15

Introduction to more advanced topics

This book has covered many of the primary topics in perturbative QCD, with a focus on certain inclusive processes for which particularly systematic treatments are available. It should provide the reader with a sound conceptual framework for further study and research. However, hadronic interactions form a vast subject, and there is an enormous literature where perturbatively based methods have been applied.

This chapter gives a summary of a selection of important areas of further application of perturbative QCD.

One common theme, a prerequisite for actual perturbative calculations, is that the reactions have in some sense a controlling hard subprocess, occurring on a short distance scale, i.e., a distance scale significantly less than 1 fm, or, more-or-less equivalently, a momentum transfer significantly larger than the typical hadronic scale of a few hundred MeV.

Another recurring idea, perhaps the closest to a unifying motif, is the idea that one should try to separate (factor) phenomena on different scales of distance and momentum. This refers not just to scales of different virtuality, but also to a separation of phenomena at widely different rapidities. A characteristic here is that almost scattering processes examined in high-energy physics are ultra-relativistic. Thus time dilation and Lorentz contraction of fast-moving hadrons by themselves provide a wide range of distance scales. For example at the Tevatron collider we have proton and antiproton beams of energy almost 1 TeV. This allows the measurement of hard processes with momentum scales of several hundred GeV. Therefore distances as small as 10^{-3} fm can be probed. Now the intrinsic distance scale of phenomena in a proton is about 1 fm in its rest frame. So time dilation of the beams indicates that there are phenomena relevant to the collisions occurring on the much larger scale of 10^3 fm. Thus relevant distance scales span 6 orders of magnitude (the *square* of E/M). Such a big ratio allows for many simplifications and useful approximations, and not just those that directly impinge on the applicability of perturbative methods.

This train of thought leads to one common (but not universal) theme, that of light-front methods. Most systematically, one can represent the states of fast-moving hadronic systems in terms of their light-front wave functions. As we have seen throughout this book, one cannot take the elementary formulations of light-front quantization etc. literally; many of the basic ideas must be considerably distorted to be applied correctly in QCD. Nevertheless this area gives concepts and methodology that underlie much of the work.

The significance of light-front methods goes beyond that of perturbative applications to relatively short-distance phenomena. There is a close relation to phenomena in soft

https://doi.org/10.1017/9781009401845.015 Published online by Cambridge University Press

hadronic physics (Gribov, 1973, 2009). This is an area often characterized as the domain of Regge theory. Although Regge theory was extremely influential in the pre-QCD era, and although one can still see its effects on current research, there is not yet a properly established connection with QCD from first principles. This is an area that deserves more investigation now that QCD is a very mature subject.

Many of the topics listed in this chapter concern some of the most difficult parts of QCD. It is not surprising therefore that their justification from fundamental principles is not always sufficient. It is generally difficult for an outsider, even for one experienced in perturbative QCD, to acquire an full understanding of these areas from the published literature. Whether or not scepticism in any particular case is justified, I will leave to the future to decide.

15.1 Light-front wave functions and exclusive scattering at large momentum transfer

One natural application of hard-scattering methods is to elastic scattering at large momentum transfer. The classic early reference is Lepage and Brodsky (1980). Standard examples include elastic hadron-hadron scattering $H_A + H_B \to H_C + H_D$ at wide angle, and electromagnetic form factors of hadrons at large momentum transfer.

The standard methods of region analysis apply: Sec. 5.9.3. An obvious kind of region was shown in Fig. 5.34(a), where essentially the partonic content of each external hadron collapses to a small configuration at a single hard scattering. The wide-angle hadronic scattering is then controlled by a kinematically equivalent scattering of valence quarks from each hadron. If we follow the same logic as for inclusive scattering, the non-perturbative factors are light-front wave functions (with an integral over transverse momentum). They are obtained from matrix elements of light-front annihilation operators between a single hadron and the vacuum, e.g.,

$$\langle 0 | b_{k_1,\lambda_1} b_{k_2,\lambda_2} b_{k_3,\lambda_3} | P \rangle \,, \tag{15.1}$$

where the operators are as in Secs. 6.6 and 6.7. One expects the usual QCD complications, of course.

But because the hard-scattering subgraph has more external partons than in inclusive scattering, the cross sections fall with a higher power of the hard scale Q than corresponding inclusive cross sections. So it is hard to probe very large Q experimentally. In addition, this strong decrease allows the possibility of other regions contributing with either the same power law or a less-suppressed power. See Sec. 5.9.3 for a brief discussion of one example, the Landshoff process. See the citations to Lepage and Brodsky (1980) and Landshoff (1974) for subsequent work.

