THEORETICAL PROCESSES IN STAR FORMATION

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1. INTRODUCTION: FRAGMENTATION VERSUS DISSIPATION

To bring out the basic problem of star formation, consider a non-rotating cosmical gas cloud which is free to contract and so to convert most of its gravitational energy into kinetic energy of mass motion. Fluid dynamical systems in general tend to dissipate their kinetic energy, and a diffuse gas cloud easily radiates excess heat. If the cloud did not break up into fragments, then the steady conversion of gravitational energy into radiation would lead ultimately into the formation of a compact massive body - perhaps a quasar - but not of a star cluster, which is maintained at a comparatively low mean density by random stellar kinetic energy. Thus the transition from a gravitationally-bound cloud of gas and dust into a bound star cluster - or a fortiori into an expanding O-B association - requires that gravitational energy released during collapse be conserved as the random macroscopic kinetic energy. It is very likely that star formation is often triggered by processes that are essentially dissipative, such as shocks following passage of the spiral wave, or cloud-cloud collisions; however, it is equally important to emphasise that we require at some stage a cut-off in kinetic energy dissipation. And indeed, once a cloud has been able to fragment into a group of self-gravitating blobs, each of small geometrical cross-section, the chance of inelastic collisions between blobs is sharply reduced.

The radius of the ultimate cluster is closely related to the amount of energy conserved during collapse and break-up. As an extreme case, suppose a cool, roughly spherical cloud of mass M, initially at rest at radius R_o, collapses and fragments completely and with negligible energy dissipation into a cluster of proto-stars. The random kinetic energy T_r that maintains the cluster at its ultimate radius R is then determined by the virial theorem $T_r = -V/2$, where $V \simeq -GM^2/R$ is the gravitational potential energy at radius R; combined with the conservation of energy $T_r + V = V_o$, this yields $R = R_o/2$. As the density of a typical open cluster is in fact similar to that of a molecular cloud, it is clearly desirable that the first fragmentation should occur while

the cloud is still of moderate density.

There will be a similar competition between fragmentation and dissipation when the cloud has a realistic angular momentum. In particular, it can be argued that what determines whether a proto-galaxy becomes disk-like or elliptical is the epoch at which energy dissipation is cut off. This in turn is related to the overall spin Ω of the protogalactic cloud, in that if $\Omega^2 R^3/GM$ is close to unity, not only will the cloud as a whole be unable to contract in two dimensions, but any subcondensations trying to form will have large geometrical cross-sections, so that dissipative collisions remain probable until the cloud has substantially flattened along the rotation axis (cf. Section 4).

A non-rotating, non-magnetic gas cloud can in fact begin to contract only if its self-gravitation can overcome its internal pressure. A cool, dense, roughly spherical cloud at temperature T, with mean density ρ and mean molecular weight μ, immersed in a hot, low-density intercloud medium will collapse if its mass M exceeds a critical mass $M_J \simeq 2\left[(\mathcal{R}T/\mu)^3/G^3\rho\right]^{\frac{1}{2}}$ (Ebert 1955; Bonnor 1956; McCrea 1957). This "Jeans mass" is close to that of a sphere of density ρ and radius equal to the "Jeans length" that emerges from the classical but non-rigorous treatment of gravitational instability. If we parametrize the subsequent temperature variation by a quasi-adiabatic relation $T \propto \rho^{(\bar\gamma-1)}$, then $M_J \propto \rho^{(3\bar\gamma-4)}$; hence if $\bar\gamma < 4/3$, M_J decreases as ρ increases. In a diffuse HI or molecular cloud, compressional heating is negligible, and the balance between heating and cooling processes implies that $\bar\gamma < 1$. In a hypothetical primeval gas cloud of pure hydrogen and helium, T remains close to 10^4 °K, and $\bar\gamma \simeq 1$. In either case, M_J will in fact decrease as the cloud collapses. This is the basis of the simplest fragmentation model (Hoyle 1953; Mestel and Spitzer 1956; Gaustad 1963). If M is only a little greater than the initial value of M_J, the cloud will begin by converting nearly all the gravitational energy released into thermal energy, which is immediately radiated away; and likewise the cloud cannot fragment, since $M \simeq M_J$. At later epochs the cloud goes over into nearly pressure-free collapse, with most of the further gravitational energy released becoming macroscopic kinetic energy of inflow. At the same time, since now $M \gg M_J$, the cloud can break up into a number of fragments, each of mass $M' \simeq M_J$, and each exerting enough self-gravitation to withstand its internal pressure; and the kinetic energy of collapse is available to be converted via elastic encounters into random kinetic energy. Further, as each fragment itself collapses under its self-gravitation, then as long as $\bar\gamma$ stays well below 4/3, by the same argument sub-fragments can separate out. The hierarchy would terminate at densities sufficiently high for heat of compression to be effectively trapped rather than radiated away, so that $\bar\gamma$ becomes closer to 5/3 rather than 1.

