IS THE CHANDLER PERIOD STABLE?

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ABSTRACT. The problem on Chandler period is an unsolved one. Several authors suggested a hypothesis that the Chandler wobble is only one free period which slightly changes in time and is amplitude-dependent. In this paper we shall make the hypothesis more rigorous than that has been carried yet. A new deconvolution method for Fourier transform is suggested. Using this method the polar motion data are analysed. The analysis results are shown; the Chandler period is not stable and is indeed amplitude-dependent. The probable explanation for the amplitude-dependent of Chandler period is that, which might be caused by non-equilibrium response of the ocean.

1. INTRODUCTION

Many authors have estimated the values of Chandler period using different methods, an identical conclusion is that the values of Chandler period is quite different in different years. In order to explain this condition three distinct opinions are suggested;

1. There are two or more natural free periods near the Chandler period (Colombo and Shapiro 1968, Chao 1983). The difficulties met by this opinion are: 1. There is still no good geophysical theory that could explain why the Chandler wobble(CW) have two or more periods. 2. The analytical results for BIH and IERS data show CW has only one peak.

2. The Chandler period should be a stable value(Munk and MacDonald 1960, Wilson and Vicente 1981, Okobo 1982). The observed Changes are either due to the errors of observation or a consequence of a random excitation of unknown. This opinion has a big difficulty which makes the short Chandler period in 1920-1940's inexplicable.

3. There is only one natural free period which slightly changes in time, and is amplitude-dependent. This opinion was suggested by Melchior (1957), Carter(1981) and Vondrak(1989). Which could explain all conditions mentioned above, but the problems are whether this hypothesis is correct and why the Chandler period is amplitude-dependent. For the purpose first we must investigate the varying process of Chandler period and amplitude. This is quite difficult for traditional spectral analysis methods, because it is impossible to separate the Chandler peak from the annual peak if the length of polar motion data is shorter than eight years. So we suggest a new deconvolution method for Fourier transform. Here we express the polar coordinates $\bar{x}(t)$ and $\bar{y}(t)$ of instantaneous as,

I.I. Mueller and B. Kołaczek (eds.),
Developments in Astrometry and Their Impact on Astrophysics and Geodynamics, 303–308.

$$\bar{x}(t)=Xa+Xcc\cdot\cos\omega_c t+Xcs\cdot\sin\omega_c t+Xac\cdot\cos\omega_a t+Xas\cdot\sin\omega_a t$$
$$\bar{y}(t)=Ya+Ycc\cdot\cos\omega_c t+Ycs\cdot\sin\omega_c t+Yac\cdot\cos\omega_a t+Yas\cdot\sin\omega_a t \quad (1)$$

here (Xa,Ya) is the position of inertia pole and ω_c and ω_a are the Chandler and annual circle frequency. Xcc, Xac, Xcs, Xas, Ycc, Yac, Ycs and Yas are parameters of Chandler and annual wobble, By using deconvolution method these parameters can be determined.

2. DECONVOLUTION METHOD FOR FOURIER TRANSFORM

We may think the amplitude and phase of a series as variable quantities, so definite X(f,t) as the actual Fourier Transform at the moment t, and it will satisfy the relation;

$$x(t)=\int_{-\infty}^{\infty} X(f,t)\exp(2\pi jft)df \quad (2)$$

When analysing a practical series, only the time finite series could be used. It looks like truncating a unlimited series by a window function, so that,

$$x'(t)=w(t)x(t) \quad (3)$$

In order to guarantee the zero-phase shift, w(t) must be symmetrical with respect to the origin. Also x(t) could be symmetrical regarding the origin through transform of coordinate . Let X'(f) be the Fourier transform of x'(t), then we have,

$$X'(t)=\int_{-\infty}^{\infty} w(t)x(t)\exp(-2\pi jft)dt \quad (4)$$

Define X(f) as the average of X(f,t) about window function w(t), so

$$X(f)=\int_{-\infty}^{\infty} X(f,t)w(t)/\int_{-\infty}^{\infty} w(t)dt \quad (5)$$

