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ALONZO CHURCH HARTLEY ROGERS, JR ARTHUR N. PRIOR ALICE A. LAZEROWITZ JULIA ROBINSON

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ERRATA

VOLUME 20

Page 197, line 18. For « H. S. CALDWELL », read « S. H. CALDWELL ». Page 321, column 2, line 10. For « H. S. », read « S. H. ».

VOLUME 22

Page 348, line 17. Delete the negation sign and for = 0, read = 1.

VOLUME 23

Page 134, line 5. For e_i read \bar{e} . Page 314, line 6 from the bottom. For $R_{e_i}^a$ read $R_{z_i}^a$.

VOLUME 24

Page ii, line 5. For « Maart », read « March ».

Page 190, line 17. For «Łuszczeweska», read «Łuszczewska».

Page 287, the unnumbered footnote. For 1959, read 1958.

Page 347, column, next-to-last line. For « Semantics », read « semantics ». Page 353. The first line at the top of column 2 should be indented.

VOLUME 25

Page ii, line 5. For õõ, read 9.

Page 158, line 7 from the bottom. For \sim , read \sim .

Page 179, line 23. For XIII, read XXIII.

Page 182, line 14. For \sim , read \sim .

Page 250, line 9. For « "Función », read « "Función" ».

Page 265, line 26. For « f. », read « ff. ».

Page 265, last line. For «f. », read «ff. ».

Page 266, line 17. For «f. », read «ff. ».

Page 269, line 1. Add a hyphen at the end of the line.

Page 284. Insert space between lines 7 and 6 from the bottom.

Page 297, line 4. For « consequences », read « conséquences ».

Page 330, line 6 from the bottom. For the first P_2 read P_0 .

Page 334, line 2. Delete parentheses, twice.

Page 354, line 24. For « systeme », read « système ».

Page 358, line 10 from the bottom. For], read [.

Page 360, line 3. For (x/p)(y/q), read $\binom{x}{p}\binom{y}{q}$.

Page 361, line 22. For XXV 000(0), read XXV 356(1).

Page 361, line 27. For a < b, read a < A b.

Page 362, line 24. For «HILLARY », read «HILARY ».

ERRATA

Page 370, last line. For « for the synthesis », read « in synthesis ».

Page 376, line 31 from the bottom. For « pure », read « "pure" ».

Page 377, line 19. For « ZIEMBINSKI », read « ZIEMBIŃSKI ».

Page 380, line 9. For XXV 000, read XXV 299(1).

Page 380, line 7 from the bottom. For « PAWLOWSKI », read « PAWŁOWSKI ».

Page 381, line 3. For « Bochenski », read « Bocheński ».

Page 381. Insert space between lines 6 and 7.

Page 381, line 10. For XXV 271, read XXV 269.

Page 383, line 15. For « Ferruccio Rossi-Landi », read « FERRUCCIO ROSSI-LANDI ».

VOLUME 27

Page 16, line 11 and line 16. For \vdash , read |.