PART II : POLAR MOTION

ON THE COORDINATE SYSTEMS USED IN THE STUDY OF POLAR MOTION

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IAU Symposium No. 78 "Nutation and the Rotation of the Earth" held in Kiev in 1977 revealed a certain lack of precision in the fundamental concepts and some looseness of terminology employed in the treatment of this problem. When talking about polar motion we should give, first of all, rigorous conceptual definitions of both the pole and a reference frame in which it moves. The selection of a reference system was the topic of an IAU Colloquium held in Torun in 1974. Although the discussion there was thorough and comprehensive, it did not result in the removal of all ambiguities which have tarnished discussion of the problems in the understanding of the Earth's rotation.

"Since distances are not directly measured in classical astronomy but have to be inferred by indirect methods, the systems of coordinates in common use are those that specify only directions" (Clemence, 1963). Since direction is completely given by a pair of angular coordinates, the space in which such systems are realized is a two-dimensional space. To emphasize this point Brandt (1975) suggested that astronomers may make use of the terminology employed in geometrical optics, where the real three dimensional space in which physical bodies are located is called the object space and the space in which images of these bodies are located is called the image space. Three dimensional coordinate systems in the object space may be either inertial or non-inertial, but the very conception of "inertiality" has no meaning for systems realized in the image space. In that case we may only discuss rotating and non-rotating systems. The image space used in astronomy is the two dimensional surface of a unit sphere. Recently, however, some authors have used other representations (Fedorov, 1976a; Zhongolovitch, 1977; Murray, 1978).

Assume that we have in the object space several direct lines connecting some points of different celestial bodies (Figure 1). Let us take in the image space an arbitrary point, O, and draw from it unit vectors, $\overline{s}_i$, parallel to these lines. Obviously, several parallel lines in the object space will be represented by only one vector in the image space.

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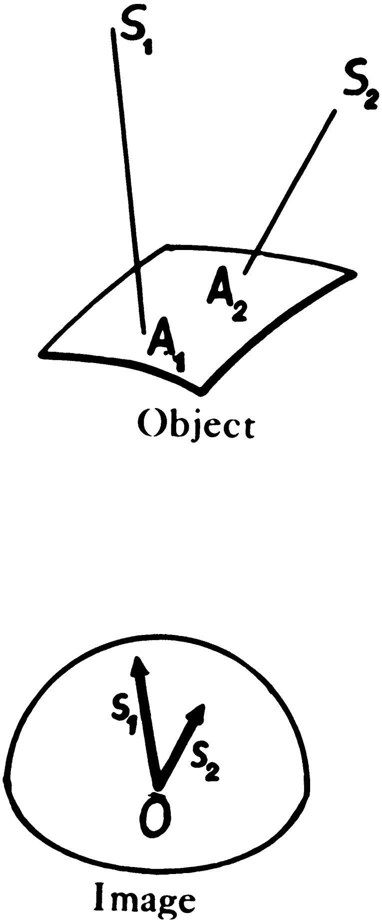

Figure 1. Object and image spaces.

Directions from the observer to extragalactic sources are practically independent of the observer's motion on the Earth, together with the Earth, or in space about the Earth (assuming that correction for aberration is taken into account). The proper motions of extragalactic sources also being negligible, the unit vectors along the directions from the observer to the object can be assumed to be fixed in the image space. On the other hand, we have the following directions linked to the Earth and rotating with it: geocentric position vectors of the points on the Earth's surface, plumb lines at these points, chords connecting these points (baselines of radio interferometers). We may draw from O in the image space unit vectors, $\bar{e}_j$, parallel to these directions.

Astronomical observations of stars, planets, the Moon, artificial satellites, or radio sources enable the rotation of the pencil of vectors, $\bar{e}_j$, with respect to the vectors, $\bar{s}_i$, to be monitored. Measurement of either the angles or distances (with laser ranging techniques) are capable of providing the necessary data (Fedorov, 1976b). These data would comprise sufficient information on the rotation of the Earth so that this phenomenon could be studied without the use of coordinates (Veis, 1976), but the use of coordinates simplifies analysis. Since the angles between the unit vectors, $\overline{s}_i$, are practically constant, a celestial coordinate system, XYZ, can be rigidly linked to them. The following resolution concerning this matter was adopted at the Torun Colloquium:

> The celestial system will be defined by a catalog of adopted conventional coordinates of extragalactic sources. These coordinates could be obtained from the best available observations and reduced to a given epoch in the existing celestial system (FK4 or FK5). After such a catalog is constructed and adopted, reference to the original celestial system may be dropped. Further improvements in the realization of the system would come through the compilation of better catalogs of extragalactic sources (e.g. with no reference to any plane or direction pertaining to the Earth or Solar System).

