CORRESPONDENCE.

To the Editor of the Transactions of the Faculty of Actuaries.

SIR,

SCOTTISH BANKERS' EXPERIENCE, 1903-1923.

The annuity values applicable to All Males combined given in Table VII. of this Experience (*T.F.A.*, vol. x., p. 257), are calculated, as there stated, from the totals of the l columns of the Combined Marriage and Mortality Table (Table V.). In using these annuities for the valuation of contributions it is assumed that the proportions of married men (including widowers) and bachelors in a Fund are the same as in Table V., but it was considered that any error due to variations in the proportions would be small.

Mr. Hugh W. Brown has recently valued a Fund in which he had occasion to value separately the contributions payable by bachelors, and he has kindly handed me a note of the formulas employed, and of the resulting annuity values at 4 per cent. interest. For the valuation of contributions payable by bachelors, he used (1) the value of an annuity payable while the bachelor remains unmarried, the formula for which is

$$(ba)_x = \sum_1^\infty v^{x+i} (bl)_{x+i} \div v^x (bl)_x;$$

and (2) the value of an annuity deferred until marriage, the formula for which is

$$(bma)_{x} = \sum_{0}^{\infty} \{ v^{x+t+\frac{1}{2}} (bm)_{x+t} \times \mathbf{j} | (ma)_{x+t+\frac{1}{2}} \} \div v^{x} (bl)_{x},$$

where $\psi(ma)_{x+t+1}$ is taken from Table VI., applicable to married men. (The latter formula is slightly different from that given by Dr. T. B. Sprague, who assumes that the first payment is due one year after marriage.) The sum of the two annuity values gives the total annuity for the valuation of bachelors' contributions when these are not altered on marriage.

In order to test the error in the original assumption, I re-valued (at 4 per cent. interest) the contributions in the specimen Fund, omitting for simplicity a small class of messengers and some minor special payments, and the following are the results in respect of each \pounds 1 of contribution. The contributions of bachelors were valued by Mr. Brown's combined table, and those of married men by Table VI.

Class.	Table VII.	H.W.B. & VI.	Difference.	
Bachelors	$\pounds9,576.1$	£9,464·3	+£111.8	
Married and Widowers	6,007.2	6,131 0	- 123·8	
Total	£15,583·3	£15,595·3	- £12·0	

The net difference was thus less than one-tenth of one per cent., which may be regarded as negligible. In order to ascertain whether the closeness of the approximation was due to the membership in the particular Fund being very near in distribution to the figures in Table V., the respective proportions of bachelors and married were found for quinquennial groups of ages, and the differences were sufficiently large to indicate that the closeness of the approximation was not accidental. Mr. Brown calculated the values of contributions in the other Fund by the same method, and found that there also the difference between the results was quite negligible.

I append the tables of annuity values at 4 per cent. interest calculated by Mr. Brown, giving the separate values, and the sum of the two. At ages 20 and under the combined figures should in theory equal the annuity values in Table VII., and the actual differences do not exceed .002.

I am,

Yours faithfully,

ALEX. FRASER.

15 SOUTH LEARMONTH GARDENS, EDINBURGH, January 1928.

BACHELORS' ANNUITIES.

