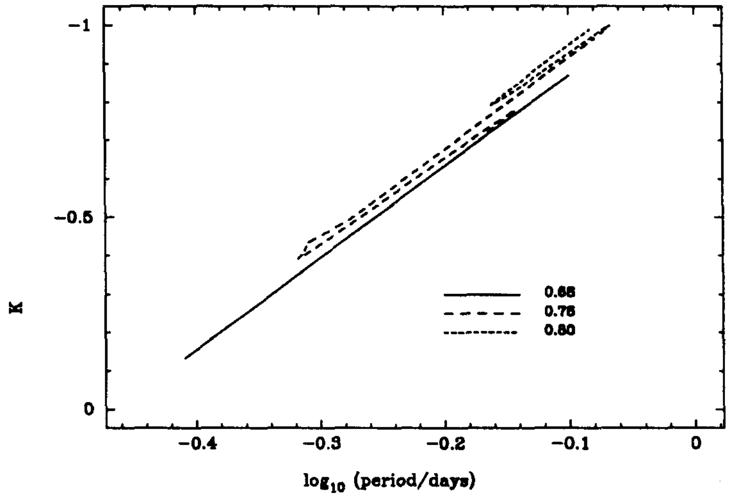

Fig. 3. Theoretical evolutionary tracks in the $\log P$ vs K-mag plane for horizontal branch stars of mass 0.68, 0.76 and 0.80 $M_\odot$.

LFJ also suggested that the nature of the $\log P$ vs K-mag relation should be insensitive to metallicity, a great advantage for a distance indicator. This is substantiated by the tight relationship in ω Cen, despite a range in metallicities of the individual stars $-2.2 \leq$ [Fe/H] ≤ -0.5. No correlation between metallicity and residual was found by Longmore *et al.* (1990).

3. Baade-Wesselink analysis

Since 1985, three groups have been primarily responsible for developing, in parallel, the infrared version of the BW analysis. Two main variations of methodology have emerged: the infrared-flux method (eg. Fernley *et al.* 1989, Skillen *et al.* 1989, Fernley *et al.* 1990a,b), and the surface-brightness method (Jones *et al.* 1987a,b, 1988a,b; Jones 1988; Liu & Janes 1989, 1990a). Moffett (1988) reviewed these techniques. Independent non-IR BW analyses have continued (Burki & Meylan 1986, Cacciari *et al.* 1989). Of particular note is the BW inversion technique introduced by Simon (1987, 1989). Despite the extremely intensive observational requirements of this type of programme (full phase coverage for highly accurate optical and IR light curves and radial velocities good to 1-2 km/s) the technique has now been applied to the much fainter RR Lyrae stars in nearby globular clusters (M4, Liu & Janes 1990b; M5 & M92, Storm *et al.* 1991, Storm 1991, Storm *et al.* 1992; M5, Cohen & Matthews 1992a,1992b, J.Cohen 1992 private communication). 29 different field stars and 13 different globular cluster RR Lyraes have been measured, six of the field stars by more than one group. Some of the globular cluster data are not yet fully analyzed.

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The following list summarises the literature to date:

- Carney, Jones, Latham, Kurucz & Storm. The Baade-Wesselink Method and the Distance to RR Lyrae Stars, Papers I-VIII. Nine field stars;
- Fernley, Skillen, Longmore, Jameson, Marang, Kilkenny, Lynas-Gray & Stobie. The Absolute Magnitudes of RR Lyrae Stars, Papers I-V. 10 field stars;
- Liu and Janes. The Luminosity Scale of RR Lyrae Stars with the Baade-Wesselink Method, Papers I-III. 13 field stars, four globular cluster stars;
- Cacciari, Clementini & Fernley. Three field stars
- Storm, 1991, three globular cluster stars;
- Storm *et al.* - relevant observational data papers;
- Cohen & Matthews. Six globular cluster stars.

There are at least three parameters which are not dealt with uniformly between the groups: (a) correction for the projection factor used to convert observed to true radial velocity (values from 1.30 to 1.36 have been used); (b) the zero point of the surface brightness method (Carney's group use one 0.04 mag brighter than that used by Liu & Janes); (c) the optimum phase range to use. (a) is known to depend on the spectroscopic dispersion used (Parsons 1972) but would greatly benefit from a re-analysis specifically for RR Lyrae stars, using modern model atmospheres and advanced simulation techniques. (b) reflects the problem common to both the techniques mentioned - the conversion from colour and flux to temperature. Both methods need to invoke model atmospheres at some stage. A weakness is that these are static models. (c) is a problem because shocks generated near phase 0.95 distort the colours around maximum light. Most of these and other residual uncertainties in the BW method would be resolved if suitable non-static model atmospheres could be constructed.

