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On the use of the bispectrum to detect and model non-linearity

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Informally a discrete time series is a set of repeated and, normally, equally spaced observations from the same process over time. The statistical analysis of time series has two functions: to understand better the generating process underlying the time series, and to forecast future values. The first analytical methods developed were based upon linear series. A linear series can be represented as a linear function of its own past and current values and the past and current values of some noise process, which can be interpreted as the innovations to the system. A non-linear series has a generally more complex structure that depends upon non-linear interactions between its past and current values and the sequence of innovations. Existing linear statistical methods can only approximate non-linear series. As there is evidence to show that non-linear series are common in real life, two important problems are to detect and then to classify non-linearity. In moving from a linear to a non-linear structure the choice of possible models has moved from a countably infinite to an uncountably infinite set. Hence the need for methods that not only detect non-linearity, but classify the non-linear relationship between the past and current values and innovations.

The third order moment is the expectation of the product of three series values lagged in time. The bispectrum is the double Fourier transform of the third order moment. Both statistics are useful tools for eliciting information on non-linear time series. There are concerns with the assumption of asymptotic independence between the values of the bispectrum estimate used by an existing test of non-linearity. We develop a method with a greater power than this existing method to detect non-linear series by using a model-based bootstrap. Further we show how patterns in the bispectrum are useful for classifying the locations of the non-linear interactions [**1**].

To understand better tests of non-linearity and related inference, we investigate the variance of two estimates of the bispectrum. The two estimates are shown to have different inferential properties. One estimate is generally better able than the other to detect non-linearity and give information on the location of the non-linear interactions.

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The third order moment is statistically equivalent to the bispectrum. A particular estimate of the bispectrum is the double Fourier transform of all the estimated third order moment values in a specified region. When using the third order moment to test for non-linearity we can examine any subset of these values in the specified region. Hence an advantage to using the third order moment, instead of the bispectrum, when testing for non-linearity is a greater flexibility in the range of values selected. We show an improved test for non-linearity over the bispectrum-based test, using a reduced set of the third order moment and a phase scrambling-based bootstrap [2].

Time series can often be observed in a multiple or repeated form, such as the exchange rate between a set of currencies. There is then interest in summarising the common features of the grouped series. An existing linear method based on the spectrum assumes that an observed series (within a population) can be described as a common population spectrum perturbed by an individual effect. The observational noise in the spectrum is modelled using the known asymptotic properties of the spectral estimate. By modelling (and then removing) the individual effects and noise, the method summarises the population linear characteristics through the spectrum. We modify and then extend this method to summarise the common features of the underlying non-linear generating process of a set of repeated time series using the bispectrum normalised by the spectrum [3].

References

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