CORRESPONDENCE.

REPLY TO "A CHALLENGE".

To the Editor of the *Mathematical Gazette.*

DEAR EDITOR,—This reply to "Wrangler's" challenge in the current number of the *Gazette* is sent off post-haste, as I believe that the acceptors of such challenges strove to be first in the field.

To find p the probability that a man of age a reaches age b, multiply $(b-a)$ by the mean of the reciprocals of the expectations of life from a to b, add the logarithm of the expectation at b and subtract the logarithm of the expectation at a, thus obtaining the logarithm of $1/p$ (natural logarithms are to be used or the product should be multiplied by μ).

If $f(x)$ is the chance that a man of age a reaches the age $a+x$, and $\phi(x)$ the expectation of life at that age,

$$\phi(x)=\frac{1}{f(x)}\int_x^\infty f(x)dx.$$

or if $f(x)$ is the derivative of a function $F(x)$,

$$\frac{1}{\phi(x)}=\frac{f(x)}{F(\infty)-F(x)}, \qquad \text{(i)}$$

and, integrating, we have

$$\int_0^x \frac{dx}{\phi(x)}=-\log\frac{F(\infty)-F(x)}{c},$$

where c is a constant. Thus making use of (i),

$$\int_0^x \frac{dx}{\phi(x)}=-\log\frac{f(x)\,.\,\phi(x)}{c}.$$

As $f(0)=1$, we have $c=\phi(0)$ and

$$\log f(x)=\log\phi(0)-\log\phi(x)-\int_0^x\frac{dx}{\phi(x)}.$$

For example, if the expectations of life at yearly intervals from 50 to 60 are

$$20{\cdot}3,\ 19{\cdot}5,\ 18{\cdot}9,\ 18{\cdot}2,\ 17{\cdot}6,\ 16{\cdot}9,\ 16{\cdot}2,\ 15{\cdot}6,\ 15{\cdot}0,\ 14{\cdot}4,\ 13{\cdot}8,$$

then

$$\log\phi(0)-\log\phi(x)=\log 20{\cdot}3-\log 13{\cdot}8=\cdot 7080-\cdot 3221,$$

while for the integral Simson's rule gives $\cdot 5979$, so that $\log p=\bar{1}{\cdot}7880$ and $p=\frac{4}{5}$ approximately. Consequently the chance that two men of 50 reach 60 is about $\frac{16}{25}$, that neither do so about $\frac{1}{25}$, and that one only does so about $\frac{8}{25}$.

C. H. HARDINGHAM.

July 3, 1932.

IS THE EARTH ROUND OR FLAT?

To the Editor of the *Mathematical Gazette.*

SIR,—Has the above question any meaning? If it is not possible for human beings to prove that the Earth is either round or flat, surely the question becomes meaningless. I give below reasons for thinking that we cannot answer the question one way or the other.

Let us take a system of three unit vectors, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$, at right angles to each other and use spherical polar coordinates, viz. ϕ for the co-latitude measured from $\mathbf{e}_3$, θ for the meridian angle measured from $\mathbf{e}_1$, r for the radius vector.

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The differential vector $d\mathbf{r}$ of Euclidean 3-space using these coordinates is

$$(1)\quad d\mathbf{r}=r(\cos\phi\cos\theta\ \mathbf{e}_1+\cos\phi\sin\theta\,.\,\mathbf{e}_2-\sin\phi\,.\,\mathbf{e}_3)d\phi$$
$$+r(-\sin\phi\sin\theta\,.\,\mathbf{e}_1+\sin\phi\cos\theta\,.\,\mathbf{e}_2)d\theta$$
$$+(\sin\phi\cos\theta\ \mathbf{e}_1+\sin\phi\sin\theta\,.\,\mathbf{e}_2+\cos\phi\,.\,\mathbf{e}_3)dr.$$

Squaring (1) we get for the square of the line element (or ground form)

$$(2)\quad ds^2=(d\mathbf{r})^2$$
$$=r^2\,d\phi^2+r^2\sin^2\phi\,d\theta^2+dr^2.$$

Putting $r=a$ in (1) we get for the differential vector of a sphere of radius a, in 3-space,

$$(3)\quad d\mathbf{r}=a(\cos\phi\cos\theta\,.\,\mathbf{e}_1+\cos\phi\sin\theta\,.\,\mathbf{e}_2-\sin\phi\,.\,\mathbf{e}_3)\,d\phi$$
$$+a(-\sin\phi\sin\theta\,.\,\mathbf{e}_1+\sin\phi\cos\theta\ \mathbf{e}_2)d\theta,$$

with ground form

$$(4)\quad ds^2=a^2\,d\phi^2+a^2\sin^2\phi\,d\theta^2.$$

Next consider the non-Euclidean 3-space whose differential vector is, with ϕ, θ and r as parameters,

$$(5)\quad \mathbf{d}\sigma=r\,.\,\mathbf{e}_1\,d\phi+r\sin\phi\ \ \mathbf{e}_2\,d\theta+\mathbf{e}_3\,.\,dr.$$

Squaring it, we get its ground form :

$$(6)\quad ds^2=r^2\,d\phi^2+r^2\sin^2\phi\,d\theta^2+dr^2.$$

Consider the Riemannian 2-pole elliptic plane with constant $\frac{1}{a}$, lying in this non-Euclidean 3-space. It is obtained by putting $r=a$ in (5). Its differential vector is

$$(7)\quad \mathbf{d}\sigma=a\,.\,\mathbf{e}_1\,d\phi+a\sin\phi\ \ \mathbf{e}_2\,d\theta.$$

Its ground form is

$$(8)\quad ds^2=(\mathbf{d}\sigma)^2$$
$$=a^2\,d\phi^2+a^2\sin^2\phi\,d\theta^2.$$

By comparing their ground forms (2) and (6), we see that the Riemannian 3-space is " applicable " to Euclidean 3-space.

By comparing (4) and (8) we see that the Riemannian plane is " applicable " to the Euclidean sphere. Let us now suppose that two persons E and N move about the Earth in company with each other. Any measurements they may make will be the same, e.g. if they measure the sides and angles of a geodesic triangle, they will get the same relations connecting the sides and angles as given in spherical trigonometry. E chooses to interpret such measurements as proving that the surface is a sphere of radius a, lying in Euclidean 3-space. N chooses to interpret them as proving that the surface is the above-mentioned Riemann plane lying in the non-Euclidean 3-space (5). The geometries of these surfaces and spaces are the same. Therefore no possible experiment can decide between them.

The proofs given in books on geography and astronomy beg the question by assuming our 3-space Euclidean. A corresponding argument applies to the case of a spheroid. A. W. KING.

Imperial College of Science and Technology,
10th June, 1932.

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897. (Madame du Châtelet) was intellectual and sensuous—an agreeable blend. She liked books, diamonds, algebra, petticoats, and physics. In this she was like Voltaire. . . —A. Maurois, *Voltaire*, p. 53. [Per Mr. E. H. Lockwood.]