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ON t-SPREADS OF $PG((s+1)(t+1)-1,q)$

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In this thesis the theory of 1-spreads of $PG(3,q)$ is generalised to a theory of t-spreads of $PG((s+1)(t+1)-1,q)$. There is a well developed theory for t-spreads of $PG(2t+1,q)$, but so far there are limited results in other cases. This thesis extends much of the existing theory to the general case of t-spreads of $PG((s+1)(t+1)-1,q)$.

After a short Introduction containing a literature review, Chapter One of the thesis gives a brief account of the concepts involved.

In Chapter Two the theory of t-spreads of $PG(2t+1,q)$ is revised, setting the scene for the generalisation to come in Chapter Three. Most of the work in this Chapter is well known, but in order to facilitate the later generalisation, some of the presentation is different from the original. For example the concept of regularity is presented in the light of the connection between a regulus of $PG(2t+1,q)$ and the classical Segre Variety which is the product of a line and a t-dimensional space of $PG(2t+1,q)$. In addition, a new and straightforward construction is given for a spread set (originally defined in [**3**]) corresponding to a t-spread of $PG(2t+1,q)$. This new construction uses the space $\mathcal{S}_m(\mathcal{M}_n(GF(q)))$ introduced in [**6**].

Chapter Three gives results for t-spreads of $PG((s+1)(t+1)-1,q)$ suggested by the theory studied in Chapter Two. A generalised t-spread set of matrices for certain of these t-spreads is found and, in addition, the new construction of a spread set discussed in Chapter Two generalises naturally to give a new but related entity, to be called a projective t-spread set. This new entity is more general because any t-spread of $PG((s+1)(t+1)-1,q)$ admits a projective t-spread set, but not every t-spread admits a t-spread set. Regularity of a t-spread of $PG((s+1)(t+1)-1,q)$ is explored using the properties of the classical Segre Variety. Different subvarieties produce different reguli of a t-spread, and therefore corresponding different types of regularity. It is shown, however, that all these types are equivalent and coincide with the usual notion of regularity in the few cases where a definition has previously been given. The approach developed in this Chapter leads to the construction of an indicator

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set for a t-spread of $PG((s+1)(t+1)-1,q)$, extending the work of [**4**]. It also yields a representation for regular t-spreads of $PG((s+1)(t+1)-1,q)$, generalising that due to [**2**] for 1-spreads of $PG(3,q)$. Examples are given to illustrate the ideas presented.

The next Chapter considers certain partial t-spreads, and in particular those called k-sets of t-dimensional subspaces. Some new concepts and results are given. The definition of k-sets is then extended to (k,n)-sets of $PG(3t+2,q)$, and connections with work already done by [**1**] and [**5**] (in the case of $s=2$) are explored. A maximal (k,n)-set is defined, and its size is determined. A condition guaranteeing that such a set arises from the construction of [**6**] is found, and applied to maximal $(k,3)$-sets of $PG(5,3^h)$ and maximal (k,n)-sets of $PG(3t+2,2)$ when $t>1$. An example of a 4-set ((4,2)-set) of lines of $PG(5,2)$ is given, which does not arise from the construction due to [**6**]. This set is contained in a spread which contains no regulus.

A short conclusion and suggestions for further research appear in Chapter Five.

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