It is quite natural that a three dimensional reference coordinate system in the object space is needed for description of various phenomena dealt with in Earth dynamics. However, to study the Earth's rotation one can use systems realized in the same two dimensional space as the non-rotating system, XYZ. Then relative orientation of the terrestrial and celestial systems would be defined only by the angles between their axes. The phenomenon called the rotation of the Earth is, in essence, the variation of these angles.

Any terrestrial system in the image space may be attached to the pencil of the unit vectors $\overline{e}_j$, but not rigidly since these vectors (in the case of a non-rigid Earth) do not maintain their directions relative to one another. For the non-rigid attachment to be realized, certain conditions should be imposed on the relationship of a system, xyz, with respect to the vectors $\overline{e}_j$. Such a system may be called the conventional terrestrial system. Its rotation is not exactly predictable since it is affected by excitation by some geophysical processes. So it is convenient to introduce an intermediate system, $\xi\eta\zeta$, whose rotation approximates as close as possible that of the system, xyz, and at the same time is precisely predictable. It may be called the terrestrial ephemeris system. This system as well as the system, xyz, can be transformed into the non-rotating celestial frame by means of the equations:

$$(X,Y,Z) = M_0\ (\xi, \eta, \zeta) = M\ (x,y,z), \tag{1}$$

where M_0 is the matrix of precession, nutation, and the "ephemeris" diurnal rotation of the Earth. We may write

$$M = M_0\ (I + \sigma), \tag{2}$$

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where I is the unit matrix, and

$$\sigma = \begin{vmatrix} 0 & -w & v \\ w & 0 & -u \\ -v & u & 0 \end{vmatrix} , \tag{3}$$

u, v, w being small rotation angles about the x, y, z-axes respectively (Fedorov et al., 1972). From (2) we have

$$M - M = M_0 \sigma . \tag{4}$$

It should be noted that the elements of the matrix change with nearly diurnal periods.

Proceed now to possible realizations of different coordinate systems, and consider first the selection of the ζ- axis of the ephemeris terrestrial system. For this axis the following directions may be adopted.

1. The axis of the total angular momentum of the Earth, $\overline{H}$

We donote the unit vector parallel to $\overline{H}$ by $\overline{h}$. The motion of $\overline{h}$ with respect to the non-rotating system XYZ is independent of all properties of the Earth other than its moments of inertia. This means that the motion of the angular momentum vector derived for a rigid Earth may be applied to any other reasonable model.

2. The instantaneous rotation axis of the Earth

A rigorous definition of this axis is valid only in the case of a rigid Earth. The equations of motion of this axis can be obtained by small changes of the coefficients of the periodic terms in the equations governing the motion of the angular momentum vector. In just this manner the ζ - axis of the terrestrial ephemeris has been defined in textbooks and astronomical ephemerides.

3. The Jeffreys-Atkinson axis

The gravitational torques exerted by the Moon and the Sun on the Earth's equatorial bulge not only force the angular momentum vector of the Earth to change its orientation in space, but they also cause a small departure of the axis of figure (i.e. the axis of maximum inertia) from the unit vector $\overline{h}$. The action of each of these bodies can be treated separately.

Let $\overline{m}$ be the unit vector in the direction of the Moon or Sun, and $\overline{f}$ be the unit vector along the axis of figure of the Earth. The vectors $\overline{m}$, $\overline{h}$, and $\overline{f}$ can be shown to lie approximately in the same plane, whose position is defined by the known coordinates of the disturbing body (Figure 2). In the case of a rigid Earth the angle between the unit vectors $\overline{h}$ and $\overline{f}$ can be computed for any moment of time. This enables the direction of $\overline{f}$ to be predicted and used as the ζ - axis of the terrestrial reference system. This idea was first suggested by Jeffreys (1963), and then elaborated by Atkinson (1973, 1975). Murray (1978) thinks that the same ephemeris axis should if possible be retained for other Earth models.

