The gravitational lens effect

of galaxies and black holes

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This thesis examines the multi-imaging aspect of the gravitational
lens effect of both galaxies and black holes. Galaxies are modeled by
spheroids, the density of which is assumed to have spheroidal symmetry,
that is, the density is constant on the surface as well as all the other
spheroidal shells that make up the spheroid. If such a spheroid is
projected onto a plane then the resultant solid ellipse's density has
elliptical symmetry.

It is shown that the equation $z = z_0 - 4DGc^{-2}I^*(z_0)$ which relates
the position of an image z_0 with the position of the source z, must
satisfy certain symmetry properties. In particular,

$$z = z_0 - 4DGc^{-2}I^*(z_0) \Rightarrow -z = -z_0 - 4DGc^{-2}I^*(-z_0) .$$

The solution for $I(z_0)$ presented by Bourassa and Kantowski [1]
violates this property, as their $I(z_0) = I(-z_0)$. Thus a corrected $I(z_0)$
is computed. This contains a proof of the fact that if the projected
density has elliptical symmetry, then there is no contribution to the
bending angle due to mass lying outside the ellipse formed by the impact

parameter. The new $I(z_0)$ is used to present diagrams which show how many images of a single source will result as a function of the position of the source with respect to the galaxy.

Similar diagrams are presented for the case where it is the Kerr black hole that behaves as a gravitational lens. Unlike other massive objects black holes have r = constant orbits for null geodesics. This implies that a single source of light will have infinitely many images at the observer. These consist of the direct rays and rays which orbit the black hole once, twice, etcetera. We concern ourselves with the direct rays as these are the ones which are least deviated, hence least deformed and most intense, and therefore, are the ones which are most likely to be observed.

Approximate solutions to the equations of motion for direct null geodesics in the Kerr metric are derived. These are correct up to and including second order terms in m/r_{min} and a/r_{min} , where m is the Kerr mass, a is the Kerr spin and r_{min} the distance of closest approach, and are sufficient for most astronomical purposes. Second order terms are included as spin comes in only at this order and our primary interest is the effect of spin on image multiplicity. It is found that the derived expressions can be combined with numerical integration to give excellent results for rays which have relatively large deviations.

References

[1]   Bourassa, R.R. and Kantowski, R., *Astrophys. J.* 13 (1975) 195.

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