Annals of Glaciology 6 1985

SETTLEMENT FORCE ON A BEAM IN SNOWPACK BY COMPUTER MODELLING

by

THEODORE E. LANG

Montana State University, Bozeman, Montana 59717, USA

HIDEOMI NAKAMURA, OSAMU ABE

National Research Center for Disaster Prevention, Shinjo, Japan

ABSTRACT

Computer analysis is carried out to predict the force that bears upon a horizontal beam embedded in settling snowpack. Use is made of extensive experimental data on snow depth, layering and density, and on measured force on a test beam, in order to evaluate the internal structural arrangement in a finite element representation of the region in the vicinity of the beam. The compressive secondary-creep viscoelastic properties and the densification properties of the snow are accounted for in the computer modelling. Results show that the bearing force on the beam develop from direct snow weight, from shear transfer by adjacent snowpack, from basal weakening below and to the side of the beam, and from snow densification above and to the side of the beam producing a bridging mechanism. These findings corroborate with related experimental results and point out the need for further refinement in definition, particularly of the bridging mechanism, which produces large force incrementation on the beam compared to the other mechanisms noted.

INTRODUCTION

Snow accumulation in central Japan is of sufficient magnitude that forces developed on structures buried by the snowpack must be considered. Direct measurement of this type of force is not always readily feasible, so that means are sought to be able to make analytical estimates. However, simple estimates with many attendant approximations do not generally yield accurate predictions of force. The reasons for this are that snowpack is a layered continuum of viscoelastic material which in the vicinity of a structure has a complex pattern of flow and force transmission. To incorporate these physical effects into an analysis of settlement, taking into account the local intrusion of a structure, requires the use of a numerical method such as the finite element method.

Extensive experimental data have been taken by H. Nakumura at Shinjo Branch over several winters, of settlement force on an instrumented horizontal beam that is buried in the snowpack. Data are known on the settlement force on the beam, and on new snow deposition and density measured in the area once each day after initial burial of the beam. Also known are the distribution of density in the snowpack from pit data taken at 10 day intervals at a location in the vicinity of the beam test site. These data are of sufficient detail that computer modelling of this configuration of beam and snowpack can be attempted.

A computer program that uses the finite element method in analysis of planar elasticity problems, including material orthotropy is used to analyze the beam settlement problem. While ordinarily used for elastic material problems, the computer code can also be applied to secondary or steady creep problems by simple redefinition of the material constants. The program does not incorporate material densification, which occurs in snow settlement, but with the density distribution known at 10 day intervals, stepwise linear interpolation on a daily basis can be made.

The approach is to use the density and force data as reference information and to investigate the conditions by which a finite element representation of the beam-snow configuration fits the data. In studying this problem, the beam data obtained by H. Nakamura in the winter of 1980-81 are selected as the experimental model. During this winter the snow beam that was monitored at Shinjo Branch was buried to a maximum depth of 1.0 m from about the middle of January to the middle of March. In addition, the reported density measurements in the vicinity of a beam in snowpack by I. Furukawa (1953) will be used as a guide. With regard to snow densification, an equation developed for typical winter snow in Japan by T. Nakamura (1984) will be used.

EXPERIMENTAL DATA

A horizontal steel beam 4.0 m long, of rectangular cross-section (10 cm x 5 cm) supported 80 cm above the ground surface by vertical posts affixed to electronic force gauges is the basic configuration used by Nakamura. Data for this beam were taken each morning at 9.00 am following the date the beam was buried by natural snowfall. For the experiment of winter 1980-81 the beam was initially buried between 4 and 5 January 1981. Depth of the snowpack that year at Shinjo Branch

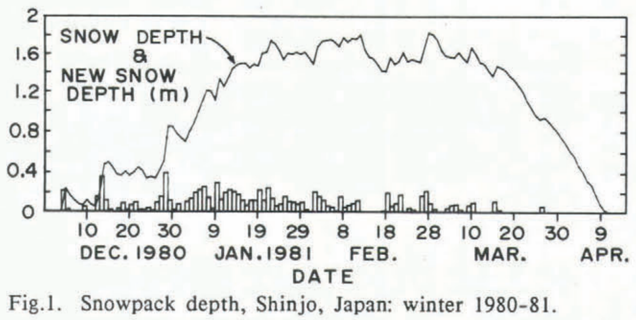


Fig.1. Snowpack depth, Shinjo, Japan: winter 1980-81.