From the definitions (2) and (5), it is easy to prove that the convolution relation will still be kept.

$$X'(f)=W(f)*X(f)=\int_{-\infty}^{\infty} X(f')W(f-f')df' \quad (6)$$

In above equation, X'(f) and W(f) are known quantities and "*" denotes convolution. For an ill-posed equation (6), it is impossible to solve X(f). But by using the special attributes of polar motion, we can compute X(f) directly. The attributes are;

1. The principal periodic components of polar motion are the CW and the annual polar motion, as to other periodic components they are very small.

2. We may think the form of the actual transform of CW and annual polar motion are known. If we express the movement of CW as,

$$x_c(t)=Mc\cdot\exp(-\alpha_c t)\cdot\cos(2\pi f_c t) \quad (7)$$

then according to the Fourier transform the actual spectral transform of Xc(f) will be the following form,

$$Xc(f)=\frac{M_c\alpha_c}{\alpha_c^2+4\pi^2(f-f_c)^2} \quad (8)$$

The actual spectral transform of Xa(f) will be the same form

$$Xa(f)=\frac{M_a\alpha_a}{\alpha_a^2+4\pi^2(f-f_a)^2} \qquad (9)$$

Hence we may express the actual Fourier transform of polar motion as follow,

$$X(f)=Xc(f)+Xa(f) \qquad (10)$$

Here $\alpha_c=\pi f_c/Qc$. Let $f_c=0.8432$/yr, $f_a=1$/yr, Qc=60 and Qa=40. So here the problem of solving X(f) will become a simple one, which is how to determine Mc and Ma. It is obvious that the four Fourier transforms of polar motion will be the same form. Because the side lobes of Hanning window are very small, we would think that Hanning window is band-limited in both time and frequency domain. It is given by

$$w(t)=\begin{cases} 0.5(1+\cos(\pi t/T_0)) & \text{when } |t|\leq T_0 \\ 0 & \text{when } |t|> T_0 \end{cases} \qquad (11)$$

We adopt $T_0=3$yrs, so the lower and upper limits of integral (4) are -3 and 3 respectively. Then we can know the width of the main lobe of W(f) is 0.54/yr. Let $f_1=0.30$/yr and $f_2=1.67$/yr, and consider the symmetry of the Fourier transform about the origin, we may have,

$$X'(f) = 2\int_{f_1}^{f_2} X(f')W(f-f')df \qquad (12)$$

and the spectral energy of CW and annual wobble only have a very little leak. Let $f_m=(f_c+f_a)/2$, and integrate the formula (12) from f_1 to f_m and from f_m to f_2 respectively, and substitute formula (8), (9) and (10) for X(f), then we got,

$$0.5\int_{f_1}^{f_m} X'(f)df= Mc\int_{f_1}^{f_m}\int_{f_1}^{f_2}\frac{\alpha_c W(f-f')df'df}{\alpha_c^2+4\pi^2(f-f_c)^2} + Ma\int_{f_1}^{f_m}\int_{f_1}^{f_2}\frac{\alpha_a W(f-f')df'df}{\alpha_a^2+4\pi^2(f-f_a)^2}$$

$$0.5\int_{f_m}^{f_2} X'(f)df= Mc\int_{f_m}^{f_2}\int_{f_1}^{f_2}\frac{\alpha_c W(f-f')df'df}{\alpha_c^2+4\pi^2(f-f_c)^2} + Ma\int_{f_m}^{f_2}\int_{f_1}^{f_2}\frac{\alpha_a W(f-f')df'df}{\alpha_a^2+4\pi^2(f-f_a)^2} \qquad (13)$$

At left side of above formula the integrated values can be obtained directly, we may express it as S_1 and S_2. And at right side the integral values can also be known, we denote them as Sc_1, Sa_1, Sc_2 and Sa_2. So we have,

$$Mc\cdot Sc_1+ Ma\cdot Sa_1= S_1$$
$$Mc\cdot Sc_2+ Ma\cdot Sa_2= S_2 \qquad (14)$$