15.2 Exclusive diffraction: generalized parton densities

A related topic concerns exclusive processes in large-Q inelastic lepton-hadron scattering. We examined the leading regions for such processes in Sec. 5.3.6. Standard examples presented there were deeply virtual Compton scattering, double deeply virtual Compton scattering, and exclusive production of mesons. The hadronic parts of these reactions are $\gamma^*(q) + P \to \gamma + P$, $\gamma^*(q) + P \to \gamma^*(q') + P$, and $\gamma^* + P \to M + P$, respectively.

Experimentally, these processes are often investigated at small Bjorken x (where the cross section is largest), so they also take on the characteristics of diffractive scattering.

In the normal case that the momentum transfer from the target-hadron end is small, the appropriate factorization property uses what are called "generalized parton densities" (GPDs). These are defined exactly like parton densities, except that the hadronic matrix element is off-diagonal, (6.90). See Sec. 11.8 for a further discussion. In exclusive production of mesons, a light-front wave function of the meson is needed, the same quantity that appears in elastic scattering of the meson. See Diehl (2003) for a good review.

15.3 Small-x, BFKL, perturbative Regge physics

In DIS much work deals with the region of small x. There is considerable experimental data from the HERA collider, where the high center-of-mass energy allowed *ep* collisions to go to small x while maintaining Q in a perturbative region, e.g., Q of a few GeV with x as small as 10^{-5}. The standard treatment of DIS involves the limit of large Q at fixed x, so the small-x regime introduces another large ratio in addition to the ratio of the hard scale to the hadron mass, Q/M.

In the small-x region, the ideas of Regge theory become relevant. Regge theory concerns asymptotic behavior where s is large and momentum transfer is fixed. This includes the total hadronic cross section at large s.

Now DIS structure functions correspond to a cross section for scattering of a virtual photon on a hadron: $\gamma^* P \to X$. At small x, the mass of both the photon and the hadron are much less than their center-of-mass energy, which is $Q\sqrt{(1-x)/x}$. When Q is in a perturbative region, one can hope that Regge theory can be usefully approximated by perturbative methods. Investigations of a Regge limit in perturbation theory for non-abelian gauge theories led to the equation of Balitsky, Fadin, Kuraev, and Lipatov (BFKL) (Fadin, Kuraev, and Lipatov, 1975; Balitsky and Lipatov, 1978). For a review, see Lipatov (1997).

This and a number of closely allied developments have had many applications, in situations where a Regge limit is appropriate. If the DGLAP equation is regarded as governing the Q dependence of DIS structure functions and parton densities, then the BFKL equation governs the x dependence, at small x.

For partonic scattering at high energy and small angle, the BFKL equation gives a ladder structure that is very similar to the multiperipheral model we mentioned in Sec. 14.3. However, the actual Feynman graphs that give the leading behavior are gauge dependent and need not be actual ladder graphs. Primarily the derivations use the leading-logarithm method, and therefore concern the situation where the gluons are strongly ordered in rapidity. But important work concerns NLO corrections.

There is interesting work by Balitsky (e.g., Balitsky, 1999), who relates the BFKL equation to the evolution of Wilson-line matrix elements with respect to the rapidity of the Wilson-line directions; thus his work is related to our treatment of TMD functions in Ch. 13.

One characteristic of the BFKL equation is that it implies that its approximation to the pomeron has an intercept well above unity. The pomeron was originally characterized as the Regge exchange that gives the highest power of energy in elastic hadronic scattering. Its intercept $\alpha(0)$ gives a total hadron-hadron cross section proportional to $s^{\alpha(0)-1}$. But the Froissart bound requires that the cross section rise at most like $\ln^2 s$. Phenomenologically hadronic total cross sections do rise slowly. Thus something with an intercept far above unity cannot be the true pomeron. However, there does appear to be a transition in DIS between soft pomeron behavior at low Q, with approximately constant $\gamma^* p$ cross sections, and a "hard pomeron" behavior at higher Q, with a substantial rise with energy (e.g., H1 Collaboration, 2010)

Related issues concern the CCFM equation (Ciafaloni, 1988; Catani, Fiorani, and Marchesini, 1990a, b; Marchesini, 1995) for parton densities, etc. at small x.

15.4 Resummation, etc.

The basic method of using perturbation theory in QCD for a quantity with a large momentum scale Q is to use the RG to set the renormalization mass μ of order Q. This removes large logarithms of Q. But in many cases there are other parameters which can also give large logarithms. One way of viewing the problem is to observe that the quantity being calculated depends on multiple momentum scales rather than just Q.