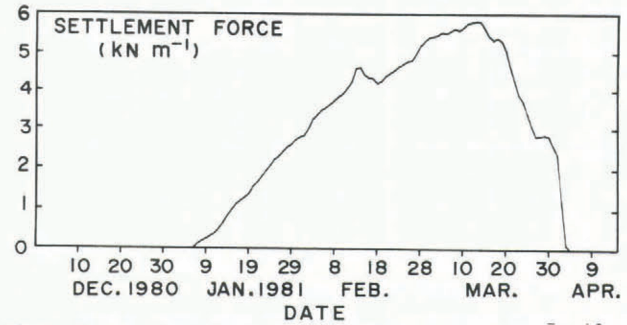


Fig.2. Snowpack settlement force on beam.

https://doi.org/10.3189/1985AoG6-1-95-99 Published online by Cambridge University Press

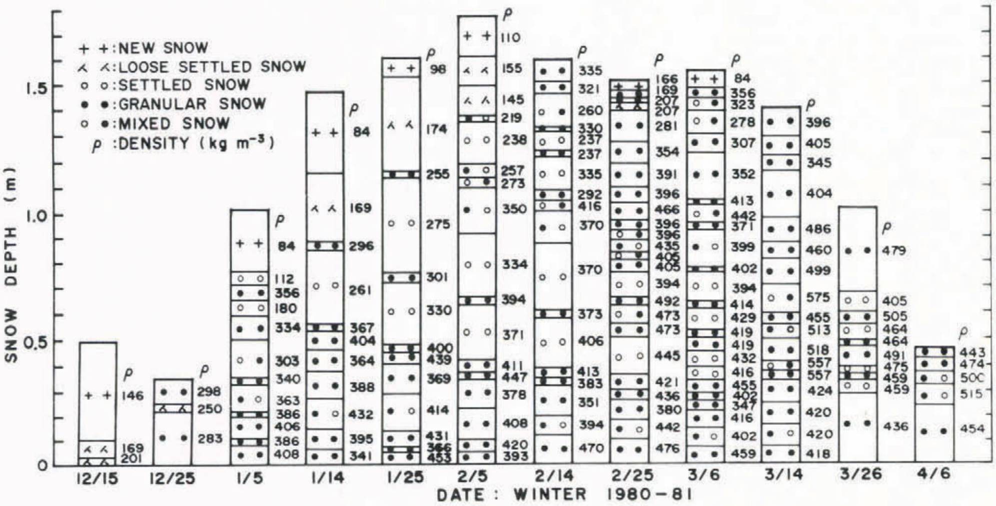


Fig.3. Ten day intervals of density distribution in the snowpack at Shinjo Branch for the winter 1980-81, Shinjo, Japan.

in the vicinity of the beam test site (but not at the test site) is shown in Figure 1. With the top of the beam 80 cm above ground, the beam was buried to upwards of 1.0 m depth for over 2 months starting about 15 January. The depth and average density of new snowfall each day, measured at 9.00 am is tabulated in Appendix A. The measured settlement force on a unit length of the beam is shown plotted in Figure 2. From pit data taken nominally at 10 day intervals the distribution of snow density in the snowpack in the vicinity of the beam test site is shown in Figure 3.

FINITE ELEMENT REPRESENTATION

Several considerations dictate the finite element model that is selected to analyze the snow-beam settlement problem. Knowing that material flow should vary most rapidly in the local region surrounding the beam, the finite element array should account for this gradation. Also, at some horizontal distance away from the beam, snow settlement should approach the infinite slab condition of simple vertical deflection. Advantage is also taken of symmetry about the vertical axis through the centre of the beam. In assimilating these factors, a grid of finite elements was selected in the study as shown in Figure 4, with 115 gridpoints and 107 elements. In this representation, the beam extends from an elevation of 80 to 75 cm on the right-hand side of the grid. The boundary condition of no horizontal displacement is imposed at the gridpoints on the vertical axis through the beam and at the vertical boundary 1.0 m to the left of the beam. At the ground surface and on top of the beam no vertical displacement is permitted. Gridpoint and element numbering increment horizontally from the ground surface.