1513 :9456 :71820 :663559 :56997910 :5481613 :5387 :00420 :542569 :318:96810 :1861713 :1147 :30220 :416579 :053.7679 :8201812 :6737 :61320 :2416579 :053.7679 :8201912 :2137 :93820 :151598 :480:5849 :0542011 :7338 :27620 :009608 :174:5048 :6782111 :2538 :61119 :864617 :862:4318 :2932210 :7828 :93119 :713627 :546:3657 :9112310 :3189 :23819 :556637 :225:3077 :532249 :8719 :95519 :052666 :264:1666 :430278 :38510 :03718 :872675 :962:1316 :043288 :6099 :82618 :486695 :405:0745 :767308 :7069 :57318 :279705 :149:0525 :201318 :3299 :23518 :064714 :905:0344 :33930:8 :7069 :57318 :279705 :149:052:271318 :3299 :23516 :78876:3 :74:3 :7443610 :1636 :42517 :7592734 :430:0104 :440<	AG:	• (ba) _x	(bma) _x	$(ba)_x + (bma)_x = a_{(b)e}$	AGE	(ba) _z	(bma) _x	$(ba)_x + (bma)_x = a_{(b)x}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	15 16 17 18 19	13 •945 13 •538 13 •114 12 •673 12 •213	6·718 7·004 7·302 7·613 7·938	20.663 20.542 20.416 20.286 20.151	55 56 57 58 59	9.569 9.318 9.053 8.774 8.480	979 868 767 672 584	10°548 10°186 9°820 9°446 9 064
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	20 21 22 23 24	11.733 11.253 10.782 10.318 9.871	8·276 8·611 8·931 9·238 9·523	20.009 19.864 19.713 19.556 19.394	60 61 62 63 64	8·174 7·862 7·546 7·225 6·901	[•] 504 [•] 431 •365 •307 •254	8.678 8.293 7.911 7.532 7.155
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	25 26 27 28 29	9 • 454 9 • 097 8 • 835 8 • 699 8 • 660	9·772 9·955 10·037 9·983 9·826	19·226 19·052 18·872 18·682 18·486	65 66 67 68 69	6·579 6·264 5·962 5·676 5·405	·207 ·166 ·131 ·100 ·074	6 786 6 430 6 093 5 776 5 479
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	30 31 32 32 34	8 706 8 829 9 027 9 302 9 621	9·573 9·235 8·808 8·290 7·712	18·279 18·064 17·835 17·592 17·333	70 71 72 73 74	5·149 4·905 4·664 4·430 4·201	·052 ·034 ·020 ·010 ·003	5°201 4°939 4°684 4°440 4°204
	38 36 37 38 39	9 909 10 163 10 384 10 572 10 729	7.157 6.625 6.118 5.636 5.178	17.066 16.788 16.502 16.208 15.907	75 76 77 78 79	3 ·974 3 ·745 3 ·513 3 ·273 3 ·028	···· ···· ···	3·974 3·745 3·513 3·273 3·028
45 11 019 2 978 13 997 85 1 631 1 631 46 10 967 2 700 13 667 86 1 424 1 424 47 10 895 2 438 13 333 87 1 227 1 227 48 10 803 2 193 12 996 88 1 042 1 042 49 10 690 1 964 12 654 89 *867 *867 50 10 555 1 753 12 308 90 *700 *700 51 10 398 1 561 11 959 91 *536 *536 52 10 220 1 389 11 609 92 *360 *360	40 41 42 42 44	10°856 10°953 11°015 11°048 11°047	4.742 4.332 3.953 3.599 3.276	15 [.] 598 15 [.] 285 14 [.] 968 14 [.] 647 14 [.] 323	80 81 82 83 84	2.785 2.545 2.309 2.079 1.852	 	2·785 2·545 2·309 2·079 1·852
50 10·555 1·753 12·308 90 '700 '700 51 10·398 1·561 11·959 91 '536 '536 52 10·220 1·389 11·609 92 '360 '360	48 46 47 47 48	11.019 10.967 10.895 10.803 10.690	2.978 2.700 2.438 2.193 1.964	13.997 13.667 13.333 12.996 12.654	85 86 87 88 89	1.631 1.424 1.227 1.042 .867	····	1.631 1.424 1.227 1.042 .867
53 10.021 1.237 11.258 93 ·186 ·186 54 9.804 1.101 10.905 1 ·186 ··· ·186	50 51 52 58 54	10°555 10°398 10°220 10°021 9°804	1 ·753 1 ·561 1 ·389 1 ·237 1 ·101	12:308 11:959 11:609 11:258 10:905	90 91 92 93	•700 •536 •360 •186	····	-700 -536 -360 -186

Scottish Bankers' 1903-1923 Experience—Interest 4%.