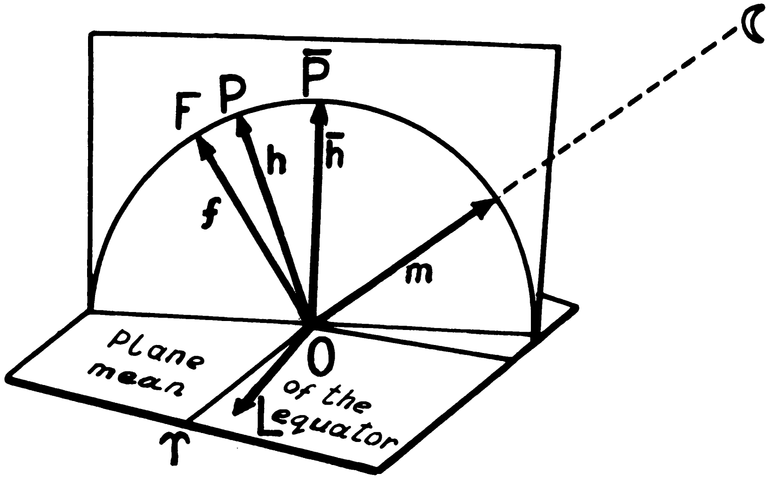

Figure 2. Relationship among the axis of figure, angular momentum vector, and the direction to the Moon or Sun.

The benefit of such a choice is that the unit vector $\bar{f}$ does not move rapidly with respect either to the xyz system rotating with the Earth or to a non-rotating frame attached to remote sources. The IAU General Assembly in Grenoble, 1976, recommended that the ephemeris axis, ζ, should be redefined in the manner proposed by Atkinson but avoided using the term "axis of figure". This was done because the motion of this axis is known to consist of free and forced components while the ζ-axis defined in accordance with Jeffreys' and Atkinson's suggestions is affected by only the forced motion and does not coincide with the axis of figure. Atkinson (1975) sometimes uses the term "axis of figure" in referring to the forced motion of the axis of figure and at other times to refer to the conventional z-axis. He writes, "We now define as the 'pole of figure' the adopted mean pole from which meridian observers reckon their colatitudes ... any constant adopted colatitudes will be adequate, assuming that they are roughly correct." It may be due to this lack of consistency in terminology that the following resolution was adopted in Kiev in 1977:

> IAU Symposium No. 78 recommends that the decision of the sixteenth General Assembly of the IAU that "the tabular nutation shall include the forced periodic terms listed by Woolard for the axis of figure" shall be annulled and that the nutation of

the true pole of date with respect to the mean pole of date should be computed for the motion of the instantaneous axis of rotation.

It has been mentioned that the direction of the angular momentum vector, $\overline{H}$, (or of the unit vector, $\overline{h}$) is the same for the rigid, elastic, or any other reasonable Earth model. However, the relative motion of the vectors $\overline{h}$ and $\overline{f}$ substantially depends on the mechanical properties of the Earth. According to McClure (1973) the effect of tidal deformation of the elastic Earth manifests itself in the relative motion of these vectors with an amplitude reaching two seconds of arc.

It has been pointed out already that the Jeffreys-Atkinson axis can be defined as an axis that would have no short-period motion either in space or relative to the terrestrial frame. This conceptual definition was first applied to the rigid Earth, but it can be extended to elastic models with a liquid core. The problem now is to replace the rigid Earth in the theory of precession and nutation with another model better fitted to our current knowledge of the mechanical properties of the Earth and to derive for this model the equations of motion of the axis which satisfy the requirement that it should only change its direction slowly on the time scale of a day with respect to both the celestial and terrestrial frames of reference. It is to be expected that the equations of transition from the vector $\overline{h}$ to this axis be somewhat different from Woolard's (54). To monitor the motion of the ephemeris ζ-axis with respect to the conventional xyz-system special observations are conducted.

We may pass to the more familiar geometrical representation by constructing the auxiliary unit sphere of Figure 3.

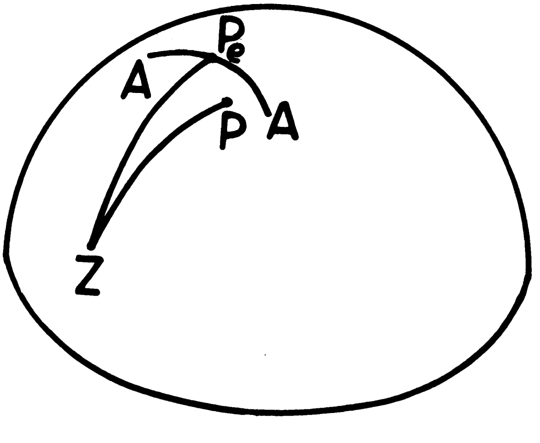

Figure 3. The auxiliary unit sphere.

