

Stereological Characterization of Structural Anisotropy of Rolled Steel

L.M. Karlsson* and L.M. Cruz-Orive**

*Dalarna University, SE-781 88 Borlänge, Sweden

**Department of Mathematics, Statistics and Computation, Faculty of Sciences, University of Cantabria, Avda. Los Castros s/n, E-39005 Santander, Spain

The purpose of this study was to assess the structural anisotropy of the elongated austenite phase in a rolled duplex stainless steel, applying the concept of star volume distribution [1]. This concept is an elaboration of the *star volume*, which is the mean phase volume that can be seen unobscured from an interior point in the phase, as the point moves over the entire phase [2].

Common approaches for characterizing structural anisotropy concentrate on the boundary surface of the structure. Many strongly anisotropic structures, however, have nearly isotropic boundaries. To improve the analysis, the concept of volume orientation distribution was proposed but this distribution is difficult to estimate from sections without model assumptions [3].

Take a uniform random point x inside the phase of interest and consider a straight line through x with orientation u in 3D. Let $l(x, u)$ denote the length of the unbroken intercept containing x determined by the straight line. The *star volume distribution* is then defined as

$$\bar{v}_V^*(u) = \frac{\pi}{3} \cdot \mathbb{E}_V \mathbb{E}_x \{l^3(x, u)\}, \quad u \in \text{unit hemisphere},$$

where \mathbb{E}_x denotes expectation of x over uniform random positions inside the phase, and \mathbb{E}_V denotes expectation under the volume weighted condition (for a phase of separated objects). Clearly, if the phase is strongly elongated along some direction u then $\bar{v}_V^*(u)$ will also be large.

In practice $\bar{v}_V^*(u)$ may be approximated via the *empirical star volume distribution*. For this purpose the unit hemisphere \mathbb{H}^2 is split into n regions of equal area. Let $\bar{v}_{V_i}^*$, $i = 1, 2, \dots, n$, denote the average of $\bar{v}_V^*(u)$ over the 3D orientations that constitute the i th region on \mathbb{H}^2 . The empirical star volume distribution is then defined as the set of n pairs consisting of $\{\bar{v}_{V_i}^*\}$ and the corresponding regions on \mathbb{H}^2 . The n star volume components $\{\bar{v}_{V_i}^*\}$ can be estimated from a set of vertical sections (Fig. 1) [4]. These must be oriented in such a way that the set of 3D orientations making the desired region on the hemisphere can be sampled. A test system of points on parallel lines is overlaid on a set of properly oriented vertical sections, with the test lines along an orientation systematically sampled in the region of interest. If m_i test points hit the target phase in the i th region, then an unbiased estimator of the i th star volume component is

$$\text{est} \bar{v}_{V_i}^* = \frac{\pi}{3} \cdot \frac{1}{m_i} \sum_{j=1}^{m_i} l^3(i, j), \quad (i = 1, 2, \dots, n),$$

where $l(i, j)$ is the length of the unbroken intercept through the j th point measured along the test line.

Plotting the estimated star volume components along the corresponding orientations on the unit hemisphere (and their symmetric images on the opposite hemisphere) gives an idea of the required star volume distribution (Fig. 2). These data were reduced further by computing the corresponding ellipsoid of inertia, an intuitive graphical summary. The procedure was applied to three plates produced under different conditions. The analysis discriminated the degree of anisotropy of the investigated structures in accordance with the perceived anisotropy. This study therefore suggests that the proposed method may constitute a simple and quantitative summary of structural anisotropy.

References

- [1] L.M. Cruz-Orive et al., *Micron and Microscopica Acta* 23 (1992) 75
- [2] J. Serra, *Image Analysis and Mathematical Morphology*, Academic Press, London, 1982
- [3] A. Odgaard et al., *J. Microsc.* 157 (1990) 149
- [4] A.J. Baddeley et al., *J. Microsc.* 142 (1986) 259

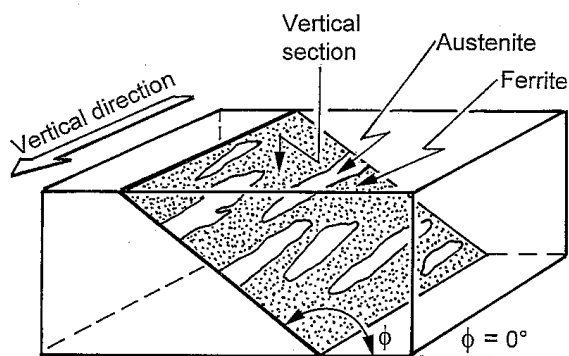


Fig. 1: Vertical section from a rolled duplex steel plate for assessing the anisotropy of the elongated austenite bands.

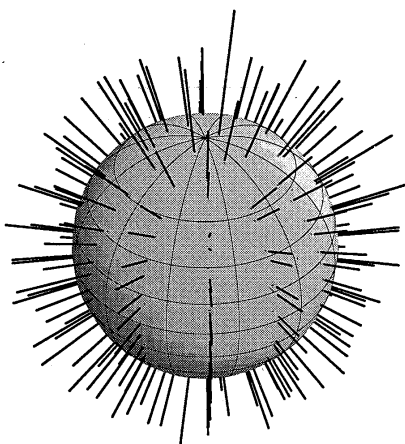


Fig. 2: A visual impression of the empirical star volume distribution.