From (14) it is very easy to solve Mc and Ma, then the actual Fourier transform X(f) can be obtained by (8), (9) and (10). But because the noise, the existence of other periodic spectral components and the error of peak shape, values of X(f) have a

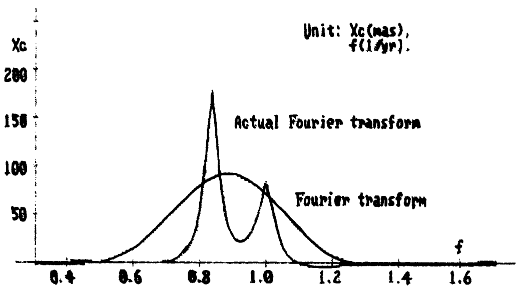

Figure 1, Shapes of Fourier transform and the actual one.

big error, so we only take it as initial values. Then using successive approximation method we can compute the precise values of X(f), For the four components of Fourier transform of polar motion we operate four times and get four group solutions. As the Chandler and annual peaks of the actual Fourier transform are very narrow, from formula (2) it is easy to prove that the values of parameters of Xcc,Xac(Xcs,Xas; Ycc,Yac; Ycs, Yas) equal to the areas under the Chandler and annual peaks respectively.

Because the weight of the taken value of Hanning window concentrates chiefly on the middle series, that is to say the actual Fourier transform chiefly reflects the average state of the middle series. For example, if we take the polar motion data from 1982.0 to 1988.0, then parameters of Xcc, Xac, Xcs, Xas, Ycc, Yac, Ycs and Yas which we obtain by using above deconvolution method will reflect chiefly the movement of pole at the epoch of 1985.0.

3. THE ANALYSIS FOR POLAR MOTION DATA

In order to research the variations of Chandler period and amplitude, the polar motion data of ILS(1900-1978), BIH(1962-1982) and IERS(1983-1992) are analysed. We first use sliding method year by year within every six years to find out subsets. For ILS data there are 20 samples in one year, so every subset has 120 samples. The sampling time interval for BIH and IERS are 5 days, so every subset has 438 samples. Let N denote the number of samples of subset, the position (Xa, Ya) of inertial pole for every subset are;

$$Xa=\sum_{i=1}^{N} x_i /N \qquad Ya=\sum_{i=1}^{N} y_i /N \qquad (15)$$

Subtracting (Xa, Ya) from every datum, we get the new subsets with zero average values.

Using deconvolution method for every subset, we can obtain the Chandler and annual parameters that corresponding to different epoch (For ILS, the epoch is from 1903.0 through 1975.0. For BIH and IERS the epoch is from 1965.0 through 1989.0). In order to examine the reliability and precision of this method, we suggest three different methods. 1. Taking different initial values of fc, Qc and Qa, we find out the results are similar. 2. Comparing $\bar{x}(t)$, $\bar{y}(t)$ that calculate from formula (1) with x(t), y(t) that given by polar motion data, we find they are very identical with each other and the error is only 12 mas (B. Gao 1990). 3. Making a spectral analysis for $x(t)-\bar{x}(t)$ and $y(t)-\bar{y}(t)$, we find that the Chandler and annual spectral components are vanished for any sub-interval. The Chandler amplitude Ac and phase φ are calculated by following formula,

$$\tan\varphi = 0.5(Xcs/Xcc-Ycc/Ycs)$$

$$Ac=(Xcc^2+Xcs^2+Ycc^2+Ycs^2)^{1/2} \qquad (16)$$

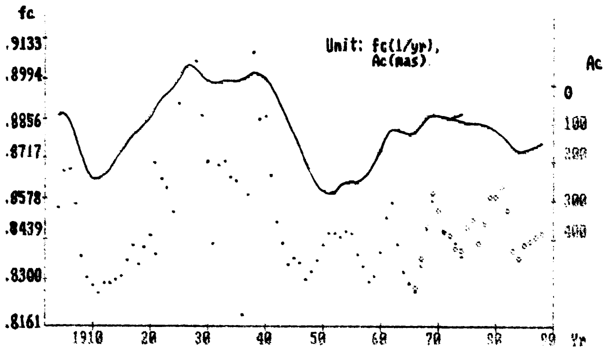

Figure 2. The variations of Ac and fc with time.