The point at which the ζ-axis passes through this sphere is the instantaneous ephemeris pole, P. Any conventional system in which the

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motion of this pole is monitored is linked to the unit vectors $\bar{e}_j$. To define the orientation of the z-axis of this system it is sufficient to adopt two angles which this axis forms with each unit vector $\bar{e}_j$. If we use more than two vectors the values of the angles cannot be prescribed arbitrarily. These angles should be derived from observations.

THE CONVENTIONAL INTERNATIONAL ORIGIN

The IPMS uses the conventional axis attached to the plumb lines at five points on the Earth's surface. The unit vectors, $\bar{e}_j$, parallel to these plumb lines define the zeniths, $\bar{z}_j$, on the auxiliary sphere. Let us take one of the latitude stations and assume that its longitude is known. Then, deriving from an observation at time, t, the instantaneous latitude, ϕ, of the station, we obtain the position of the zenith in the ephemeris system $\xi \eta \zeta$ measuring from the pole P the arc PZ along the meridian of the station equal to its colatitude, $90^\circ - \phi$. By definition the CIO is the point P_e located at the angular distance, $ZP_e = 90^\circ - \Phi$, from the zenith Z, where Φ is the adopted constant value of the mean latitude of the station. In other words, the locus of the conventional pole P_e is a circle, AA described on the auxiliary sphere with Z as the center and spherical radius, $90^\circ - \Phi$.

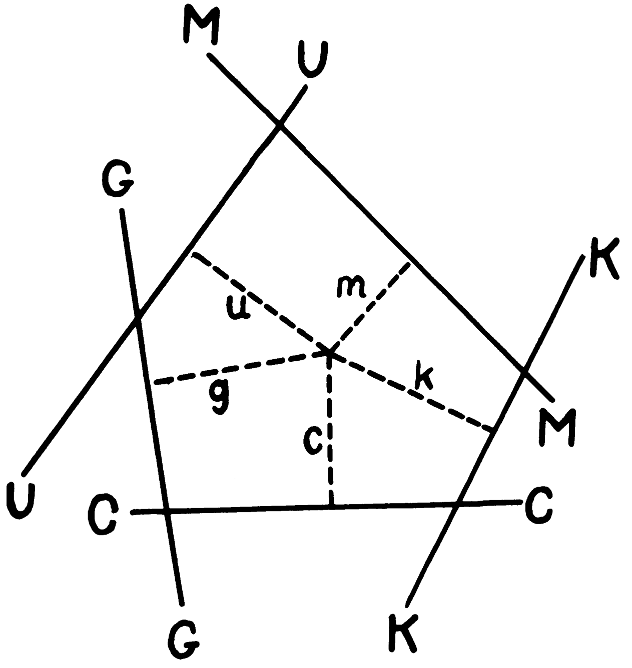

Figure 4. Lines of position defining the CIO.

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Considering a small region on the sphere in the vicinity of P (Figure 4) we may replace the arc AA by a straight line located at a distance $(90^\circ - \phi) - (90^\circ - \Phi) = \Phi - \phi$ from the instantaneous ephemeris pole, P, and called the line of position. The CIO is believed to be the point at which all the lines of position of the five international stations always cross. However, from inspection of the observational data we can satisfy ourselves that such a point does not exist at all. Thus, the CIO cannot be defined as the point situated at the constant defining angular distances from the zeniths of the five International Latitude Service stations. The very definition of the CIO should be changed.

It is easy to show that the CIO is the point for which the following always exists:

$$m^2 + k^2 + c^2 + g^2 + u^2 = \text{minimum}. \tag{5}$$

In this condition m, k, c, g, and u represent the distances from the lines of positions of the stations Mizusawa, Kitab, Carloforte, Gaithersburg, and Ukiah respectively.

THE POLE OF THE BIH 1968 SYSTEM

Time observations are also capable of giving the position of the conventional pole, P_e, relative to the ephemeris pole, P. Taking two stations we can obtain the line of position, BB, (Figure 5) such that any point on the line assumed for P_e will preserve the longitude difference of the selected stations. If the zeniths of the stations were fixed to one another all such lines of position will cross at a single point which may be taken for the conventional pole, P_e. This is not the case, and we have to determine the position of P_e by means of a condition similar to (5).

