of several variables are included, but not Lebesgue integration or Fourier series. The treatment of the selected topics is careful and thorough, sometimes even laboured; the only omissions which the reviewer would question are those of Abel's and Dirichlet's tests for convergence and the classification of conditional stationary values.

In his preface to the second edition, the author remarks that experience has convinced him that the book is difficult enough for those using it as a first introduction to rigorous analysis. The reviewer feels that the book is unsuitable for a first introduction, since no attempt has been made to separate the easier topics from the harder ones or to use elementary arguments. It may prove a useful stimulus to an able student, especially as the text includes frequent summaries of the difficulties inherent at particular stages.

P. HEYWOOD

KOLMOGOROV, A. N., AND FOMIN, S. V., *Elements of the Theory of Functions and Functional Analysis*, vol. 2: *Measure, the Lebesgue Integral, Hilbert Space*, translated by H. KAMEL AND H. KOMM (Graylock Press, Rochester, N.Y., 1961), ix+128 pp., 34s.

Also published as *Measure, Lebesgue Integrals, and Hilbert Space*, translated by N. A. Brunswick and A. Jeffrey (Academic Press, New York and London, 1961), xii+147 pp., $4.

The Graylock Press set a high standard when they published the English translation by L. Boron of the first volume of this course. The standard is fully maintained in their version of the second volume. The other translation, published almost simultaneously by the Academic Press, is not a serious rival: it is in fact a remarkably inept piece of work, but fortunately there is no need to consider its faults in detail, most of them being absent from the Graylock version.

Although this second volume can be read independently of the first, the Graylock version follows the Russian original in having the chapters and sections numbered consecutively: Volume 2 consists of Chapters V-IX. Chapter V is an excellent introduction to measure theory. It starts with the construction of Lebesgue outer measure for bounded plane sets. The fundamental properties of measurable sets are then discussed in a way that illuminates the concept of measurablity very clearly. There follows a discussion, in an abstract setting, of the extension of measures from semi-rings to rings and to sigma-rings (a " measure " being defined here as a non-negative, finite-valued, additive function on a semi-ring of sets). The connexion between the full extension of a measure and the completion of an associated metric space is pointed out.

Measurable functions are discussed in Chapter VI in preparation for Chapter VII in which the Lebesgue integral is introduced. Integration is defined in the first place with respect to an arbitrary (finite-valued) measure on a sigma-algebra, and the usual theorems are proved. The Lebesgue integral over a finite interval is considered as a special case and is compared with the Riemann integral. There is then a discussion of product measures, leading to a general version of Fubini's theorem. Chapter VII ends with a brief reference to the Radon-Nikodym theorem.

Chapter VIII is concerned with the space L_2 of real-valued square-integrable functions with respect to a finite measure. The fundamental properties of this space are established, including completeness with respect to the norm, and separability when the measure satisfies a countability condition (equivalent to separability of the metric space associated with the measure). Orthogonal systems of functions are then considered, with the Riesz-Fischer theorem and the mean-square theory of Fourier series. (" Fischer " is spelt " Fisher " in the Graylock edition.) At the end of the chapter, the isomorphic isometry between l_2 and a separable L_2 is established, in preparation for Chapter IX.

In Chapter IX, real Hilbert space is defined axiomatically, and after some discussion of subspaces and of bilinear functionals, the concept of a self-adjoint operator is introduced. The representation of a completely continuous self-adjoint operator in terms of its eigenvalues and eigenvectors is established, and, finally, the usefulness of this is illustrated by a brief consideration of integral equations with symmetric L_2 kernels.

It seems odd that there is no mention of complex Hilbert space (even though there is a reference, at the end of Chapter VIII, to quantum theory); but as with Volume 1, it is surprising to find so much material in so few pages, without undue compression. The authors have again achieved a nice blend of the abstract and the concrete. The Graylock translation benefits from the inclusion of a number of exercises, prepared by H. Kamel, which are distributed through the book and usefully amplify the text. An appendix, translated from the Russian edition of this volume, corrects some minor errors in Volume 1.

Here, in the Graylock translation, is a very good book: an excellent companion to the first volume, and also to Kolmogorov's well-known little book on probability (of which an up-to-date English edition would now be welcome).

J. D. WESTON

AHLFORS, L. V., AND OTHERS, *Analytic Functions* (Princeton University Press, 1960), vii+197 pp., 40s.

This book contains the principal addresses delivered at a Conference on Analytic Functions held at the Institute for Advanced Study, Princeton, in September, 1957. As one would expect, the papers are of a specialised character and they are as follows: *On differentiable mappings* by R. Nevanlinna; *Analysis in non-compact complex spaces* by H. Behnke and H. Grauert; *The complex analytic structure of the space of closed Riemann surfaces* by L. V. Ahlfors; *Some remarks on perturbation of structure* by D. C. Spencer; *Quasiconformal mappings and Teichmüller's theorem* by L. Bers; *On compact analytic surfaces* by K. Kodaira; *The conformal mapping of Riemann surfaces* by M. Heins and *On certain coefficients of univalent functions* by J. A. Jenkins.

The value placed by the reader on any particular article will naturally depend on his knowledge and predilections but, in the opinion of the reviewer, the book is well worth possessing if only for the article by Behnke and Grauert. This, when taken in conjunction with a lecture given by Behnke at the Amsterdam congress (*Funktionentheorie auf Komplexer Mannigfaltigkeiten*, Proceedings of the International Congress of Mathematicians 1954 (Amsterdam), 3 pp. 45-57) provides an excellent survey of the work done on complex manifolds by H. Cartan, Serre, Stein and Ahlfors since 1950. The value of this paper is enhanced, too, by four pages of references at the end.

The printing and layout of the book are first class.

D. MARTIN

AHLFORS, L. V., AND SARIO, L., *Riemann Surfaces* (Princeton University Press, 1960), xi+382 pp., 80s.

With the appearance of this book a comprehensive and modern treatment of the subject in English has become available for the first time. The first chapter gives a thorough and extensive treatment of the topology of surfaces. Particular attention is paid to bordered surfaces, open polyhedra (triangulated surfaces) and to compactification. The terminology is in a few instances non-standard. Thus an unlimited covering surface is called regular and a regular covering surface is called normal; the latter change of usage certainly seems preferable. Riemann surfaces appear first in the second chapter. The problem of constructing harmonic functions with given singularities on an open Riemann surface is solved in the third chapter by means of Sario's