PART II

ARITHMETIC AND AESTHETICS

Part I addressed counting as a means of interrogating the relationship between poetic content (the 'stuff' that a poem contains) and the space that is needed to express it. There I demonstrated that counting had an important role to play in poetic criticism of the Hellenistic period and that later poets were aware of this, incorporating and developing counting criticism in their own programmatic poetic statements. In early mathematical education, after counting there came more complex operations: multiplication, but also calculations that in modern mathematical notation would be written as equations and solved algebraically. These mathematical procedures today form part of arithmetic. The focus of Part II is thus on how the 'stuff' of poetry is expressed and arranged so as to require an arithmetical interpretation and solution.

In antiquity, the domain of modern arithmetic was divided into the λογιστικὴ τέχνη ('the art of calculating') and the ἀριθμητικὴ τέχνη ('the art of number').[1] The former dealt with tangible objects and their manipulation; the latter dealt with the theory of numbers per se. The clearest source for the nature of 'logistic' is a *scholium* to Plato's *Charmides* (165e), which is worth quoting at length.[2]

λογιστική ἐστι θεωρία τῶν ἀριθμητῶν, οὐχὶ δὲ τῶν ἀριθμῶν, μεταχειριστική, οὐ τὸν ὄντως ἀριθμὸν λαμβάνουσα, ὑποτιθεμένη τὸ μὲν ἓν ὡς μονάδα, τὸ δὲ ἀριθμητὸν ὡς ἀριθμόν, οἷον τὰ τρία τριάδα εἶναι καὶ τὰ δέκα δεκάδα· ἐφ' ὧν ἐπάγει τὰ κατὰ ἀριθμητικὴν θεωρήματα. θεωρεῖ οὖν τοῦ<το> μὲν τὸ κληθὲν ὑπ' Ἀρχιμήδους βοϊκὸν πρόβλημα, τοῦτο δὲ μηλίτας καὶ φιαλίτας ἀριθμούς, τοὺς μὲν ἐπὶ φιαλῶν, τοὺς δ' ἐπὶ ποίμνης, καὶ ἐπ' ἄλλων δὲ γενῶν τὰ πλήθη τῶν αἰσθητῶν σωμάτων σκοποῦσα, ὡς περὶ τελείων ἀποφαίνεται. ὕλη δὲ αὐτῆς πάντα τὰ ἀριθμητά· (*Scholium* on *Charm.* 165e Cufalo)

[1] For further on logistic, see Heath (1921) I, 14–15; Klein (1968) 6–8; Taub (2017) 44–5.
[2] The *scholium* is late, but it evidently draws from Hero's first-century CE *Definitions* (135.5); see Heath (1921) II, 13–15 and Cufalo (2007) 173. However, Plato in the *Laws* (819b) provides further evidence for arithmetical handling and manipulation of objects (see the introduction to Chapter 4). He says this goes back to the Egyptians (cf. 819a), as does the *scholium* in the remainder of the passage, not quoted.

Logistic is the science which deals with numbered things, not numbers. It does not take number in its essence, but it presupposes 1 as a unit and the numbered object as a number, so that 3 is taken to be a triad and 10 to be a decad. To these it applies the theorems of arithmetic. It investigates on the one hand what is called by Archimedes the cattle problem and on the other hand *mêlites* and *phialites* numbers, the latter concerning bowls, the former concerning flocks of sheep.[3] It investigates the number of sensible bodies in other kinds of things too and treats them as absolutes. Its subject is everything that is numbered.[4]

The priority of λογιστική is to treat real world objects in a numerical manner, rather than to think abstractly about numbers. Numbered bowls and sheep, that is, are treated *as* these objects and are thus indivisible units: one is not allowed to chop up the sheep.[5] Part II tackles poetry that incorporates such arithmetical challenges where the configuration of the poetic content would have been solved by logistic and treated as such rather than simply a series of abstract numbers.

A prime example of setting arithmetic in poetry is a scene from the *Contest of Homer and Hesiod* that I briefly discussed in the Introduction, which can be traced back to the fifth century BCE.[6] Homer and Hesiod meet and compete at the funeral games held for Amphidamas, the king of Euboea. There, they competitively exchange verses from both of their poems, as well as verses not otherwise known to have been composed by either poet, but certainly based on them. They alternate between posing challenges of wisdom to each other (e.g. 'what is the best thing for mortals?')

[3] Heath (1921) I, 14 wants to correct 'flock of sheep' to 'apples'. As I suggest below (Chapter 4, Section 4), however, there is good reason to think that there was no consensus regarding the interpretation of μηλίτες ἀριθμοί and that indeed later poets will be seen to play with the ambiguity.