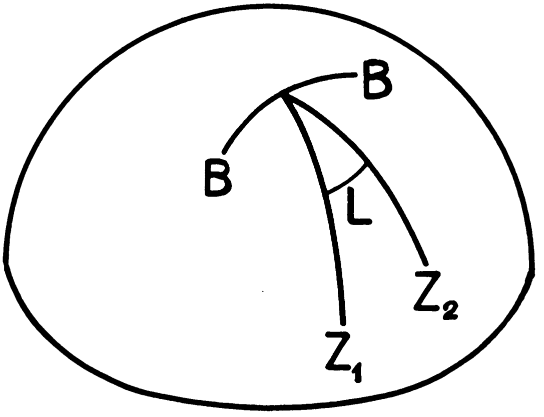

Figure 5. Line of position for the BIH 1968 System.

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The realization of the terrestrial reference system adopted by the BIH is achieved by assigning mean longitudes and latitudes to a number of stations. This means that the z-axis is related to the lines of position derived from both time and latitude observations. However, it is not fixed with respect to the unit vectors $\bar{e}_j$ since they do not maintain their directions relative to one another.

THE MEAN POLE OF THE EPOCH OF OBSERVATION (ORLOV'S POLE)

The fact that sum (5) nearly always differs significantly from zero is considered an indication of a nonpolar component in the latitude variations. To separate this component Orlov (1941) compared observations at observatories with nearly the same (or differing by 180°) longitudes. Periodic variations of latitudes proved to be nearly identical, while variations of the mean latitudes (obtained by filtering out periodic components) were quite different.

This has been confirmed by Mironov (1974) who obtained correlation coefficients for a number of pairs of stations with nearly equal longitudes. For periodic variations the correlation coefficients proved to be always positive and only in rare cases smaller than 0.75. On the other hand, divergent values ranging from -0.90 to +0.90 have been obtained for the non-periodic variations. That is a forceful argument in favor of the opinion that variations of the mean latitude are of a nonpolar origin and that these variations should be excluded from observations prior to using them for the computation of polar motion. Then, proceeding in the same way as in the case of the determination of the CIO, we shall arrive at the mean pole of the epoch of observation, P_O. The BIH used this pole from 1959 to 1967.

The relative displacement of the CIO and P_O is called the secular polar motion. Several authors have tried to determine the rate and direction of this motion. The agreement of their results is easily explained since all of them have applied the same methods to the same initial data from the ILS Stations. To estimate the reliability of the results obtained we have derived linear trends from observations from 1900 to 1972 (Fedorov, 1975). The following centennial rates have been obtained:

Mizusawa	-0".330,
Carloforte	+0.049,
Gaithersburg	+0.241,
Ukiah	+0.345.

Observations at Kitab commenced in 1931 and have been found to be useless for our discussion.

The following null hypothesis has been considered: the observed linear trends are independent random values. Using known methods of statistical testing we have found that the probability of this hypothesis is equal to 0.38 which means the hypothesis does not contradict observations at

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these four international stations. These data are too scanty for a definite answer as to whether or not the secular motion of the pole has taken place during the last century.

These considerations lead me to believe that the motion of the ephemeris pole should be related to the mean pole of epoch rather than to the CIO. In other words, the z-axis of the conventional terrestrial system should be directed toward P_o. Now we shall consider the general principles of the observations from which the polar motion can be derived.

CLASSICAL ASTRONOMICAL METHODS

Let $\bar{e}$ be the unit vector parallel to the vertical defined in the conventional terrestrial system by its coordinates. The matrix M(t) is not known in advance. Thus, to convert to the non-rotating frame XYZ we must use the "ephemeris" matrix M_o. If $\bar{s}$ is a unit vector directed towards the observed star we may obtain the "ephemeris" or predicted cosine of the angle between $\bar{s}$ and $\bar{e}$ by a scalar multiplication of $M_o\bar{e}$ by $\bar{s}$:

$$\cos \gamma_o = \bar{s}\cdot M_o\bar{e}.$$

The observed value of the cosine is

$$\cos \gamma = \bar{s}\cdot(M\bar{e} + \bar{\Sigma}),$$

where $\bar{\Sigma}$ is the sum of errors independent of the relative orientation of the coordinate systems. Therefore

$$\cos \gamma - \cos \gamma_o = \bar{s}\cdot(M - M_o)\bar{e} + \bar{s}\cdot\bar{\Sigma} = \bar{s}\cdot M_o\sigma\ \bar{e} + \bar{s}\cdot\bar{\Sigma}. \qquad (6)$$

From (6) one can obtain equations for deriving coordinates of the pole from astronomical observations as well as the difference between universal and atomic time.

