Abstract prepared by Athipat Thamrongthanyalak
E-mail: `athipat.th@chula.ac.th`
URL: `https://escholarship.org/uc/item/3046c8hr`

Erik Walsberg, ***Metric Geometry in a Tame Setting***, University of California, Los Angeles, 2015. Supervised by Matthias Aschenbrenner. MSC: Primary 03C64. Keywords: o-minimal structures, metric spaces, tame geometry.

Abstract

The thesis is about the topology and geometry of metric spaces definable in an o-minimal expansion $\mathcal{R}$ of an ordered field $(R, <, +, \cdot)$. A definable metric space is a pair (X, d) consisting of a definable set $X \subseteq R^k$ and a definable $(R, +, <)$-valued metric. If $X \subseteq R^k$ is definable and e is the restriction of the usual euclidean metric on R^k to X then (X, e) is a definable metric space, in this way the geometry of definable sets may be considered as a special case of the geometry of definable metric spaces. Examples of definable metric spaces whose geometry is unlike that of any definable set are given by the hyperbolic plane ($\mathbb{R}_{\exp}$-definable) and certain subriemannian spaces ($\mathbb{R}_{\text{an}}$-definable). The main theorem of the thesis is the following: *Let (X, d) be a definable metric space. Then one of the following holds*:

1. *There is an infinite definable $A \subseteq X$ such that (A, d) is discrete.*
2. *There is a definable set $Z \subseteq R^l$, for some l, such that (X, d) is definably homeomorphic to Z equipped with its induced euclidean topology.*

If $(R, <, +, \cdot)$ is the ordered field of real numbers, then a definable set A is infinite if and only if it is uncountable. As a separable metric space cannot contain an uncountable discrete subset the theorem above shows that a separable metric space definable in an o-minimal expansion of the real field is definably homeomorphic to a definable set equipped with its induced euclidean topology. This reduces the topology of separable definable metric spaces in o-minimal expansions of the real field to the topology of definable sets. Perhaps surprisingly, there are interesting examples of nonseparable metric spaces definable in $(\mathbb{R}, <, +, \cdot)$, geometric realizations of Cayley graphs of "definable group actions".

Later in the thesis, the theory of imaginaries in real closed valued fields is used to prove the following: *If $\mathcal{X}$ is an $(\mathbb{R}, <, +, \cdot)$-definable family of compact metric spaces then the collection of Gromov–Hausdorff limits of sequences of elements of $\mathcal{X}$ forms an $(\mathbb{R}, <, +, \cdot)$-definable family of metric spaces.* This theorem is an analogue of a result proven by van den Dries on Hausdorff limits of definable families of sets. Its proof gives a connection between the model theory of valued fields and the geometry of definable metric spaces.

Abstract prepared by Erik Walsberg
E-mail: `erikw@illinois.edu`
URL: `https://escholarship.org/uc/item/9z92f967`

Anton Freund, ***Type-two well-ordering principles, admissible sets, and Π^1_1-comprehension***, University of Leeds, UK, 2018. Supervised by Michael Rathjen. MSC: 03B30, 03D60, 03F05. Keywords: well-ordering principles, admissible sets, Π^1_1-comprehension, dilators, beta-proofs, Bachmann-Howard ordinal, primitive recursive set theory, slow consistency, proof length, Paris-Harrington principle.

Abstract

This thesis introduces a well-ordering principle of type two, which we call the Bachmann-Howard principle. The main result states that the Bachmann-Howard principle is equivalent to the existence of admissible sets and thus to Π^1_1-comprehension. This solves a conjecture of Rathjen and Montalbán. The equivalence is interesting because it relates "concrete" notions from ordinal analysis to "abstract" notions from reverse mathematics and set theory.

A type-one well-ordering principle is a map T which transforms each well-order X into another well-order $T[X]$. If T is particularly uniform then it is called a dilator (due to Girard). Our Bachmann-Howard principle transforms each dilator T into a well-order $\mathrm{BH}(T)$.

The latter is a certain kind of fixed-point: It comes with an "almost" monotone collapse $\vartheta : T[\mathrm{BH}(T)] \to \mathrm{BH}(T)$ (we cannot expect full monotonicity, since the order-type of $T[X]$ may always exceed the order-type of X). The Bachmann-Howard principle asserts that such a collapsing structure exists. In fact we define three variants of this principle: They are equivalent but differ in the sense in which the order $\mathrm{BH}(T)$ is "computed".

On a technical level, our investigation involves the following achievements: a detailed discussion of primitive recursive set theory as a basis for set-theoretic reverse mathematics; a formalization of dilators in weak set theories and second-order arithmetic; a functorial version of the constructible hierarchy; an approach to deduction chains (Schütte) and β-completeness (Girard) in a set-theoretic context; and a β-consistency proof for Kripke–Platek set theory.

Independently of the Bachmann-Howard principle, the thesis contains a series of results connected to slow consistency (introduced by S.-D. Friedman, Rathjen, and Weiermann): We present a slow reflection statement and investigate its consistency strength, as well as its computational properties. Exploiting the latter, we show that instances of the Paris–Harrington principle can only have extremely long proofs in certain fragments of arithmetic.

Abstract prepared by Anton Freund (coincides with thesis abstract)
URL: `http://etheses.whiterose.ac.uk/20929/ (PhD thesis) and https://arxiv.org/abs/1809.06759 (paper version)`

GIANLUCA PAOLINI, ***Independence in Model Theory and Team Semantics***, University of Helsinki, Finland, 2016. Supervised by Tapani Hyttinen and Jouko Väänänen. MSC: Primary 03C45, Secondary 03B48, 81P10. Keywords: model theory, classification theory, independence calculi, independence logic, team semantics, probability logic, quantum logic.

Abstract

The subject of this doctoral thesis is the mathematical theory of *independence*, and its various manifestations in logic, mathematics, and computer science. The topics covered in this study range from model theory and combinatorial geometry, to database theory, quantum logic and probability logic. The thesis consists of seven articles ([1–6], and [7]), grouped along two main themes:

(1) Independence calculi and combinatorial geometry ([1–3], and [4]);
(2) New perspectives in team semantics ([5–6], and [7]).

The first topic is a classical topic in model theory, which we approach from different directions (implication problems, abstract elementary classes, and unstable first-order theories). The second topic is a relatively new logical framework where to study nonclassical logical phenomena (dependence and independence, probabilistic reasoning, and quantum foundations). The fundamental thesis defended in this work is that these two themes are deeply intertwined, under the guiding thread of independence.

Abstract prepared by Gianluca Paolini
E-mail: gianluca.paolini@mail.huji.ac.il
URL: `https://helda.helsinki.fi/handle/10138/163784`

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