The dynamics of this type of fragmentation differs from Jeans-type gravitational instability, in that we demand not just that a density fluctuation in a medium at rest should amplify when its mass exceeds M_J, but that it separate out from a medium that is itself in a state of

overall collapse. Analytical studies of both the linear and post-linear domains (Hunter 1962, 1964; Lynden-Bell 1973) show that Hoyle's intuitive transition from "can fragment" to "will fragment" is very plausible. The danger that fragments may dissipate their energy in collisions and so coalesce (Layzer 1963) is probably only serious if the overall contraction of the cloud is strictly spherical - a highly improbable case (Lin, Mestel and Shu 1965).

In fact, when a realistic angular momentum is introduced it is no longer clear that Hoyle-Hunter fragmentation is relevant to the star formation problem (cf. Section 4). However, the idea of opacity-limitation to systematic break-up reappears in one form or another in most current discussions: we return to it in Section 7. But first we re-discuss the dynamical problem, with more realistic physical conditions.

2. GRAVITATIONAL COLLAPSE: THE ROLE OF THE GALACTIC MAGNETIC FIELD

Consider now a cool, roughly spherical cloud, of mass greater than M_J, but threaded by the lines of the local galactic magnetic field $\underset{\sim}{B}$. As the cloud collapses, the frozen-in magnetic field is distorted and begins to exert forces opposing gravity. Intuitively one expects that if the field is not to be able to prevent collapse, then the mass must exceed a new critical value given approximately by replacing $\mathcal{R}T/\mu$ by $(\mathcal{R}T/\mu + B^2/4\pi\rho)$. In the limiting case with $\mathcal{R}T/\mu$ negligible, the virial theorem (Chandrasekhar and Fermi 1953; Mestel and Spitzer 1956; Mestel 1965; Strittmatter 1966) does indeed yield a critical mass $M_c \propto (B/\rho^{2/3})^3/G^{3/2}$ where B and ρ are simultaneous values in a roughly spherical cloud. However, as a cold spherical cloud will in fact flatten along the field, it is best to write this at once in terms of the magnetic flux $F = \pi B R^2$ through the equator. The <u>mass-flux relation</u> $GM_c^2 = 5kF^2/9\pi^2$ fixes the minimum mass required to enforce gravitational collapse of a cloud containing a large-scale field with flux F: the factor k varies from 1 to $\simeq 1/4$ as the cloud flattens from a sphere into a highly oblate spheroid (Strittmatter 1966). Note that the density and the length-scales both along and across the field have disappeared.

At this point we digress and emphasise that quite generally the relation between B and ρ - with strict flux-freezing assumed - depends critically on the kinematics and so also on the dynamics of the gas flow. The erroneous idea is still abroad that flux-freezing necessarily implies the relation $B \propto \rho^{2/3}$, whereas in fact this holds only if the gas motion is <u>isotropic</u>. This is clearly correct during the contraction of a slowly rotating star, for which linear dimensions both along and across the field change by the same scale-factor λ: flux and mass-conservation then yield respectively $B \propto 1/\lambda^2$ and $\rho \propto 1/\lambda^3$, so that $B \propto \rho^{2/3}$. This is the formula from which one can deduce an internal magnetic field within a white dwarf or a pulsar in terms of a corresponding field within the core of an evolving main sequence star. By contrast, the gas in a cool spherical cloud forming e.g. by thermal instability must flow preferentially along the field because of the anisotropy of the Lorentz force $\underset{\sim}{j} \times \underset{\sim}{B}/c$: the contraction factor