RADIO INTERFEROMETRY

The conventional terrestrial system may also be attached to the unit vector, $\bar{e}$, of the baseline of a radio interferometer. Zhongolovitch (1976) emphasizes that such a system will be based on a much more rigid foundation than any using the directions of the plumb lines of several observatories.

The method of observation is based on the same equation (6). If D is the length of the baseline connecting the two antennas and c is the velocity of light, then we immediately obtain from (6)

$$\tau - \tau_o = s\cdot\frac{D}{c}M_o\sigma e + s\cdot\Sigma, \qquad (7)$$

where $\tau - \tau_o$ is the difference in the time of arrival of the signal at the two antennas.

LASER RANGING TO SATELLITES AND REFLECTORS ON THE MOON

In this case $\bar{e}$ refers to the unit vector along the geocentric position vector of the observing station. The construction shown in Figure 6 is made in the object space, but the only information required for Earth rotation is the variation in the direction of $\bar{e}$ (or the attached xyz system) in the image space.

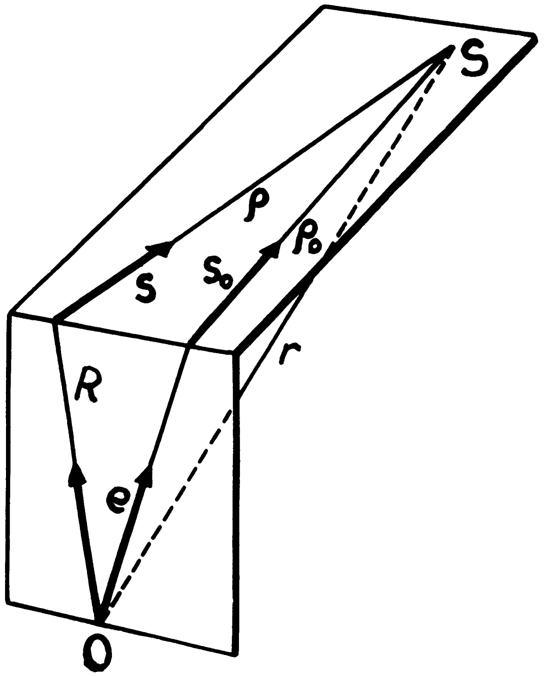

Figure 6. Geometry used in the determination of Earth rotation by laser ranging.

We denote by ρ the distance from the station to the satellite and by R the modulus of the position vector of the station. Then we may write the basic equation,

$$\rho\bar{s} = \bar{r} - RM\bar{e} + \bar{\Sigma}. \tag{8}$$

For the "ephemeris" value of the "station-satellite" vector we have

$$\rho_o s_o = r_o - RM_o\bar{e}. \tag{9}$$

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Hence

$$\rho\bar{s} - \rho_o\bar{s}_o = \rho(\bar{s} - \bar{s}_o) + (\rho - \rho_o)\bar{s}_o = - RM_o\sigma e + \bar{\Sigma}, \qquad (10)$$

where the difference $r - r_o$ is included in $\bar{\Sigma}$. Scalar multiplication of (10) by s_o taking into account that $s \cdot (s - s_o) = 0$ leads to

$$\rho - \rho_o = - \bar{s}_o \cdot RM_o\sigma\, e + s_o\bar{\Sigma}. \qquad (11)$$

This equation underlies the methods of deriving the coordinates of the pole from laser ranging results. Scalar multiplication of (10) by a vector, $\bar{n}$, normal to $\bar{s}$ leads to the expression,

$$n \cdot (\bar{s} - \bar{s}_o) = - \frac{R}{\rho}\,\bar{n} \cdot M_o\sigma\, e + \frac{\bar{n}}{\rho} \cdot \bar{\Sigma}. \qquad (12)$$

The dynamical method by which polar coordinates can be determined from observations of the directions to satellites are based on the general expression (12).

It should be realized that the coordinates of the pole enter equations (7), (11), and (12) through the matrix σ. Since the right-hand parts of these equations contain the matrix M_o quasi-diurnal variations should be seen in the differences $\rho - \rho_o$, $\tau - \tau_o$, and $\bar{s} - \bar{s}_o$.

Depending on the methods and techniques employed, the axes of the conventional terrestrial system are related to different unit vectors. Comparison of the polar coordinates obtained in different systems is capable of giving information on the relative motion of the z-axis of these systems since the position of the ephemeris pole in space is independent of the choice of the conventional terrestrial reference frame.

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