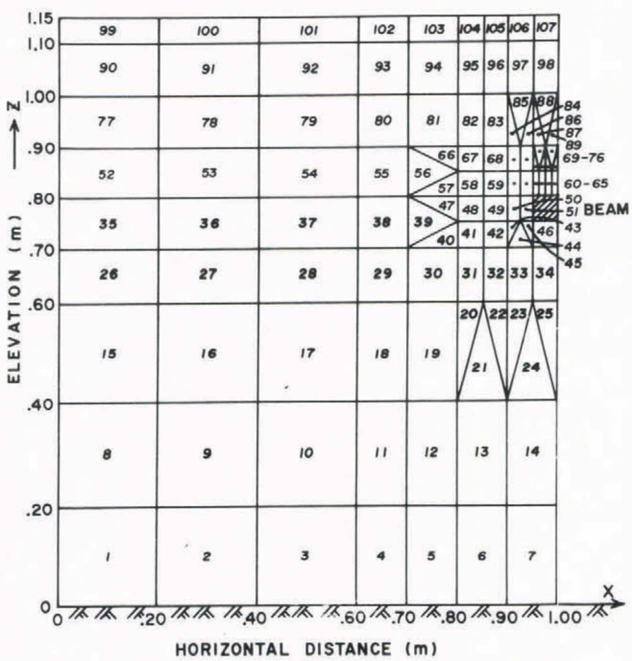


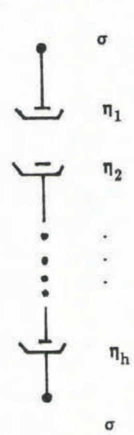
Fig.4. Finite element grid layout of beam-snow settlement problem.

The procedure in analysis is to assume that the element array is fixed, and for material to flow or settle through the elements. However, depth of the upper three layers of elements is allowed to vary in order to account for new snow deposition. For the fixed (lower) elements, the density of snow in the elements is subject to change from update of daily gridpoint deflection. If in an individual element several snow layers of different density occur, then an equivalent average density of that element is computed. This calculation is facilitated by assumed linearity between snow density, ρ, and secondary creep, η. Thus an element of several layers can be represented by a series of dashpots, and considering a compressive stress, σ, applied vertically to the dashpots, the strain rate, $\dot{\epsilon}$, of each layer is (neglecting Poissonic effects):

$$\dot{\epsilon}_1 = \frac{1}{\eta_1}\sigma$$

$$\dot{\epsilon}_2 = \frac{1}{\eta_2}\sigma$$

$$\dot{\epsilon}_n = \frac{1}{\eta_n}\sigma \qquad (1)$$



Rate of reflection, $\dot{\delta}$, of each layer is then

$$\dot{\delta}_1 = \dot{\epsilon}_1 h_1$$

$$\dot{\delta}_2 = \dot{\epsilon}_2 h_2$$

$$\vdots$$

$$\dot{\delta}_n \quad \dot{\epsilon}_n h_n \qquad (2)$$

The total rate of deflection over the entire element is:

$$\dot{\delta} = \dot{\epsilon}_{EQ}\, h = \frac{\sigma}{\eta_{EQ}}\, h = \dot{\delta}_1 + \dot{\delta}_2 + \cdots \dot{\delta}_n \qquad (3)$$

leading to the resulting equation:

https://doi.org/10.3189/1985AoG6-1-95-99 Published online by Cambridge University Press

$$\eta_{EQ} = \frac{h\ (\eta_1)(\eta_2)(\eta_3)\ \cdot\ \cdot\ \cdot(\eta_n)}{h_1(\eta_2)(\eta_3)\cdots(\eta_n)+h_2(\eta_1)(\eta_3)(\eta_4)\cdot\cdot(\eta_n)+\cdot\cdot+h_n(\eta_1)(\eta_2)\cdots(\eta_{n-1})} \quad (4)$$

Since η and ρ are related by a proportionality constant, an equation for equivalent density can be written directly as

$$\rho_{EQ} = \frac{h(\rho_1)(\rho_2)(\rho_3)\qquad(\eta_n)}{h_1(\rho_2)(\rho_3)\cdots(\rho_n) + h_2(\rho_1)(\rho_3)\cdots(\rho_n) + \cdots h_n(\rho_1)(\rho_2)\cdots(\rho_{n-1})} \quad (5)$$

Once density is assigned to each element, then the viscous description of the material must be defined, which is considered below.