https://doi.org/10.1017/S0074180900173383 Published online by Cambridge University Press

Taking $f_{co}=1/1.185$yr, converting phase values into a same epoch (For ILS to 1902.0, for BIH and IERS, to the 1964.0). From the changes of phase we can get the variations of Chandler frequency. Let fc be the average frequency from epoch Y_1 to Y_2, then we have,

$$(f_c-f_{co})(Y_2-Y_1)=\varphi_2-\varphi_1 \quad (17)$$

Let $Y_2-Y_1=2$ yrs and Ac be the average of amplitudes in this 3 years, consequently values of Ac and fc could be obtained. (For ILS, from 1904.0 to 1974.0, for BIH and IERS, from 1966.0 to 1988.0).

In Figure 2, the solid line shows the variations of Ac with time. The points and little circles show the values of fc in different epochs that got from ILS or BIH(IERS) data respectively.

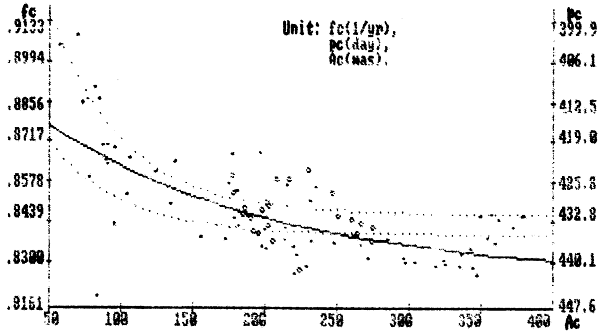

Figure 3. The variations of fc with Ac.

It can be seen that; the variations of Ac and fc with time are very coincident even in small parts. This fact denotes that Chandler frequency is indeed amplitude-dependent. From Figure 3 it is obvious that fc may have a non-linear exponential relation with Ac. Vondrak(1989) gave the formula as;

$$f_c = f_0 + d\cdot\exp(K\cdot A_c) \quad (18)$$

For the old polar motion data Vondark gave; $f_0=0.8644$/yr, d=0.1918/yr, K=-0.02; for the new technical data, $f_0=0.8394$/yr, d=0.0866/yr, K=-0.02. The two curves are shown in Figure 3 in dotted lines respectively. Using the least square method we get the statistics relation as;

$$f_c = 0.825+0.0722\cdot\exp(-0.0064\cdot A_c) \quad (19)$$

This result is shown in Figure 3 in the solid line, it may be seen that our result is more coincident with the observational results.

4. CONCLUSION

The result of this paper further demonstrates that the Chandler frequency is not stable and is amplitude-dependent. Lambeck(1980) suggested that the only explanation for this phenomenon are the unequilibrium pole tide. Smith(1977) estimate that for the elastic oceanless earth the Chandler period is about 404 days, for the earth which have ocean and the pole tide in equilibrium the Chandler period is 433 days. Dickman(1977) had investigated the pole tide in considerable detail. He find that the pole tide could be observed only in some region. So he conclude that the ocean tide may have a big departure from the equilibrium state, and this departure may have a big effect on the Chandler period. The surface of Earth is covered by ocean in the ratio 7/10. For the small Chandler amplitude it may only cause a very small tide, and when the Chandler amplitude become larger, the pole tide will become larger and larger in exponential relation. Naito(1977) reported that the enhancement of amplitude of pole tide in Honolulu is generally larger during periods of larger polar motion than that of small polar motion. As the amplitude of pole tide is extremely small(about 0.5 cm), so the observing and analysing of realistic pole tide is a difficult problem.

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