[4] Translation adapted from Heath (1921) I, 14.

[5] For another clear distinction between arithmetic and logistic, in similar language, see Proclus *In Euc*. 39.7–40.9.

[6] The text in the manuscript tradition is a Hadrianic recension, but the tradition and even large portions of the text date back to the Hellenistic period and quite probably to the *Musaion* of Alcidamas, active in the second half of the fifth century. For a clear study of the tradition see Bassino (2019) 1–82. Alcidamas' influence on the tradition of the contest is undoubtedly strong, but it does predate him. See Richardson (1981), *pace* West (1967). For the likelihood that Aristophanes' *Frogs* knows the *Contest*, see Rosen (2004). Thanks to the papyrus *PPetrie* I 25, a fair proportion of Alcidamas' work prior to the Hadrianic recension can be securely reconstructed. This passage is not definitively connected to Alcidamas, but since it follows only a few lines after the previous exchange that *is* preserved in the papyrus and seems to be part of a wider run of questions which challenge Homer's ability from a range of angles, I think it is probable.

and responding to each other's individual sentences. Following on from Hesiod's presenting of 'ambiguous propositions' (τὰς ἀμφιβόλους γνώμας, *Contest* 102–3 Bassino) to Homer, Hesiod presents him with a mathematical challenge.

πρὸς πάντα δὲ τοῦ Ὁμήρου καλῶς ἀπαντήσαντος πάλιν φησὶν ὁ Ἡσίοδος·

> τοῦτό τι δή μοι μοῦνον ἐειρομένῳ κατάλεξον,
> πόσσοι ἅμ' Ἀτρείδῃσιν ἐς Ἴλιον ἦλθον Ἀχαιοί;

ὁ δὲ διὰ λογιστικοῦ προβλήματος ἀποκρίνεται οὕτως·

> πεντήκοντ' ἦσαν πυρὸς ἐσχάραι, ἐν δὲ ἑκάστῃ
> πεντήκοντ' ὀβελοί, περὶ δὲ κρέα πεντήκοντα·
> τρὶς δὲ τριηκόσιοι περὶ ἓν κρέας ἦσαν Ἀχαιοί.

τοῦτο δὲ εὑρίσκεται πλῆθος ἄπιστον· τῶν γὰρ ἐσχαρῶν οὐσῶν πεντήκοντα ὀβελίσκοι γίνονται πεντακόσιοι καὶ χιλιάδες β´, κρεῶν δὲ δεκαδύο μυριάδες ͵ε †ϋν†[7]

(*Contest of Homer and Hesiod* 138–48 Bassino)
(50 × 50 × 900 = 2,250,000)[8]

Since Homer had replied well to all these things [*sc*. challenges], Hesiod said again:

'Detail to me only this which I ask: how many Achaeans went to Ilium with the Atreids?'

He answered with a logistic problem as follows:

'There were fifty hearths of fire, in each were fifty spits and around each were fifty pieces of meat: three times three hundred Achaeans were around one piece of meat.'

But this results in an unbelievable number; for if there are fifty hearths then there are 2,500 spits and 125,000 pieces of meat. †

Homer's outline of the spits in each fire and the men around each piece of meat is specifically designated by its author as a response

[7] Kwapisz (2020b) proposes, in contrast to the recent edition of Bassino (2019), that the original form of Homer's reply is probably that preserved in *AP* 14.147; see Chapter 4, Section 4. It is a convincing suggestion that deserves serious consideration. Since I am quoting more than Homer's reply here, I have chosen to keep to Bassino's edition for consistency. In any case, the difference between the two versions does not affect the present discussion.

[8] I follow Kwapisz (2020b) 193 in understanding the verse to mean, though not unambiguously, that each hearth has 50 spits and so 50 pieces of meat, rather than 50 pieces of meat on each spit.

to Hesiod in the form of a 'logistic problem' (λογιστικοῦ προβλήματος, *Contest* 142 Bassino).[9] Homer is effectively made to treat the Greek soldiers as those units which can be manipulated and arranged in a number of ways, but must stay as – and fundamentally are – indivisible bodies. From an early point in time poets were well able to adapt their abilities to versifying logistic challenges.