MATERIAL DESCRIPTION

Several assumptions are made to reduce the complexity of the material description and make a solution tractable. One assumption is that of plane strain, that a section of the beam cross-section and snow-slab (Figure 4) is "typical" in the sense of constancy in the third coordinate direction. Designating the plane of elements in Figure 4 as the X,Z plane, then for this plane to be a typical plane, it is necessary for strain in the Y direction to be constant. Another assumption is that transient material response is negligible. so that only secondary (steady) creep of the material needs to be defined, and this in turn limited to linear viscoelastic representation. By this assumption the correspondence principle of linear viscoelasticity can be used, so we consder first the elastic constitute law, then convert to the viscoelastic analog. The isotropic elastic equation assuming plane strain ($\epsilon_y = o$) is:

$$\begin{bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & G\end{bmatrix}\begin{bmatrix}\epsilon_x\\ \epsilon_z\\ \gamma_{xz}\end{bmatrix} \quad (6)$$

where $G = \frac{1-2\nu}{2}$. Assuming next that most of the domain surrounding the beam is in a state of compression, the compressive viscoelastic analog is used. Detailed derivation of this analog is reported by Lang, Nakamura (1983), so only the resulting equation is repeated here. The derivation entails the incorporation of the long duration uniaxial compressive viscous coefficient, η, experimentally measured by Shinojima (1967), and a value of the secondary creep Poissonic coefficient in compression, designated ν. Dividing the material creep response into volumetric and deviatoric components, the derived constitutive equation for steady creep reduces to the simple form:

$$\begin{bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{bmatrix} = \frac{\eta}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & 0\\ \nu & 1-\nu & 0\\ 0 & 0 & (1\text{-}2\nu)\end{bmatrix}\begin{bmatrix}\dot{\epsilon}_x\\ \dot{\epsilon}_z\\ \dot{\gamma}_{xz}\end{bmatrix} \quad (7)$$

The relationship between η and material density is obtained from Shinojima (1967), shown in Figure 5, wherein the dashed long duration line is used, based upon 5 data points extrapolated from the Shinojima data at temperature T = 0 °C. For snow in compression the Poissonic coefficient is set at $\nu = 0.27$.

A refinement in equation development to the above would be to consider both tensile and compressive stresses separately, since for snow the viscoelastic analogs are somewhat different. The theoretical development of the governing equations for a combined tension-compressinn stress state proceed from an orthotropic elastic model to the viscoelastic model, as worked out by Lang, Nakamura (1983); however, these equations were not used in the present analysis.

The linearity assumed in the material representaion for this large-deflection nonlinear differential settlement problem is justified in the fact that the computer analysis will be updated on a daily basis. Thus, the analysis methodology comes under the procedure of stepwise linearization of the basic nonlinear process.

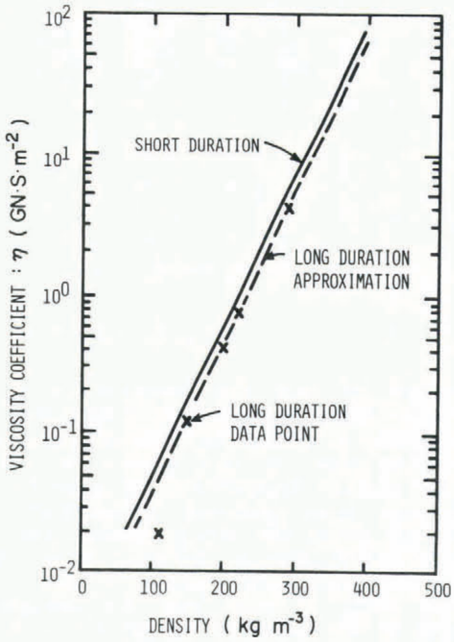


Fig.5. Secondary compressive viscosity-density relationship for short and long duration creep tests of dry snow at temperature T = 0°C (after Shinojima).