Now that there is a significant body of data it is useful to collate all the results. This has been done by Jones *et al.* (1992), Carney *et al.* (1992, CSJ), Skillen *et al.* (1992) and Cacciari *et al.* (1992). CSJ give an especially detailed discussion, also applying the unified results to comparisons of globular cluster distances and ages. They find ages $\geq$14 Gyr, depending on the assumed [O/Fe] ratio. Using the BW M_V results they determine that $M_V = 0.16[\mathrm{Fe/H}] + 1.02$ (see Fig.1 in their paper).

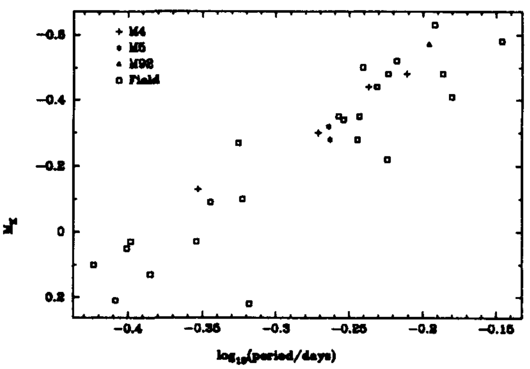
The existence of an independently determined $\log P$ vs K-mag relation can be used to test the relative accuracy of individual BW results. Fig.3 of CSJ shows that all stars (field and globular cluster) lie on a relation $M_K = -2.33 \log P - 0.88$ within the observational errors. Fig. 4 below is an up-dated version of that figure including the most recent results. Four stars (SS Leo, BB Pup, AR Per & V445 Oph) are omitted from the figure and subsequent analyses because of reddening or other uncertainties mentioned in the respective original papers. The linear regression on 25 field stars gives $M_K = -2.56 \log P - 0.98$ with a mean residual of $0^{\mathrm{m}}.09$ (well within the quoted errors of each of the BW analyses). To test for the metallicity dependence I also carried out a multiple linear regression, finding

$$M_K = -2.38 \log P + 0.04[\mathrm{Fe/H}] - 0.88 \qquad (6)$$

The $\log P$ coefficient agrees precisely with the mean gradient in Table 1, while the

[Fe/H] term is in the same sense as, but significantly smaller than, that for M_V vs [Fe/H]. However, the correlation coefficient and the residuals were essentially unchanged by including the extra variable. Slightly modifying the sample could significantly change the coefficients. The reason for this indeterminism can be seen from Fig.5 - there is a strong $\log P$-[Fe/H] relation for this sample of stars. Such a selection effect is difficult to overcome as it is a property of the field RR Lyrae population. The argument needs to be inverted: from Table 1, −2.38 is the correct gradient for $\log P$ vs K so the best estimate for the metallicity term is 0.04. This result can only be considered indicative at present.

Using equation (6) and the five best-determined globular cluster $\log P$ vs K-mag relations, distances can be derived that are almost independent of errors in reddening determinations (Table 2). For explanation of the different M_V values see Longmore *et al.* (1990).



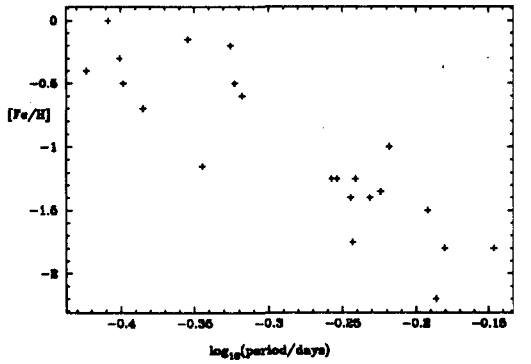


Fig. 4 (left) $\log P$ vs M_K from BW analysis of field and cluster RR Lyrae stars.
Fig. 5 (right) $\log P$ vs [Fe/H] for field RR Lyraes in Fig.4.

Table 2. Distances and HB absolute magnitudes for five globular clusters.

Cluster	$(M-M)_0$ (from field star BW)	$(M-M)_0$ (BW direct)	M_V (HB)	M_V (RR)
M3	14.88		0.72	0.78
M4 *	11.21	11.19	0.73	0.71
M5	14.21	14.19	0.83	0.79
M15	14.95		0.52	0.59
ω Cen	13.54		0.62	0.67
(M92)		(14.49)		

* E(B-V) = 0.37, R = 3.8

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4. Other applications of RR Lyrae IR photometry.