$\lambda_\perp$ across the field is less than $\lambda_\parallel$, and $B/\rho^{2/3}$ decreases. If the gas flow is initiated by the passage of the spiral shock wave, the B-ρ relation will depend on the mutual orientation of the shock and the field. A shock front aligned across the field will cause flow down the field, so that B stays constant while ρ increases; whereas a shock parallel to the field will cause both B and ρ to increase like $1/\lambda_\perp^2$, so that B $\propto$ ρ; and a shock inclined to the field will cause refraction of the lines of force. The Parker Rayleigh-Taylor instability yields flow down the field, so that $B/\rho^{2/3}$ will again decrease.

   The situation is summed up succinctly by Vrba and the Stroms (1976): "The shape of a cloud and the field structure bear witness to the cause of collapse". As a further example, consider a cool cloud which forms as a highly prolate spheroid about the field direction. Self-gravitation always acts so as to increase eccentricity (Lin, Mestel and Shu 1965); just as an oblate spheroid tries to flatten, so a prolate spheroid tries to become cylindrical. However, the consequent compression of the field yields a magnetic pressure $B^2/8\pi \propto \rho^2$, so that the spontaneous increase in eccentricity is now checked (cf. McCrea 1957). The collapse is thus more-or-less isotropic, with $B/\rho^{2/3}$ consequently staying constant (Mestel 1965). However, this depends on special initial conditions: flattening and anisotropic collapse is more likely.

   Note that a "uniform" field - i.e. one with a scale much larger than the cloud dimensions - exerts negligible force, and so does not interfere with the initial collapse. Clearly, if the mass M < M_c(F), the cloud must try and reach a state of magneto-thermo-gravitational equilibrium, essentially with gravity balanced by thermal pressure along the field, and by the magnetic pressure gradient and curvature force across the field (Mestel 1965; Strittmatter 1966). A very rough estimate for the radius across the field of such a model is R = $R_o(1-(M/M_o)^2)$, where R_o is the radius from which the collapse across the uniform field began. The dimension along the field is essentially the thermal Jeans length. More recently, these approximate virial theorem treatments have been replaced by detailed accurate models of cool, isothermal gas clouds, immersed in a hot inter-cloud medium, and threaded by a magnetic field which becomes uniform at infinity (Parker, D.A. 1973, 1974; Mouschovias 1976a,b). Continuity of thermal pressure at the cloud surface ensures that the external density is low, so that only the cloud's self-gravitation is important; and as the tangential as well as the normal components of B are continuous, there are no surface currents and consequent unbalanced surface stresses. Mouschovias's technique ensures the invariance of an initially pre-scribed distribution of mass within flux-tubes, as required by flux-freezing: one can thus follow the evolution of a given cloud through a series of equilibria as the temperature falls; or compare the parameters of clouds of different mass, all forming from the same initial state. Such models can be plausibly used to describe clouds in which a locally enhanced field, as inferred from the Zeeman effect on the 21 cm line, is associated with a high density - due both to flow down the field and simultaneous lateral motion of gas and field-lines - and with a high

cloud mass, exerting sufficient gravitation to balance the lateral
magnetic forces (Mestel 1969; Verschuur 1969). However, it should be
already clear that interpretation of these observations in terms of the
"law" $B \propto \rho^{2/3}$ - even if it could be assumed that all clouds formed
from regions with the same distribution of magnetic flux with respect
to mass - is highly implausible; and Mouschovias argues reasonably that
the observations are equally consistent with a lower index, as
suggested by theory. After equilibrium models have been constructed,
there remains the question of their stability, especially against non-
axisymmetric perturbations. A beginning to this formidable problem has
been made by Garlick (1976) who has extended the hydromagnetic energy
principle to non-rotating clouds with non-zero field at infinity.
Although so far no definite case of instability has yet been found, there
are hints that instability will set in at least for some highly flattened
models. It is surmised that rotation will encourage the spontaneous
transformation into a non-axisymmetric structure, as in the Maclaurin-
Jacobi problem.