One final consideration with regard to the material description is a densification equation that is needed to extrapolate element densities for each daily increment. The equation used is derived from measurements of snow densification in central Japan, as reported by Nakamura, Lang (1984), namely

$$\rho^2 = \rho_o^2\ (1 + \frac{0.006}{\rho_o^2}\ t) \quad \text{(e.g. } t < -30) \qquad (8)$$

where ρ is the snow density at day t, based upon snow density at t = 0 of ρ_o in [g/cm^3].

SETTLEMENT ANALYSIS

The density and vertical distributions of snow on 5 January and 14 January 1981, from pit data, are shown at the left and right ends of Figure 6, respectively. These data are referenced to morning measurement of the data noted, as is true of all other recorded data, unless specified otherwise. From 5 January to 6 January new snow deposited to a depth of 23 cm, having density $\rho_N = 0.084$. However, densification occurs continuously so by the morning of 6 January the distribution of snow labeled "6 January" in Figure 6 is assumed. Based upon this configuration of snow density and depth the force on the beam is computed. This was done by a trial and error process in which densities in elements near the beam were varied until force prediction by the computer was approximately the same as the measured value. By this procedure it is intended that the computer solution will yield information on the structure of the snow in the vicinity of the beam. After

https://doi.org/10.3189/1985AoG6-1-95-99 Published online by Cambridge University Press

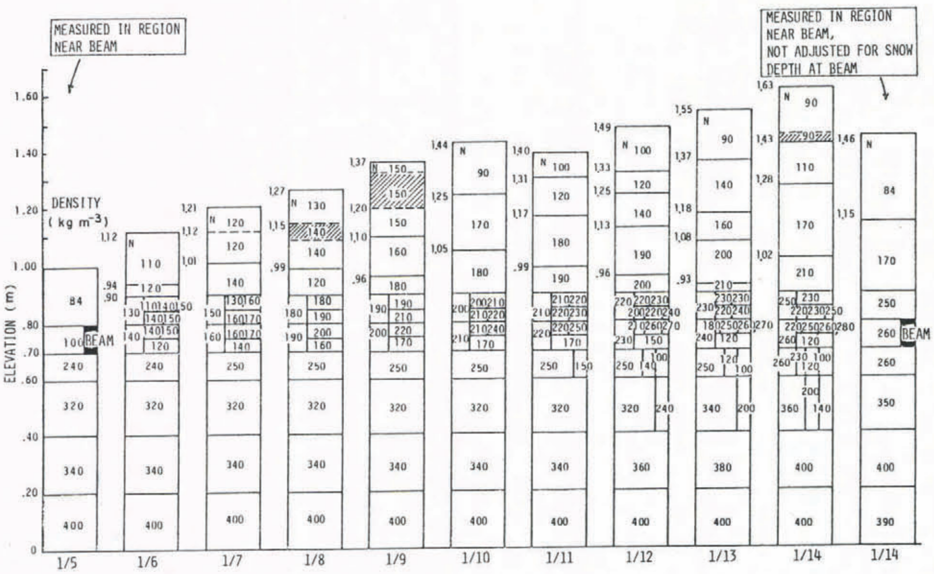


Fig.6. Snow depth and densification from 5 - 14 January 1981.

each calculation of force was completed for a given date, new snow deposition and snow densification were imposed in order to predict the material configuration of the following date. In first attempts at modelling the snow, the assumed densification was generally taken too small, and large gridpoint displacements were obtained in the upper, low density layers. This result indicates that densification of new snow occurs rapidly, or that the secondary creep viscosity of new snow is greater than what is read from the straight line representation of the Shinojima data of Figure 5. From another snow densification study by Lang, Nakamura (1983), the straight-line representation at low densities is suspect. In the present computer study, in order to prevent excessively large deflections of the upper layers, densification is assumed on the high side of the curve shown in figure 5 for snow densities below ρ = 0.10 gm$\cdot$cm^{-3}. To show the type of results obtained from the computer evaluation, the density distribution and displacement pattern in the local region around the beam computed for 9 January are shown in Figure 7. It is seen that large displacements occur in elements 48, 49, 50 and 51, so that densification should occur rapidly in this region. For this configuration, the force transmitted to the beam through elements 62, 63, 64 and 65 totals to 86 N$\cdot$m^{-1}. However, shear force transmitted to the beam through element 51 is 158 N$\cdot$m^{-1}, indicating the strong shear transfer mechanism within the snowpack. This is