But there is also literary sophistication to this exchange of verses. Hesiod asks a question which cannot but recall Homer's Invocation prior to the Catalogue of Ships. The first line is formulaic, and the verb κατάλεξον functions as something of a technical term for recalling and cataloguing information.[10] The second line is calqued from verses in which Homer is appealing directly to the Muses for knowledge. The first phrase (πόσσοι ἅμ' Ἀτρείδῃσιν) reworks the relatively rare ἅμ' Ἀτρείδῃσιν used by Homer during the Catalogue when requesting to know in addition who were the best of the Achaeans 'who followed the Atreids' (οἳ ἅμ' Ἀτρεΐδῃσιν ἕποντο, *Il.* 2.762), and the final words echo the conclusion of Homer's Invocation (ὅσοι ὑπὸ Ἴλιον ἦλθον, *Il.* 2.492), where he signalled his dependence on the Muses in handling the mass of tradition (488–92).[11] Hesiod uses Homer's own poetry to question the extent to which his claim to be supported by the Muses is true when it comes to numerical information.

Homer's reply, however, differs from the Iliadic Invocation. These lines of the *Contest* appear to have been borrowed from the conclusion to *Iliad* 8 where, in a similar fashion to the Invocation in *Iliad* 2, the poet juxtaposes a simile with a numerical approach to the mass of warriors, this time the mass of Trojans. He first describes the Trojan camp's many fires

9 While without parallel – Bassino (2019) 157 – it is a perfectly understandable phrase, especially in light of the later prose discussions of logistic.

10 LSJ *s.v.* A.I.3. It also seems to have an affiliation with counting, cf. e.g. *Od.* 16.235, where Odysseus commands Telemachus: ἀλλ' ἄγε μοι μνηστῆρας ἀριθμήσας κατάλεξον ('but come recount and number for me the Suitors').

11 ἅμ' Ἀτρείδῃσιν appears at *Od.* 17.103 and 19.182 in the same *sedes* in Hesiod's verse, whereas at *Il.* 2.762 it occurs in a different *sedes*. However, Hesiod's second verse also resembles Odysseus' words to Thersites earlier in *Iliad* 2, that there is no man worse than him 'among as many as those who went with the Atreids to Ilium' (ὅσσοι ἅμ' Ἀτρεΐδῃς ὑπὸ Ἴλιον ἦλθον, *Il.* 2.248). I therefore see this a deliberate connection to *Iliad* 2.

'[as when] the infinite air is broken open in the heavens and all the stars are seen' (*Il.* 8.558–9) and then adds further qualification, 'a thousand fires burned on the plain and beside each sat fifty in the brightness of the burning fire' (*Il.* 8.562–3). The *Contest* therefore does not echo a Homeric catalogue here, but a Homeric calculation.[12] The Homer of the *Contest* in this sense is even more calculating than the poet of the *Iliad.* He does not allow room for addition at all, whereas in the Catalogue it is necessary to add together the troops under each leader in order to reach a sum for the entire Achaean contingent, in the manner that Thucydides had theorised. If Hesiod's echoing of invocatory language intends to test the Muses' support of Homer, then Homer's reply is strategic. He does not offer a catalogue, which might display the extent of the Muses' knowledge through the poet, but rather offers a multiplication which explains the number of the host in only a few lines. This Homer responds to with a display of his own – and not the Muses' – calculating capacity.

Important to observe here is that the poet of these new verses has not adapted any old Homeric verses or provided a calculation with any chance objects, but instead has excavated the *Iliad* itself for a scene and for a set of objects which might easily be adapted to arithmetic and form an equally knotty challenge for Hesiod in turn. What is more, the coincidence of the subject matter and the arithmetic is turned to reflect again on Homer's capacity as a poet but also – since it is a 'logistic problem' left unsolved and addressed to Hesiod in response – to challenge the literary and arithmetic capacities of the reader. It is this practice of seeking for ways to integrate arithmetic into poetry, and the particular configuration of the poet and the reader which results, that is my focus in the second half of the book. My overarching claim in Part II is that the objects – the 'stuff' – that are arranged into ratios in other arithmetical poems are not arbitrary either, nor are the language and imagery used to describe them. That is to say, the way poets chose to verbally encode arithmetical challenges demonstrates an awareness that they are composing poems as much as calculations,

[12] See also Agamemnon's calculation of the opposing forces at *Il.* 2.119–28, discussed in the Introduction p. 3.

but also attests to their interrogation of how that very arithmetic shapes the poetic form. Whether consciously or not, these poets articulate a literary aesthetic appropriate to arithmetic.

