PRECISE PROPER MOTIONS AND POSITIONS OF STARS FROM A COMBINATION OF FUNDAMENTAL CATALOGUES WITH THE HIPPARCOS CATALOGUE

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ABSTRACT. A combination of the HIPPARCOS catalogue with fundamental catalogues, such as the basic FK5, shall provide, for the common stars, proper motions (and derived positions) which are significantly more accurate (by a factor of up to 5) than those in the original catalogues. For many other stars, the ground-based observations would also be suited for improving the HIPPARCOS proper motions considerably.

1. INTRODUCTION

Stars in fundamental catalogues provide the most accurate astrometric data as far as absolute proper motions and positions are concerned. These data have been obtained mainly from observations with meridian circles over a long period of time (more than a century). The European Astrometry Satellite HIPPARCOS is expected to measure extremely accurate positions of about 115 000 stars for a mean epoch of about 1990, and proper motions for these stars based on observations over the active lifetime of the satellite, about 2.5 years. Due to this relatively short time interval, the individual errors of the HIPPARCOS proper motions will be significantly larger than those of the ground-based proper motions for many fundamental stars.

The purpose of this paper is to show that a combination of the HIPPARCOS observations with the data contained in fundamental catalogues will provide extremely accurate proper motions for thousands of stars. These 'combined' proper motions of fundamental stars will be significantly more accurate than the available fundamental proper motions or the expected HIPPARCOS proper motions alone. Positions of these stars, which can be predicted with the help of such 'combined' proper motions, for epochs which differ by more than a few years from the mean HIPPARCOS epoch, will also be of superior quality.

2. COMBINING TWO CATALOGUES OF POSITIONS AND PROPER MOTIONS

Let us suppose that we have two astrometric catalogues (i = 1,2)

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available both of which provide proper motions as well as mean positions at two different epochs. We assume that the two catalogues are based on completely independent observations, and that the mean epochs T_i are defined in such a way that the proper motions and positions within each catalogue are uncorrelated. We denote the mean errors of the proper motions by $\varepsilon_{\mu,i}$, and those of the positions at the mean epochs by $\varepsilon_{p,i}$.

We shall discuss here only the random error in the resulting proper motions. In fundamental astrometry, this is called the 'individual accuracy' of the proper motions. The discussion of systematic errors is beyond the scope of this paper. It is clear, however, that both catalogues, or sometimes even their basic observational data, have to be reduced to a common reference system before the individual proper motions of stars can be derived from a combination of the two catalogues within such a common system.

First, we derive the mean error $\varepsilon_{\mu,0}$ of that proper motion μ_0 which can be obtained from the two positions at the mean epochs T_i alone:

$$\varepsilon_{\mu,0} = (\varepsilon_{p,1}^2 + \varepsilon_{p,2}^2)^{1/2}/|T_2-T_1| \quad . \qquad (1)$$

We are now asking for the mean individual error $\varepsilon_{\mu,tot}$ of the proper motion μ_{tot} which can be derived from combining the total information provided by both catalogues. From a discussion, similar to that carried out by Kopff, Nowacki and Strobel (1964), of the normal equations of both catalogues and of the combination of these two sets of normal equations into one combined system of normal equations, we derive the following simple expression for the mean error $\varepsilon_{\mu,tot}$ of the total proper motion μ_{tot}:

$$1/\varepsilon_{\mu,tot}^2 = 1/\varepsilon_{\mu,0}^2 + 1/\varepsilon_{\mu,1}^2 + 1/\varepsilon_{\mu,2}^2 \quad . \qquad (2)$$

This expression is strictly derived from the normal equations. Nevertheless, the result is also very plausible: $1/\varepsilon_\mu^2$ represents the 'weight' of each proper motion. According to Eq.2, the total weight of the combined proper motion μ_{tot} is simply the sum of the weights of the three independent proper motions, namely the proper motion μ_0 (derived from the two mean positions alone) and the two proper motions μ_1 and μ_2 given in the catalogues.

We can also derive the combined mean epoch T_{tot} of both catalogues:

$$T_{tot} = (\varepsilon_{p,1}^{-2}\, T_1 + \varepsilon_{p,2}^{-2}\, T_2)/(\varepsilon_{p,1}^{-2} + \varepsilon_{p,2}^{-2}) \quad . \qquad (3)$$

This means that the combined epoch T_{tot} is a weighted mean of the two mean epochs T_i, where the appropriate weights are now the weights $1/\varepsilon_{p,i}^2$ of the catalogue positions at the two epochs T_i.

For the mean error $\varepsilon_{p,tot}$ of the combined position at the combined mean epoch T_{tot}, we find:

$$1/\varepsilon_{p,tot}^2 = 1/\varepsilon_{p,1}^2 + 1/\varepsilon_{p,2}^2 \quad . \qquad (4)$$

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3. COMBINATION OF FUNDAMENTAL CATALOGUES WITH THE HIPPARCOS CATALOGUE

We shall now apply the formulae derived in Section 2 to a combination of accurate ground-based catalogues, such as the FK5, with the HIPPARCOS catalogue.