3. MAGNETIC FRAGMENTATION

We now suppose that $M \gtrsim M_c(F)$ so that the cloud will contract
indefinitely (as long as centrifugal forces do not interfere). If the
contraction were isotropic, then sub-condensations could not form at
any stage; for a blob of mass $M' = \lambda^3 M$ ($\lambda < 1$) would contain flux
$F' = \lambda^2 F$, so that $GM'^2/F'^2 = \lambda^2 GM^2/F^2$ would be sub-critical; or
equivalently $B/\rho^{2/3}$ would stay constant. But if the cloud spontaneously
flattens along the field, $B/\rho^{2/3}$ decreases and smaller masses can
separate out. If a cool spherical cloud of initial density ρ and radius
R has reached a thickness $\bar{z}$ and density $\bar{\rho} = \rho R/\bar{z}$, then a blob of radius
$\bar{z}$ at the cloud centre, containing flux $F' = \pi B \bar{z}^2 = (\bar{z}/R)^2 F$, has mass
$M' = 4\pi\bar{\rho}\bar{z}^3/3 = M(\bar{z}/R)^2$, so that now $GM'^2/F'^2 = GM^2/F^2$. Because of the
flattening, sufficient mass has agglomerated by flow down the field to
ensure that a quasi-spherical blob exerts enough self-gravitation to
dominate over the forces exerted by a field of flux F' (Mestel 1965).

The argument so far ignores thermal pressure - we implicitly assume
that the Jeans length is small compared with the scale of the cloud. It
is well known (McCrea 1957) that isothermal, one-dimensional collapse is
limited, since the pressure gradient $\simeq a^2\bar{\rho}/\bar{z}$ increases like $1/\bar{z}^2$, while
the z-component of the gravitational force density is a disk
$\simeq G(\bar{\rho}\bar{z}^3)\bar{\rho}/\bar{z}^2 \propto 1/\bar{z}$. The minimum mass that can form, following the
flattening of a magnetic cloud into a state of hydrostatic support along
the field, will therefore be somewhat greater than $M(\bar{z}/R)^2$ - perhaps by
an order of magnitude (Mestel 1965). However, the important points are:
(1) fragmentation of an isothermal magnetic cloud can occur following
preferential flow along the field, and the blobs that can form have
masses determined by the ratio $(\bar{z}/R)$; (2) subsequent gravitational con-
traction across the field will reduce $\bar{z}/R$ still further, so allowing
further break-up; (3) the magnetic flux does not appear explicitly in
the mass of the fragments forming at each stage; (4) as the fragments
are all "strongly magnetic", in the sense that the magnetic and gravi-

tational energies are comparable, the masses forming at each density are certainly greater than those in the non-magnetic problem; (5) if the field is effectively frozen-in all the way to the opaque, pre-main sequence phase, it is again opacity effects that will fix the minimum stellar masses (cf. Section 7); (6) again we require conservation of a substantial fraction of the energy released by contraction as random kinetic energy of the fragments.

Tentative support for the picture of star formation following flow down the field-lines comes from the work of Vrba (1976) and Vrba and the Stroms (1976), summarised in the Symposium proceedings.