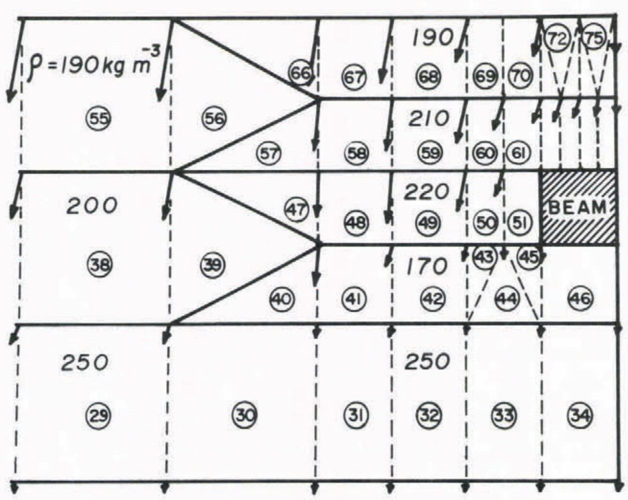


Fig.7. Gridpoint displacements per day in local region around beam, 9 January 1981.

consistent with the experimental findings by Furukawa (1953) that a higher density region develops outward from the beam. Note that the above forces are total force; that is, the contribution from both sides of the beam is included in the values cited. Thus total computed force is 244 N$\cdot$m^{-1} compared to 255 N$\cdot$m^{-1} measured experimentally.

From 5 January through 10 January the settlement force on the beam could be matched by the computer model from simple densification of the snow. Note however that by 8 January the reference depth of snow h_R = 115 cm and the depth assumed for the computer modeling (h = 127 cm) differ. Depth h_R is the value measured in the vicinity of the snow beam but not at the test site. Starting on 8 January snow was added to the test site depth by the shaded areas in Figure 6 in order to account for local mounding. Local mounding was estimated at about 10%, which by 9 January is the approximate difference between h_R and the depth used in the computer model.

By 11 January it was no longer possible to match the settlment force on the beam by a simple densification schedule assigned to the snow layers. At this time it was necessary to introduce weakening of layers beside and below the beam, which allowed continued matching between the experimental and computed forces. The process of weakening in lower layers around the beam is also indicated by Furukawa. For continued force matching it was necessary to enlarge the weakened region as is indicated by the succession of elements with lower density between 11 January and 14 January (Figure 6).

By 14 January continued lower layer weakening would not produce sufficient settlement force. On this date considerable numerical experimentation was carried out, by successively changing the density (and hence the viscosity) of single elements at a time in order to obtain some understanding of the process going on. Eventually, what was observed was that by increasing the density in elements 56, 57, 58 and 59 by small amounts, with slight reduction in density of elements 31, 39, 40 and 41 that successively larger settlement forces could be obtained (Figure 7). This trend takes on the characteristic of a bridging effect, wherein the weight of snow in the upper regions at increasing distances from the beam feeds into the beam. The computer results show an extreme sensitivity in this process.

As has been described, the experimentally measured beam settlement force was used as a means of finding the internal arrangement of finite elements in order to predict the force correctly. However, on some dates the force prediction by the computer model matched the experimental value by simple extrapolation of the snowpack geometry to the next day by simple account of densification. The resulting comparison between the experimental and computer predicted settlement force from 5 January through 14 January is shown in Figure 8.

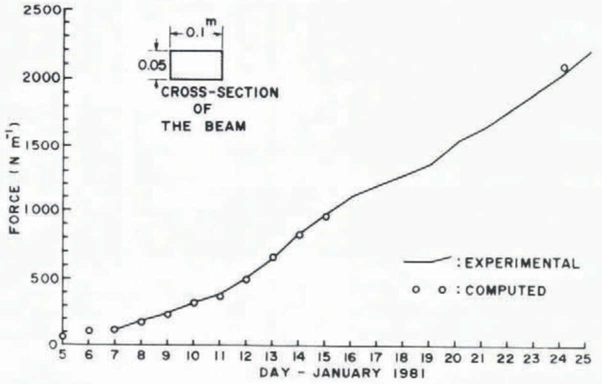


Fig.8. Experimental and computed settlement force on the beam.