One example is the hard-scattering coefficient in ordinary "collinear" factorization for the Drell-Yan process when $q_T \ll Q$. Among many other examples are processes at small Bjorken x (Sec. 15.3), and at large x (Sec. 15.8 below).

The most fundamental method of dealing with such situations is to formulate an appropriately improved factorization theorem, such as we did using TMD factorization for the Drell-Yan cross section in Ch. 14. After that the various perturbative coefficients are all single-scale quantities.

Another very common method is that of resummation. There one analyzes the source of the large logarithms. It is often not too hard to determine the leading logarithms to all orders of perturbation theory, even without a more complete treatment. This avoids exact Feynman-graph calculations at very high order. Then one sums the large higher-order corrections.

The vast literature on this subject can be sampled by searching for papers with titles containing "resummation" or "resummed".

Resummation is at its most useful when the logarithms are not too large, since it can provide an efficient way to improve the accuracy of perturbative calculations. One important example is in the use of resummed calculations of jet shapes (Gehrmann, Luisoni, and Stenzel, 2008) in e^+e^- annihilation to obtain accurate estimates of the strong coupling (Bethke *et al.*, 2009).

The method gets much harder to justify when the logarithms are large. For example, in the Drell-Yan process at small transverse momentum, the errors in the approximations giving collinear factorization include terms that are a power of M/q_T. When the transverse momentum is of order a hadronic mass, the derivation does not apply. TMD factorization solves this problem, with the outcome that more non-perturbative information is needed in the transverse momentum distributions of partons and of soft-gluon emission. In the intermediate region $M \ll q_T \ll Q$, the full TMD factorization property can be used to derive a resummation formula. In the version of TMD factorization given in (13.81), a resummation result can be obtained by omitting the non-perturbative factors in the fourth and fifth lines.

15.5 Methods for efficient high-order calculations

In many realistic applications of perturbative QCD, calculations of high-order graphs are needed. For the LHC, calculations of parton-parton scattering with many partons in the final state are used, preferably at one-loop order. Examples of the calculations are in Berger *et al*. (2009).

It is readily evident that such calculations are very complex, particularly when performed in the most direct way from the standard Feynman rules. A 3-gluon vertex has 6 terms, so that a graph with n such vertices has 6^n terms. Straightforward calculations by hand become very lengthy or impractical. Of course, intensive use of computers helps. It also helps if calculations are restricted to massless on-shell amplitudes as much as possible.

But it is also observed that the final results of a calculation are often much simpler than intermediate results, and certainly much simpler than one expects from the complications in individual graphs. This suggests that there are much better methods. See Bern, Dixon, and Kosower (2007) for a review of much of the work in this direction, with further references.

15.6 Monte-Carlo event generators

The analysis of the regions for Feynman graphs for processes with a hard scattering gives much more information on the detailed structure of the final state than we used in factorization theorems for inclusive cross sections. A contrasting approach is provided by Monte-Carlo event generators, e.g., PYTHIA (Sjostrand, Mrenna, and Skands, 2006, 2008) and HERWIG (Bahr *et al*., 2008). These are computer programs which simulate actual collisions. That is, they generate complete events with a distribution that is intended to be a useful approximation to the distribution of events in actual collisions.

Modern collider experiments generate events with many final-state particles, and the detectors are sensitive to most of the final state. The acceptance and efficiency of the detectors is quite complicated, and the signatures of many interesting signals (e.g., the Higgs particle) involve properties of whole groups of final-state particles. Therefore understanding the nature of a physics signal is greatly assisted by having a realistic simulation of the complete final state. Monte-Carlo event generators are therefore an essential tool in the analysis of experimental data in high-energy physics, not to mention the planning of future experiments.

Event generators also evade another problem. This is that the number of Feynman graphs rises with the order N of the Feynman graph roughly like $N!$. The difficulty of computing each single graph also rises with the order. Although modern methods ameliorate this

https://doi.org/10.1017/9781009401845.015 Published online by Cambridge University Press

somewhat, there is a formidable computational problem in directly computing production of final states with many particles. The situation is worse in QCD because straightforward perturbation theory is not useful without interesting reorganizations: factorization, renormalization group, etc. A Monte-Carlo event generator provides an approximation to the production of N particles that uses computational resources linear in N, instead of its factorial, thereby giving a dramatic improvement over unassisted perturbation theory.

The price is, of course, the approximation and the difficulty of justifying it.