For the HIPPARCOS catalogue, we adopt T = 1990, ε_p= 2 milliarcseconds (mas) and ε_μ= 2 mas/year. This is probably rather conservative. Bright HIPPARCOS stars, with $m_V < 7^m$, may have smaller mean errors, probably about 1.2 mas and 1.7 mas/year respectively (Perryman and Schuyer 1985).

In Table 1, we present the results of the combination of the HIPPARCOS catalogue with the different parts of the FK5. The basic part of the FK5, which is now finished (W.Fricke, H.Schwan, T.Lederle et al., in preparation), consists of the old fundamental stars, i.e. the 1535 FK4 stars. The basic FK5 is by far the most accurate part of the whole FK5. The FK5 extension will probably contain about 1000 FK4 Sup stars (bright extension) and about 2000 fainter IRS stars (faint extension), as discussed by H.Schwan elsewhere in this volume. The numbers for T, ε_p and ε_μ for the FK5 extension, used in Table 1, represent rough preliminary estimates only, since the work on the FK5 extension is still going on.

The typical mean errors quoted in this paper for the various catalogues are based on linear averages of the individual mean errors of all the stars in the catalogue, and represent also an average over the two components (right ascension and declination).

Table 1 shows that the proper motions derived from a combination of the basic FK5 and the HIPPARCOS catalogue are more accurate than those of HIPPARCOS alone by a factor between 4 and 5, depending on the assumed accuracy of the brighter HIPPARCOS stars. With respect to the

Table 1. Combination of the FK5 and of the AGK3RN with HIPPARCOS

Catalogue combined with HIPPARCOS	in the catalogue: number of stars	T	ε_p (mas)	ε_μ (mas/year)	from a combination of the catalogue with HIPPARCOS: $\varepsilon_{\mu,0}$ (mas/year)	$\varepsilon_{\mu,tot}$ (mas/year)
FK5 Basic	1535	1950	20	0.7	0.5	0.4
FK5 Bright Extension	∿1000	1950	40	2	1.0	0.8
FK5 Faint Extension	∿2000	1940	70	3	1.4	1.1
AGK3RN	20194	1943	80	5	1.7	1.3
		For comparison:		HIPPARCOS in general		2.0
				Bright HIPPARCOS stars		1.7

Note: 1 mas/year = 1 milliarcsecond/year = 0.1 arcsecond/century

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Table 2. Combination of FK3, FK4 or FK5 with HIPPARCOS

Catalogue combined with HIPPARCOS	in the catalogue: number of stars	T	ε_p (mas)	ε_μ (mas/year)	from a combination of the catalogue with HIPPARCOS: $\varepsilon_{\mu,0}$ (mas/year)	$\varepsilon_{\mu,tot}$ (mas/year)
FK5 Basic	1535	1950	20	0.7	0.50	0.40
FK4	1535	1916	35	1.7	0.47	0.44
FK3	1535	1903	44	2.7	0.51	0.48

proper motions of the basic FK5, the accuracy is improved by a factor of about 2. For the FK5 extension, the gain in accuracy by the combination of HIPPARCOS with ground-based observations is still considerable: typically a factor of 2 with respect to HIPPARCOS proper motions, and even more in comparison to the FK5 extension itself.

There do exist ground-based observations for many other, non-fundamental stars which can also be used for improving the individual accuracy of the proper motions of these HIPPARCOS stars. As an example, we have listed in the lower part of Table 1 the northern stars of the International Reference System IRS, i.e. the AGK3RN (Corbin, 1982). Even for this large number of stars, the combination of the meridian circle observations with the HIPPARCOS data leads still to an improvement of the accuracy of the proper motions by a factor of about 1.5.

A comparison of the mean errors $\varepsilon_{\mu,0}$ and $\varepsilon_{\mu,tot}$ in Table 1 shows that the main contribution to the high accuracy of μ_{tot} stems from the proper motion μ_0, which is derived from the two positions at the mean epochs alone. The contribution of the two proper motions given in the catalogues is much smaller, especially for the basic FK5 and the bright extension.

It may already be intuitively expected that the older observations with the larger epoch differences with respect to HIPPARCOS should be most valuable for the derivation of combined proper motions. This is quantitatively illustrated in Table 2, where the results for a combination of two older fundamental catalogues (FK3: Kopff 1937, 1938; FK4: Fricke, Kopff et al. 1963) with the HIPPARCOS catalogue are presented. Table 2 shows that roughly the same accuracy for the proper motions could be achieved by combining the HIPPARCOS catalogue with the rather old FK3 instead of the recent basic FK5. All the ground-based observations over the last 50 years (included in the FK5 but not, of course, in the FK3) contribute only about 30% to the total weight of the combined proper motion of the basic FK5 plus HIPPARCOS. The contribution of the older observations (already contained in the FK3) to the total weight is roughly twice as large as that of the more recent ground-based observations. The lower accuracy of the older observations is obviously more than compensated for by the larger epoch differences.