On 14 January, once bridging was established, it was possible from small local changes in the density of finite elements 57, 58 and 59 (Figure 7) to predict the beam force up to 24 January, which would amount to a simple linear interpolation from 14 January. In comparing the density distribution in the snowpack between 14 January and 24 January the evidence is that the snowpack is stable and density changes are small (Figure 3). This is reflected also in the near linearity of increase in the experimental force during these dates (Figure 2). Continued development of the bridging mechanism with attendant higher densities in the upper layers of the snowpack should account for the continued increase in settlement force beyond 24 January.

SUMMARY

From this initial computer analysis of snowpack settlement upon an embedded horizontal beam, mechanisms of force transmission to the beam have been identified. These are (1) direct pressure from overburden snowpack, (2) shear transfer from adjacent snowpack, (3) snowpack weakening beneath and to the side of the beam, and (4) snow densification in layers above and to the side of the beam to develop a bridge for intensified transfer of force. It is the bridging mechanism of step (4) that force intensification on the beam becomes as large as has been measured experimentally. Results of this computer study show the bridging region to extend horizontally from the beam by a factor of 2 to 3 times the width of the beam.

Either by further computer analysis or by experimental studies, more information is needed to further quantify this bridging mechanism, so that this physical condition may be useful in structural design. In snowy regions, as in Central Japan, the force intensification effects on structures is a serious problem that warrants technical definition.

REFERENCES

Nakamura T, Kemmosuto K, Lang T E 1985 Winter snow densification in Central Japan. *Shinjo Branch* NRCDP, in preparation

Furukawa I 1953 Study on snow settlement force. *Researches on Snow and Ice* 1, Japanese Society of Snow and Ice, Tokyo: 155-166

Lang T E, Nakamura T 1984 Finite element computer analysis of snow settlement. *Research Notes of the National Research Center for Disaster Prevention* 59

Shinojima K 1967 Study of the viscoelastic deformation of deposited snow. *Physics of Snow and Ice* 1(2): 875-907

APPENDIX A. NEW SNOW DEPTH AND DENSITY AT SHINJO BRANCH FOR THE WINTER 1980-81, SHINJO, JAPAN.

DATE	DEPTH (cm)	DENSITY ($g\ m^{-3}$)
Jan 1	8.0	0.132
2	0.0	0.0
3	11.0	0.133
4	14.0	0.164
5	20.0	0.066
6	23.0	0.084
7	26.0	0.092
8	15.0	0.099
9	3.0	0.127
10	30.0	0.063
11	13.0	0.071
12	20.0	0.081
13	23.0	0.071
14	21.0	0.072
15	18.0	0.069
16	12.0	0.065
17	7.0	0.056
18	12.0	0.059
19	12.0	0.056
20	23.0	0.066
21	12.0	0.083
22	25.0	0.061
23	14.0	0.063
24	6.0	0.089
25	8.0	0.064
26	16.0	0.055
27	11.0	0.073
28	10.0	0.089
29	8.0	0.060
30	10.0	0.070
31	2.0	0.092

DATE	DEPTH (cm)	DENSITY ($g\ m^{-3}$)
Feb 1	0.0	0.0
2	21.0	0.083
3	16.0	0.102
4	12.0	0.096
5	6.0	0.072
6	5.0	0.047
7	0.0	0.0
8	16.0	0.048
9	5.0	0.058
10	7.0	0.072
11	9.0	0.105
12	12.0	0.096
13	0.0	0.0
14	0.0	0.0
15	0.0	0.0
16	0.0	0.0
17	0.0	0.0
18	1.0	0.120
19	20.0	0.102
20	6.0	0.085
21	9.0	0.071
22	18.0	0.063
23	0.0	0.0
24	4.0	0.097
25	2.0	0.143
26	0.0	0.0
27	17.0	0.085
28	22.0	0.072

https://doi.org/10.3189/1985AoG6-1-95-99 Published online by Cambridge University Press