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On theorems of Thornton

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The topology of a topological group in which the intersection of open sets is open is uniquely determined by a normal subgroup, and the group is uniquely an extension of an indiscrete group by a discrete group. This was proved by M.C. Thornton under the additional hypothesis that the group is a torsion group. The proofs here given make the more general facts almost trivial.

Thornton in [2], proves the following theorems:-

THEOREM 4. *There is a bijective correspondence between normal subgroups of a torsion group G and the A-space topologies on G giving a topological group.*

THEOREM 6. *Any A-space torsion topological group G can be uniquely expressed as the extension of an indiscrete normal subgroup N by a discrete group D.*

The author proves these theorems by proving three lemmas. In fact a shorter proof can be given as follows. Besides throughout the paper the assumption 'torsion' can be dropped.

Since G is an A-space, the minimal open set U_e containing the identity element e of the group G exists. Also since G is a topological group and $\{U_e\}$ is a neighbourhood base at e, we have $U_e \cdot U_e^{-1} \subset U_e$ and $aU_e a^{-1} \subset U_e$ for every $a \in G$ [1, p. 18]. Thus U_e is a normal subgroup of G. Since U_e is minimal, U_e is indiscrete and $U_e = G_e$, the connected component of e. It is evident that G/U_e

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is a discrete topological group.

Also for each A-space topology one such U_e exists and for each normal subgroup N of a group G, we can define a topology on G by declaring each set to be open iff it is the union of cosets of N.

Thus both the above theorems are proved simultaneously.

References

[1] Edwin Hewitt and Kenneth A. Ross, *Abstract harmonic analysis, Vol. I* (Die Grundlehren der mathematischen Wissenschaften, Band 115. Academic Press, New York; Springer-Verlag, Berlin, Göttingen, Heidelberg, 1963).

[2] M.C. Thornton, "Torsion topological groups with minimal open sets", *Bull. Austral. Math. Soc.* 5 (1971), 55-59.

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