The combined proper motions with their higher accuracy can also be used for predicting more accurate positions of stars at various epochs.

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Table 3. Accuracy of positions predicted for an epoch T

Epoch difference $\lvert\Delta T=T-T_{HIPP}\rvert$ (years)	Mean error of position predicted from FK5 Basic $\Delta T<0$	FK5 Basic $\Delta T>0$	HIPPARCOS (arcseconds)	FK5 Basic + HIPP.
0	0.034	0.034	0.002	0.002
10	0.029	0.040	0.020	0.004
20	0.024	0.047	0.040	0.008
50	0.021	0.066	0.100	0.020
100	0.047	0.100	0.200	0.040
200	0.114	0.169	0.400	0.080
(2000	1.4	1.4	4.0	0.8)

The resulting mean error $\varepsilon_{p,T}$ of a derived position at epoch T can be calculated from:

$$\varepsilon^2_{p,T} = \varepsilon^2_{p,tot} + \varepsilon^2_{\mu,tot}\,(T-T_{tot})^2 \quad . \qquad (5)$$

In practice, $\varepsilon_{p,tot}$ is always nearly identical to the mean error of the position at the mean HIPPARCOS epoch T_{HIPP}, and T_{tot} differs from T_{HIPP} by less than half a year for the combination of the basic FK5 with HIPPARCOS. Hence in Table 3, we use the epoch difference $T-T_{HIPP}$ instead of $T-T_{tot}$. Table 3 shows the predicted accuracy of a position as a function of the epoch difference $T-T_{HIPP}$ for three cases: (1) using the proper motion and position from the basic FK5, (2) using the HIPPARCOS proper motion and position, and (3) using the combined proper motion and position based on the basic FK5 and HIPPARCOS. A comparison of the resulting mean errors of the predicted positions shows clearly the advantage of combining ground-based observations with HIPPARCOS. The combination FK5 Basic plus HIPPARCOS is outstanding, because HIPPARCOS suffers from the less accurate proper motions while the basic FK5 is handicaped by the larger errors of its positions and proper motions and partially by its earlier mean epoch.

While the older ground-based observations are able to contribute significantly to our knowledge of proper motions even after HIPPARCOS, the situation would completely change after a second HIPPARCOS mission, a decade or more after the first one. The accuracy of proper motions which could be derived from the mean positions of HIPPARCOS I and a 'HIPPARCOS II' (or a similar instrument) is illustrated in Table 4. It is obvious that such a combination of two space astrometry missions would give proper motions (and positions for various epochs) of unsurpassed accuracy. For an epoch difference of 20 years, the proper motions from two space missions would be better than those of the combination FK5 Basic plus HIPPARCOS (I) by a factor between 3 and 5. The older meridian circle observations for all the HIPPARCOS I and II stars would then be merely of historical value.

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Table 4. Accuracy of proper motions from a 'HIPPARCOS II'

Epoch difference $T_{HIPP.II}-T_{HIPP.I}$ (years)	Mean error of the proper motion ($\varepsilon_{\mu,0}$) for $\varepsilon_{p,HIPP.I} = \varepsilon_{p,HIPP.II} =$ 2 mas (mas/year)	1.2 mas (mas/year)
10	0.28	0.17
20	0.14	0.08
30	0.09	0.06
For comparison:		
HIPPARCOS (I)	2.0	1.7
FK5 Basic + HIPPARCOS (I)	0.40	0.40

4. CONCLUSIONS

We have shown that the combination of the HIPPARCOS catalogue with fundamental catalogues, or with other compilation catalogues based on accurate meridian circle observations, leads to proper motions and derived positions which are significantly more accurate (with respect to their individual random errors) than those of HIPPARCOS or ground-based catalogues alone. In the case of the basic fundamental stars, the gain with respect to HIPPARCOS amounts to a factor between 4 and 5.

The older meridian circle results carry a very large weight in such a combination, because of their early epochs and their still acceptable accuracy. Therefore, a careful treatment of these older meridian circle observations is extremely important. For this and other reasons, we will rediscuss all those older astrometric catalogues which are expected to provide important contributions to such a combination. In doing so, we will be prepared for combining all the ground-based observations with the HIPPARCOS catalogue in order to obtain proper motions of highest accuracy for a large number of stars, among which the basic fundamental stars will still be outstanding.

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Discussion:

EICHHORN These formulas slipped by me a little part. Do you start with one pair of positions at central epoch of proper motions and derive a new set of positions and proper motions and at a new central epoch?
WIELEN Yes, this is correct. The formula given is derived from the complete normal equations, and it is rigorous.

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