To understand how the methods used by the event generators arise, consider our treatment in Sec. 8.9 of factorization in a non-gauge theory. There the dominant structures were generalized ladder graphs. We can extend these ideas to analyze the structure of the jets in the final state, obtaining a structure of ladders within ladders. If we take, as is appropriate, a fixed order for each rung, we have a small number of graphs in each order, and the number of rungs is proportional to N. The structure readily maps to the linear-in-N structure in an event generator. The use of Monte-Carlo methods, i.e., probabilistic methods, is the most sensible for numerical calculations of high-dimensional integrals and maps perfectly onto how data appears in a scattering experiments.

In QCD, the ladder structure only arises after a sum implemented by Ward identities from graphs with non-local attachments of gluons. The kinematics of final states, with soft gluons filling in rapidity gaps, is also much more complicated than in a non-gauge theory.

The theory of the event generators (see Sjostrand, Mrenna, and Skands, 2006; Bahr *et al.*, 2008; Sjostrand, 2009) is based on the ideas used in ordinary factorization theorems for inclusive processes. But a full justification, which I am not sure really exists, needs to go much further. One symptom of this is in the kinematic approximations used in deriving factorization for inclusive processes. At various points we change the kinematics of partons going into the final state from their actual values, but in such a way that the inclusive cross section is not affected (at leading power). But this is not adequate for an event generator where a complete description of the final state is to be given. An event that does not obey conservation of 4-momentum is not useful in this context. Prescriptions are needed to correct this (Bengtsson and Sjöstrand, 1988), and these do not fully match how inclusive factorization theorems are derived.

Furthermore, in hadron-hadron collisions, generating complete final states goes beyond a situation in which factorization in its basic form is valid. Event generators incorporate modelling of the soft final state and this can be regarded as a model of the spectator-spectator interactions that we examined (in the context of another very simple and naive model) in Sec. 14.3.

There has naturally been much work on Monte-Carlo event generators that I cannot review here. They represent an interesting way of combining the results of perturbative calculations with other elements including modelling of non-perturbative physics to give a very useful approximation to real QCD.

15.7 Heavy quarks

At various points in this book, I have mentioned the issues that arise when heavy quark masses are not small compared with the hard-scattering scale Q. Many situations can be dealt with by minor modifications of the standard factorization method; see for example Krämer, Olness, and Soper (2000).

But there are other situations that require different techniques. One of the most important is the analysis of the decays of hadrons containing heavy quark constituents, notably B mesons. This is the domain of heavy-quark effective theory (HQET). An account of HQET is found in Manohar and Wise (2000).

15.8 Large x

Limitations on the validity of a basic factorization theorem often arise near kinematic limits. An important case is DIS at $x \to 1$. There the spectator part of a typical leading region becomes soft instead of collinear, and therefore indicates that a change in the analysis is needed. Essentially the same considerations apply to any other inclusive process in a kinematic region where the initiating partons of a conventional hard scattering must have $x \to 1$. Similar issues arise in fragmentation as $z \to 1$. Because of the restricted kinematics of the spectator system, more accurate treatment of the kinematics is needed than in the conventional factorization.

Cross sections decrease quite rapidly as $x \to 1$ because parton densities decrease roughly as $(1 - x)^3$ or a higher power. This decrease affects the accuracy of conventional factorization methods. One indication of this is in the NLO correction (9.54) for DIS, where the plus distribution implements a cancellation between real and virtual gluon emission. Where the parton densities decrease rapidly this cancellation becomes inaccurate, giving large logarithms of $1 - x$.

Recent work can be traced from Almeida, Sterman, and Vogelsang (2009).

15.9 Soft-collinear effective theory (SCET)

In recent years a new approach to perturbative QCD has been developed under the name soft-collinear effective theory (SCET) (Bauer *et al.*, 2001; Bauer and Stewart, 2001). See Fleming (2009) for a recent overview. Historically SCET arose as a generalization of heavy-quark effective theory; see Sec. 15.7.

The overall philosophy of SCET is like that of the Wilsonian renormalization group. This is to integrate out certain ranges of momentum modes for the fields of QCD and to replace them with effective fields. In SCET momentum space is divided into many bins in each of which the integrating-out is to be done. A problem that needs to be addressed in any such method is how to deal with a momentum that lies just outside a boundary of an integrated-out region.[1] There is no small parameter to expand in, unlike the case of momenta far from the boundary.

In the Wilsonian RG this problem is overcome by using an infinite set of operators. But this rather obscures the underlying simplicity of the situation, where one has a simple factorization of coefficients times a limited set of operators.

