in (2), which is conjectured to give sharp bounds for the coefficients $a_n$ and $b_n$ of $h$ and $g$, respectively. The only sharp bound known so far is $|b_2| \leqslant \frac{1}{2}$.

Clunie and Sheil-Small proved that the conjectured bounds also hold for typically real functions and starlike mappings (Chapter 6) and also obtained bounds for the general class $S^0_{\mathrm{H}}$.

The author presents all the above results and many more in his usual clear and concise way. There is a good list of references, but there is no detailed reference to the classical representation of Weierstrass and Enneper in the 1960s, 'which appears to have been the start of the study of minimal surfaces'.

I predict a growing interest in the study of harmonic mappings both for the beauty of the results already obtained and for the many open problems and conjectures. For all students in this field Duren's book will be essential reading. It will also be the classic reference book in this area.

W. K. HAYMAN

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Voisin, C. *Hodge theory and complex algebraic geometry*, Volumes I, II (transl. from the French original by L. Schneps) (Cambridge University Press, 2002, 2003), 0 521 80260 1 (hardback), £55 ($80), 0 521 80283 0 (hardback), £60 ($85).

During a period in which algebraic geometry was crying out for new techniques, Hodge produced a deep insight, realizing that harmonic forms could be used both to understand old results from a clearer perspective and to produce new results including the Hodge index theorem, one of the inspirations for the Atiyah–Singer theorem and its consequences. His existence proof was analytic in nature and thus combined differential and algebraic geometry and has encouraged many others to use analytic techniques in geometry. Some of the prominent names that have done so more recently are Yau, Donaldson, Hamilton and Perelman. Hodge theory itself has continued to develop and, in particular, work of Deligne and Griffiths over a significant part of the second half of the last century greatly increased its scope. Griffiths, in particular, often wrote in an expository style and this helped others to understand the results and as a consequence Hodge theory has become an essential tool in diverse areas.

During the 1990s other expository accounts were written but these two volumes give the most complete account to appear so far. They include a great deal of background material often seamlessly woven into the proofs of results that were very new when the course on which they are based was given in Paris in 1998–2000. A motivated reader who knows basic facts about differential forms and algebraic topology should have little difficulty in learning a great deal by reading these volumes; one can even start afresh at a number of points. The whole is written with the authority of the foremost expert on Hodge theory; some further striking results have been found by Voisin since these volumes were written but the background to these recent papers is all here. She described some of her new results at the Hodge centenary meeting held in Edinburgh in 2003. Even more recently she has found striking and simple examples of compact Kähler manifolds not homotopically equivalent to a projective variety, thus solving a very longstanding problem.

Due to the origin of these volumes, there are a number of exercises even in the more advanced chapters and hence they could easily form the basis for reading courses or seminars as well as for individual learning. Both volumes start with fairly detailed introductions, each about 15 pages long. Anyone wishing to find out what the theory is about in a general way or to understand how the material is arranged should spend some time reading these chapters. The first volume is divided into four parts: 'Preliminaries', 'The Hodge decomposition', 'Variations of Hodge structure' and 'Cycles and cycle classes'. The chapters in the part entitled 'Preliminaries' are on

'Holomorphic functions of many variables', 'Complex manifolds', 'Kähler metrics' and 'Sheaves and cohomology'. The material in this part would be an excellent basis for a beginning graduate course on these topics. The second part includes a precise statement of the 'essential theorem on elliptic operators' and refers to an article by Demailly for the proof, but there are many accounts of this theory; this apart, everything that is needed is included and the exposition is based as much on first principles as it possibly could be. This second part could similarly be used as a basis for a course on traditional Hodge theory. The Hodge decomposition, although defined using a metric, depends only on the complex structure of the manifold. It turns out that to understand how it varies with the complex structure it is necessary to think in terms of the Hodge filtration and Griffiths proved it varies holomorphically. Very clear accounts of the Kodaira–Spencer map and of the Gauss–Manin connection are included as preliminaries. The final part of the first volume discusses some classical material such as the Abel–Jacobi map as well as more recent ideas such as Deligne cohomology. The main thrust of this part is to treat topics related to the Hodge conjecture, which is one of the main topics in the second volume.

The second volume consists of three parts entitled 'The topology of algebraic varieties', 'Variations of Hodge structures' and 'Algebraic cycles'. The first part includes an exposition of Morse theory and its application to the proof of the Lefschetz theorem on hyperplane sections; it goes on to study the 'complex analogue' of Morse theory, namely Lefschetz pencils, which have been adapted so successfully in Donaldson's revolutionary study of symplectic manifolds and his proofs use a significant amount of analysis. The second part includes Griffith's transversality theory as well as the result that the primitive cohomology is generated by the residues of explicit forms. It goes as far as Nori's connectivity theorem, which is a version of Lefschetz's theorem for certain families of varieties. It also gives a great deal of information about algebraic cycles, thus providing some of the material needed in the final part, which is devoted to the relations between Hodge theory and algebraic cycles. This final part includes an account of Mumford's theorem on Chow groups and its extensions, Bloch's conjecture on the converse of Mumford's theorem, and Mukai's version of the Fourier transform on abelian varieties.

There is an obvious comparison to be made between these volumes and the hugely influential *Principles of algebraic geometry* by Griffiths and Harris (Wiley, 1978), which covers some similar ground. Both cover the basic material on Hodge theory but the emphasis is rather different and, of course, Voisin covers more recent work—much of it due to Griffiths and his students. The older book is a wonderful source for the study of specific topics and examples in classical algebraic geometry from a modern perspective; this aspect is not attempted by Voisin. Indeed, the two books are remarkably complementary and to be really expert in this area one would have to be familiar with both. I suspect that most people would find that the basic ideas are more clearly written in Voisin's book—the details seem to be written in a more accessible and elegant style. Those interested in classical algebraic geometry will find a wealth of material in the older book. I would recommend anyone interested in learning about a topic in complex differential or algebraic geometry to read Voisin's volumes. She has done a remarkably good job.

E. G. REES