4.1 Distance to the Galactic Centre

Fernley *et al.* (1987) used $H(1.65\mu m)$ photometry, assumed $(H-K)$ ~0.03 for RR Lyrae stars, and employed the $\log P$ vs K-mag relation to find the distance to the Galactic Centre. They measured the RR Lyrae stars in Plaut's (1973b and references therein, Oort & Plaut 1975) l=0 deg, b=−12 deg field. They found $R(0)$ = 8 Kpc; re-calibrating using eqn.(6) and assuming [Fe/H] = −1.0 (Walker & Terndrup 1991) a revised distance of 7.5±0.6 kpc is found. Walker (1992) deduced $R(0)$ = 8-8.5 Kpc from RR Lyrae calibrations based on LMC distance indicators, including Cepheids. This difference implies that only about 15% mag uncertainty remains between RR Lyrae and Cepheid scales, which is almost within the errors of the IR BW analysis alone. B.Carney (private communication) has completed 2.2μm observations of RR Lyraes in Baade's window. We can look forward to seeing the result of the distance calculation from this data set.

4.2 RR Lyrae Temperatures and Globular Cluster Colour-Magnitude Diagrams

For well-established reasons, $V-K$ colours of well observed RR Lyrae stars give a better RR Lyrae relative temperature determination than $B-V$. Tighter temperature - amplitude relations, for example, confirm this. However, even using the new Kurucz models, $(V-K)$ gives lower mean RR Lyrae temperatures than $B-V$ by ~170 K at $\log g$ =2.7. Fig.6 (from Dixon 1991) shows a V,$V-K$ diagram of M4. Although marginal, there is an indication that $V-K$ for the variables (triangles) is displaced redward in $V-K$ compared with the HB non-variables. If the effect is real it is still unexplained but could account for the temperature difference noted above.

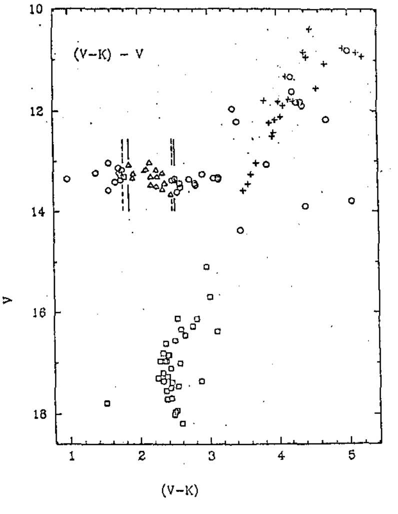

Fig.6. An optical - infrared CMD (V vs $V-K$) of the globular cluster M4, from Dixon (1991). The vertical solid lines indicate the probable range in (V-K) of the RR Lyrae stars while the vertical dashed lines indicate similarly for the non-variable HB stars.

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5. Summary

1. This work is observationally highly intensive!
2. Baade-Wesselink M_K residuals from the $\log P$ vs K-mag relation are all within observational errors.
3. The $\log P$ vs K-mag relation is essentially identical between field and cluster RR Lyraestars, in gradient and absolute calibration.
4. The absolute calibration implies globular cluster ages greater than 14 Gyr.
5. RR Lyrae temperatures are not yet satisfactorily determined.
6. Walker's (1992) RR Lyrae calibration using the LMC may be a problem for the BW results, but only at the ~15% level.

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DISCUSSION

D.HARTWICK: How do the RR Lyrae distance moduli for globular clusters compare with those from main sequence fitting?

A.LONGMORE: They are comparable, given the uncertainties. Depending on the Lutz-Kelker corrections, the RR Lyrae calibrations may give distance moduli up to $\sim 0^{\mathrm{m}}.15$ closer. I consider this to be within the overlap of likely systematic errors.

N.SIMON: Could you convert your M_K vs $\log P$ fit into a M_V vs $\log P$ fit?

A.LONGMORE: It could be done indirectly, if the temperatures of the stars are known independently. M_V could then be calculated because $(V-K)$ is very well correlated with temperature. Alternatively, if individual masses are known or assumed, equation (6) would yield M_V.

A.SANDAGE: (in reply to a question from the floor on the large scatter of $M_V =$ f(Period) compared with the small scatter for $M_K = \mathrm{f}(P)$: it is expected that M_V(RR) $= \mathrm{f}(P)$ has great scatter because the CMD in V is flat through the instability strip, yet in a given cluster there is a large spread in period because the instability strip has finite width. Therefore M_V does not change but P does (e.g., from $0^{\mathrm{d}}.4$ to $0^{\mathrm{d}}.7$ in M3). But in K, the HB is NOT horizontal, therefore there must be a much tighter relation in $M_K = \mathrm{f}(P)$ than in $M_V = \mathrm{f}(P)$.