The time has now come for a computer attack on the formidable problem of the dynamics of collapse of a self-gravitating magnetic gas cloud, analogous to Mouschovias' studies of equilibrium models. Earlier studies (e.g. Mestel 1966; Mestel and Strittmatter 1967) have necessarily been partly analytical and partly intuitive. However, some qualitative features, due essentially to the severity of the flux-freezing constraint, are likely to persist in a future detailed treatment. In any problem of gravitational collapse, there is a strong tendency for the denser inner parts to run ahead of the outer. A frozen-in field will acquire a correspondingly distorted structure: the more-or-less uniform field within the centre will be linked with the background galactic field by nearly radial field-lines, which consequently exert strong pinching forces towards the equator. The outer parts of the cloud tend to be held up by the tensions of the distorted field-lines, a point implicit in the earlier studies and recently emphasized by Mouschovias (1976b,c). This structure forces re-appraisal of the strict flux-freezing assumption. In a lightly ionized gas the field moves with the ionized component via electromagnetic induction (except for Ohmic effects). The neutral gas moves under the non-electromagnetic forces, including the frictional force due to ion-neutral particle collisions; the ionized component (plus the field) moves under the joint action of the magnetic force and the same frictional force with reversed sign. The strong magnetic pinching forces at the equator begin by compressing both ionized and neutral components; however, the neutral gas can diffuse out of the equatorial zone under its partial pressure, leaving just the ionized gas to withstand the magnetic pressures. The strongly compressed ionized gas will then largely recombine into neutral atoms, which will again drift out of the pinched zone. This run-away process leads to the mutual separation of oppositely directed field-lines becoming so small that Ohmic diffusion occurs - perhaps violently, as in a solar flare - leading to a steady detachment of the field in the cloud centre from the local galactic field (Mestel and Paris 1976).

This change in the topology of the field at some stage in the con-traction of a cloud will clearly reduce the angular momentum transport by Alfvén waves travelling along the field-lines extending to infinity (cf. Section 4). However, a new point to stress is that a field which has acquired O-type neutral points is dynamically unstable against modes analogous to the familiar pinch instabilities. The problem has been discussed by Wright (1973) and Markey and Tayler (1973) in the context of

magnetic star theory. In an opaque body such as a star, dynamically-driven motions will be nearly adiabatic, so that the unstable modes are restricted by the condition that motions be nearly perpendicular to the gravitational field. But in a diffuse gas cloud a dynamical time-scale is sure to be much longer than the temperature equilibration time; motions in any direction will be nearly isothermal, and the adiabatic stabilization term in the energy integral vanishes. It is difficult to follow the non-linear development of these instabilities, but it is at least plausible that the field rapidly tangles itself sufficiently for Ohmic destruction of flux to occur. If this is correct, then as soon as the cloud field detaches from the background field, instabilities occur which tend to restore the field to an undetached topology. The process implies a reduction in the flux threading the cloud, essentially in an Alfvén time-scale, which in a strongly magnetic cloud is comparable with the free-fall time.

Rough estimates suggest that the detachment time is shorter than the free-fall time once the cloud has shrunk by a factor $\simeq 10$, so that subsequently the cloud will tend to be magnetically isolated from the background. However, if the cloud is not in free-fall, but is held up e.g. by centrifugal forces (cf. Section 4), then this dynamical flux destruction via instabilities may allow some magnetic contact between cloud and background to persist longer. The point to note now is that in a strongly magnetic cloud or fragment, hydromagnetic instabilities can lead to dynamical flux destruction, but in a time that is comparable with the free-fall time. Such flux loss is neither "fast", i.e. occurring in a time shorter than the free-fall time, and so rendering the magnetic forces weak compared with the gravitational; nor "slow", implying effective flux-freezing, as has been assumed so far. It may enable a cloud or fragment to lose any embarrassing excess flux, so perhaps helping further fragmentation (cf. Section 4 below), without simultaneously implying that the field becomes dynamically insignificant.

4. ROTATING MAGNETIC CLOUDS

The whole problem of collapse and fragmentation becomes more complicated when we give the cloud a realistic angular momentum, derived either from the galactic rotation or from the interstellar turbulence. To illustrate this, consider the idealization of a cloud of mass M with a uniform angular velocity Ω. If the angular momentum H is strictly conserved - the analogue of flux conservation - then Ω increases under isotropic contraction like $\rho^{2/3}$; the ratio of centrifugal force to gravity $3\Omega^2/4\pi G\rho \propto (H^2/M)\rho^{1/3}$, and so increases with decreasing scale. There is thus no analogue of the mass-flux relation. Overall contraction is ultimately halted in two dimensions, but if the cloud is cool it will spontaneously flatten along the rotation axis, so reducing $\Omega^2/G\rho$. If we temporarily ignore difficulties due to the shearing effect of non-uniform rotation, then we can expect sub-condensations to form, orbiting in centrifugal balance under the overall gravitational field of the cloud, but with the ratio of centrifugal force of spin to self-gravitation reduced below unity. (The fragmentation problem is somewhat