Beyond the versified logistic problem spoken by Homer in the *Contest*, there survive from antiquity two further cases of calculations in poetry, and the following chapters will be devoted to understanding the particular aesthetics in which the poets wrapped their arithmetic. They are represented in the *scholium* to *Charmides*, which distinguishes between 'what is called by Archimedes the cattle problem' and '*mêlites* and *phialites* numbers'. Part II dedicates a chapter to each of these types in poetry. Quite what the difference is between the two kinds of logistic is unclear; the syntax of the *scholium* (μέν . . . δέ) could be either conjunctive or disjunctive. The only observable distinction in the arithmetic over the course of my discussion will be the difficulty or solvability of the problems, though this is not to make a claim about what the differences (or indeed similarities) were thought to be in antiquity.

In Chapter 3 I address the elegiac poem called the *Cattle Problem* attributed to Archimedes, which I take to be synonymous with the problem referred to in the *Charmides scholium*. The poem outlines the ratios of the Cattle of the Sun that reside on Sicily, producing a logistic problem the solution to which was only recently resolved and was most probably irresolvable in antiquity. It was supposedly addressed to Eratosthenes, the head of the Alexandrian Library. Whereas it has long been of interest to historians of mathematics, my aim in the chapter is to analyse it as a poetic work. What will emerge is a composition that knowingly intertwines poetry and arithmetic: the language and sophisticated allusions to earlier poetry set Archimedes on a par with more well-known Hellenistic poets. Particularly significant will be Archimedes' positioning of the *Cattle Problem* within literary and generic traditions both through extended reference to Homer's Catalogue of Ships and counting of the troops in *Iliad* 2 and also by modelling his count on the oracular practice of claiming possession of land through calculating the amount of agricultural produce or livestock in a given location. These two aspects will prove to be especially pointed given that it was sent to

Eratosthenes, who was a geographer as well as a mathematician and poet, and who in his geographical treatise had stripped Sicily of its Homeric past. Ultimately, the aesthetics of the *Cattle Problem* will be seen to be as much about testing the notion that one can combine mathematics and poetry as they are about challenging the idea that mathematics is a sophisticated means of gaining geographical knowledge.

In the case of the *Cattle Problem*, sufficient information exists about its context to develop a historically informed reading of its aesthetics. Yet over forty further poems survive that versify logistic problems, which are much shorter and lack such a specific context. These are the so-called arithmetic epigrams preserved in Book 14 of the *Palatine Anthology*, which I will be calling arithmetical poems since they are not all epigrammatic in either metrical or generic form. They seem to reflect in their arithmetic as well as subject matter the '*mêlites* and *phialites* numbers' mentioned in the *Charmides scholium*.[13] My intention in Chapter 4 is to develop a deeper understanding of the genesis of these poems and their aesthetic, both on the level of individual poems and as a collection. I detail the various generic affiliations of the poems and their strategies of expanding on numerical aspects in pre-existing genres. I go on to propose that the fact these poems demand input on the part of the reader in order to become interpretable, as well as the striking continuity of generic forms, locates these poems as a product of Late Antiquity. Drawing on a range of comparative works, I outline how these arithmetical poems match the period's balancing act of literary conservatism and formal experimentation. I then consider the organisation of the arithmetical poems as they were collected by a certain Metrodorus at some point in Late Antiquity and then as they were incorporated into the *Palatine Anthology*. It will become clear that in both cases the editors are alive to the particular nature of the compositions as arithmetical poetry and that this affects the orderings and

[13] Taub (2017) 40–1 connects the logistic described in the *scholium* with the passage from Plato's *Laws* (819a–c) that describes mathematical education through playing with apples, crowns or bowls. Kwapisz (2020a) 459–60 makes the connection stronger, I think, with his observation that at *AP* 14.48–50 three arithmetical poems offer problems with the same objects, in the same order.

juxtapositions of the poems and the themes that they subsequently draw out. Part II demonstrates, in other words, that over the course of more than a millennium audiences and authors alike were attuned to a whole range of images and strategies for aestheticising arithmetic.

I must here also offer a caveat regarding notation. I have presented the accompanying solutions to the poems algebraically. This is a guide for the modern reader (just as I provided the isopsephic counts in the case of Leonides' epigrams) and should not be understood to be a reconstruction of how the problems were solved in antiquity. The algebraic method does not align with ancient arithmetic practice. Moreover, in the case of the *Cattle Problem* and some of the arithmetical poems I have provided more than one unknown where necessary, so that the reader may solve the problem as a series of simultaneous equations. Based on the evidence of Diophantus' *Arithmetica*, it would seem that only one unknown symbol was used for solving arithmetical problems.[14]

[14] See Heath (1910) 39.