[1] Compare the discussion in the first few paragraphs of Sec. 13.12.

https://doi.org/10.1017/9781009401845.015 Published online by Cambridge University Press

In reality, the integrating-out in SCET is performed by integrating over all momenta, with subtractions to enforce the region conditions. This is similar to what was done in this book in Ch. 10. However, I have not been able to penetrate the SCET literature to properly understand its rationale. I just refer the reader to the literature cited above.

15.10 Higher twist: power corrections

In deriving factorization we made approximations that used the leading power of an expansion in small variables like masses relative to a hard scale Q. It is natural to ask what can be done with non-leading powers.

The basic techniques do apply to non-leading powers. In fact, the earliest of the factorization theorems, the operator product expansion (OPE), does treat all powers, leading and non-leading, in a uniform formalism. The OPE (Collins, 1984, ch. 10) expresses a suitable matrix element in a limit of large *Euclidean* momentum q as a sum of q-dependent coefficients times q-independent operator matrix elements, e.g.,

$$\int \mathrm{d}^4x\, e^{iq\cdot x}\, \langle P|\, j(x) j(0) |P\rangle = \sum_i C_i(q)\, \langle P|\mathcal{O}_i|P\rangle\,. \tag{15.2}$$

The power law for the q dependence is controlled by the dimension of the operators.[2]

When the OPE is applied to moments of DIS structure functions, the normal leading power corresponds to operators $\mathcal{O}_i$ that obey

$$\text{dimension} - \text{spin} = 2. \tag{15.3}$$

This quantity is called twist, and non-leading powers have a higher value of twist. It has therefore become a standard jargon to use “higher twist” to refer to any power-suppressed correction. Leading-power factorization for inclusive processes is then labeled “twist-2”; integer moments of integrated parton densities are exactly matrix elements of twist-2 operators.

For work on higher-twist corrections to factorization see Qiu and Sterman (1991b).

But the vast majority of applications avoid the use of higher-twist corrections, trying to stay in kinematic regions where the leading-power formalism is sufficient. There are several reasons.

One is that the relevant generalizations of parton densities use multiparton operators, and these depend on more than one fractional momentum variable. The more non-leading the power, the larger the number of variables that is needed. But it is hard to extract such a multivariable function from data. This contrasts with the twist-2 case, where, in the parton-model approximation, DIS structure functions are simple linear combinations of quark densities.

One can only do better if one is in a special situation where the non-leading power terms are particularly simple.

[2] Here I ignore the effects of anomalous dimensions.

A second reason for not using power corrections is a fundamental limitation on the accuracy of perturbative calculations in QCD. Consider the perturbation series for an IR-safe quantity

$$F = \sum_{n=0}^{\infty} c_n \alpha_s(Q)^n, \tag{15.4}$$

with purely numeric coefficients. Suppose, as is the general expectation, that this is an asymptotic series with large-order behavior

$$c_n \alpha_s^n \sim (an\alpha_s)^n \text{ as } n \to \infty. \tag{15.5}$$

We estimate the error in a truncated perturbative expansion by the first term omitted, as is appropriate for an asymptotic series. Then the minimum error in a perturbative calculation is the smallest term in the series. So, (15.5) implies that the minimum error is from the term with

$$n \propto \frac{1}{a\alpha_s(Q)}. \tag{15.6}$$

Then the minimum error itself is roughly

$$\exp\left(-\frac{\text{constant}}{\alpha_s(Q)}\right) \sim \exp\left(-\text{constant} \ln Q^2\right) = O(Q^{-p}), \tag{15.7}$$

for some positive constant p. Any higher-twist correction with a more negative power of Q is smaller than the minimum error in the perturbative calculation of the leading-twist term. It is therefore phenomenologically useless.

Another severe complication arises for higher-twist corrections in hadron-hadron collisions. Factorization has independent parton densities for each beam hadron. To obtain this independence, we needed a cancellation of interactions between the two hadrons. The proof of the cancellation, Sec. 14.4, relied on causality in the ultra-relativistic limit. In a non-relativistic situation the active partons could get correlated before the hard scattering. Such effects generally contribute to higher-twist corrections. Therefore initial-state interactions require that the non-perturbative functions in corrections of sufficiently higher twist are properties of the whole two-hadron state, rather than being multiparton correlation functions in individual hadrons.

This issue does not affect terms suppressed by $1/Q$ and $1/Q^2$ relative to the leading-power terms (Qiu and Sterman, 1991a, b). So twist-3 and twist-4 terms can be investigated in a generalized factorization framework.