relaxed: since centrifugal forces hold up the cloud, the dynamical
problem is closer to the classical Jeans problem rather than the Hoyle-
Hunter problem, which demands that the blob density increase faster
than that of the collapsing background). However, again isotropic
contraction of each blob is limited by the growth of centrifugal forces
of spin. The contrast with the magnetic case is clear. A modest degree
of anisotropic flow can resolve the magnetic problem permanently, in
the sense that subsequent isotropic contraction of a given mass M is
never halted, provided the condition $M > M_c(F)$ is satisfied. But in a
rotating system in which sub-condensations are supposed to form without
any removal of angular momentum of spin, there is always a limit to the
density which a given mass can reach by isotropic contraction. A
completely cold cloud could systematically flatten and fragment, but
with a realistic value for the initial angular velocity the masses
which could reach stellar densities would be far below stellar masses.
And if we allow the gas to have a realistic temperature, then the
hierarchical process just outlined would be halted by thermal pressure
at densities much less than stellar order - $10^{-14} gm/cm^3$ or so.

The problem has been posed in its starkest form, with no transfer of
spin angular momentum from a fragment to the surrounding medium. We can
however be much more optimistic and suppose that some powerful braking
process ensures that the spin of a contracting blob remains synchronous
with that of the surrounding gas, at least for some stage of contraction
and break-up. Then it is easy to see that with this assumption, the
angular momentum problem could be resolved early in the evolution of
the cloud. As an example, let the cloud have a density $\rho_o \simeq 30 m_H/cm^3$,
and a mean angular velocity $\Omega_o \simeq 10^{-15}$, comparable with the local
galactic orbital angular velocity, so that $3\Omega_o^2/4\pi G\rho_o \simeq 1$. Then suppose
the cloud contracts isotropically, and at constant angular velocity, by
a factor of no more than 1/30, and then subsequently at constant angular
momentum. The ratio $3\Omega^2/4\pi G\rho$ will not reach unity again until densities
of $10^{18} m_H/cm^3$ (corresponding to the top of the Hayashi curve). The
argument is easily generalised to apply to a hierarchical fragmentation
picture, in which synchronism is maintained between a blob and its
surroundings only for a limited degree of contraction, with further
break-up occurring following flattening down the rotation axis. Some
such process is envisaged in the scheme outlined by Bodenheimer (1976).

It is at this point that the presence of the magnetic field becomes
an asset rather than a nuisance. First of all, if the field has a
component perpendicular to the angular momentum vector, it will tend to
iron out non-uniform rotation within the cloud. Secondly, field-lines
linking a cloud with the surrounding galactic gas, or indeed a fragment
with the rest of the cloud, can and will transport angular momentum.
The absence of evidence for strong rotational velocities in dense mole-
cular clouds may very well be due to magnetic braking during cloud
formation. However, in making the optimistic estimate above it was
assumed that braking process was so efficient that contraction occurred
at constant angular velocity. If the cloud mass M is less than the
critical mass $M_c(F)$ for its flux, then indefinite contraction will in any

case be prevented by the magnetic forces, so that the braking will
proceed in its own time-scale, until corotation of cloud and surrounding
gas has been reached. But if $M > M_c(F)$, then removal of angular momen-
tum allows the cloud to contract further and so to spin up. (We again
note the contrast with a rotating star, which is held up essentially by
thermal pressure, with centrifugal forces a perturbation over the bulk;
braking enables the star to become more nearly spherical, but will not
lead to overall contraction). Thus if a cloud or fragment is to remain
effectively co-rotating with the environment, braking must be so
efficient that the characteristic time is short compared with the time
of free-fall. But suppose instead that the time of braking is longer
than the free-fall time. The cloud will then remain in approximate
magneto-centrifugal-gravitational balance, and its rate of contraction
will be determined not by gravity alone, but by the rate of braking.
If M were very close to $M_c(F)$, then the rotation rate would necessarily
be kept well below its centrifugal balance value $\Omega \simeq (4\pi G\rho/3)^{\frac{1}{2}}$; but with
M markedly greater than $M_c(F)$, force balance would be maintained only as
long as Ω is close to $(4\pi G\rho/3)^{\frac{1}{2}}$, so that Ω would systematically increase
as the cloud contracts. This in turn would mean that magnetic braking
would be able to resolve the angular momentum problem completely, only
if redistribution of angular momentum continues all the way to the
pre-main-sequence phase.

It is thus clear that a reliable estimate for the rate of magnetic
braking is of crucial importance. Ebert et al (1960) studied the case of
a spherical cloud of radius $\bar{R}$ and density $\bar{\rho}$, rotating about an axis
parallel to a uniform magnetic field B_0 linking the cloud with a surround-
ing medium of density ρ_0; the characteristic time found is
$\tau_b = (8/5)\bar{R}(\bar{\rho}/\rho_0)(1/V_A)$, where V_A is the Alfvén speed in the external
medium, a result which can be recovered by estimating the time for Alfvén
waves to travel through a cylindrical slab with the same moment of
inertia as the cloud. This time exceeds the free-fall time by a factor
rather greater than $(\bar{\rho}/\rho_0)^{\frac{1}{2}}$. However, a more realistic treatment must
allow for the distortion of the field as the contraction proceeds.
Calculations still in progress (Gillis, Mestel and Paris 1974; 1976)
adopt a field structure (symmetric about the rotation axis) that
simulates the (undetached) field outlined in Section 3. The results
suggest that for clouds with M greater than but close to $M_c(F)$, the
time of braking may in fact be less than the free-fall time until the
cloud has shrunk by a factor $\eta(F) < 1$. If we consider a series of
models with the flux decreasing steadily from the virial theorem
maximum, the value of $\eta(F)$ increases: the lower the flux, the earlier
the cloud changes over from a state of free-fall under negligible
centrifugal forces, to a state of mechanical balance, with contraction
determined by angular momentum loss.

A complete theory must synthesize all the different processes.
When magnetic and angular momentum axes are more-or-less parallel,
systematic fragmentation is possible via flow down the field-lines and
magnetic redistribution of angular momentum (recall that three-dimensional
contraction is required for the systematic break-up of an isothermal

cloud). At some stage of contraction, the detachment of the field of a fragment from the background will inevitably cut off further angular momentum loss; however, the process can and will be repeated at the next stage of fragmentation. It can be plausibly argued that if flux-loss never becomes "fast" at any stage, so that the magnetic field is dynamically significant at all pre-opaque phases, then this combination of anisotropic flow and magnetic braking can yield break-up into a hierarchical distribution of proto-stars. Statistical energy inter-change should tend in time to smooth out the spatial distribution within a cluster, but still leaving a sizeable proportion of binary and multiple systems.

The mathematical problems are more severe when there is a sizeable component of the magnetic field of a cloud perpendicular to the angular momentum axis. There is some evidence that magnetic braking will then be more efficient than in the parallel case (Kulsrud 1971; Mouschovias 1976d) though whether it can resolve the whole angular momentum problem early in the lifetime of the cloud again depends on the field's retaining an undetached structure for sufficiently long. Once the field has detached from the background, then a cloud with $\underset{\sim}{B}$ perpendicular to $\underset{\sim}{\Omega}$ and with mass $M > M_c(F)$, and which has retained a sizeable angular momentum, can apparently achieve equilibrium as a roughly ellipsoidal body, with gravity balanced in one dimension by magnetic force, in another by centrifugal force, and in the third by magnetic and centri-fugal forces jointly. No accurate models have yet been constructed, still less tested for stability. However, approximate studies (Mestel 1965) suggest that for a given value of F/M there will be two equilibria. The model that is less prolate about the magnetic axis is unstable, and so can flatten along the magnetic axis and fragment. The more prolate model is prima facie stable: further slow loss of angular momentum would cause _isotropic_ contraction, with B increasing like $\rho^{2/3}$. Break-up can however occur after the cloud has lost flux via hydromagnetic instabilities (cf. Section 3); the ellipsoid would become more and more cylindrical about the field direction, and ultimately fragment e.g. into two self-gravitating blobs, orbiting in their mutual gravitational field. Again, contraction of each blob will be followed by magnetic braking of the spin. One can picture a new hierarchy, with each essential process - flux destruction, subsequent fragmentation, magnetic braking - having a characteristic time-scale comparable with the instantaneous free-fall time; provided flux-loss never becomes "fast" before the opaque phase is reached, so that the magnetic force remains comparable with the gravitational within each fragment, then again the scheme seems capable of producing systematic break-up of clouds into proto-stars.

If the magnetic axis remains essentially perpendicular to the rotation axis during all stages of contraction and break-up, we may expect a mass-spectrum more heavily biassed towards massive stars (cf. Section 7). However, the magnetic stresses on a rotating cloud or fragment will normally have a component perpendicular as well as parallel to the rotation axis: the field exerts a precessional as well

as a braking torque, unless the field is either axially or equatorially
symmetric with respect to the rotation axis. Paris (1971) showed that
an oblique field coupling a fragment to a massive surrounding medium
tends to rotate the angular momentum vector of the fragment towards
parallelism with the field, though it was difficult to find a reliable
time-scale estimate. It is worth noting here the observations by
Markkanen (1976) of linear polarization of stars in the Pleiades cluster;
he concludes that the rotation axis of Pleione is parallel to the
direction of the field in the surrounding nebula.

5.   THE FLUX-FREEZING APPROXIMATION

The scenarios outlined above assume that flux-loss from a fragment
never becomes faster than the most rapid relevant dynamical process –
free-fall, braking, fragmentation. We have argued that the galactic
magnetic field will not prevent fragmentation; in fact, so far from the
field's being an embarrassment for star formation, we suggest that it is
an asset, mainly because of the transfer of excess angular momentum and
because of the suppression or control of rotational shear, and also as
a possible extra parameter for fixing the final mass spectrum. We have
noted that strict flux-freezing is a severe constraint, so that a
gravitationally-distorted magnetic field is in fact likely at some stage
to evolve into a lower energy state via local reconnection of field-lines.
The new field structure may in turn be subject to dynamically-driven
flux destruction, but the time-scale for this process is given by the
Alfvén speed, which is at most of the order of the free-fall speed.
Flux-freezing is relaxed sufficiently e.g. to allow the fragmentation of
a prolate cloud with $\underset{\sim}{B}$ and $\underset{\sim}{\Omega}$ perpendicular, but the field remains "strong"
at all stages, in the sense that the magnetic energy of a fragment is
comparable with its gravitational energy.

In his original paper on fragmentation and the formation of Type
II stars, Hoyle (1953) dealt with a primeval gas of atomic hydrogen
which is kept at a temperature of near 10^4 ^{0}K by collisional ionization
followed by radiative cooling. The significant point is that the very
processes by which compressional heat is radiated away imply a moderately
high density of ions and electrons, so that any primeval magnetic field
would inevitably remain frozen-in, in the sense that flux-loss never
becomes "fast". However, in cool HI and molecular clouds, heat loss is
primarily by processes unconnected with the degree of ionization. It is
therefore not a priori obvious that the field remains strong throughout
the whole epoch of formation of Type I stars. The question is important
because as already noted, it is not yet clear whether the galactic
magnetic field is likely to resolve the "angular momentum problem" in
the early, low-density phases, when flux-freezing is certainly a good
approximation. It may be that the magnetic stresses are able to keep
the centrifugal forces of spin close to gravity at each stage of
fragmentation, but cannot maintain for long effective corotation with
the local galactic gas. If so, and if flux-loss does become rapid at
some pre-proto-stellar phase, we shall need a different scenario to
describe the later stages of star formation, with the magnetic field

playing a much weaker role (cf. Section 6).

The time within which the electrons and ions (inductively coupled to the magnetic field) are forced out of a lightly ionized fragment of mass M and containing flux F is given by $(9\pi\alpha/4)(n_i/n_n)(M/F)^2$, where n_i, n_n are the densities of ions and neutral particles, and the frictional coupling constant α = (ion-neutral collision cross-section) (thermal velocity of neutrals)$/m_H \simeq 10^{15}$ cgs in a cool cloud   (Mestel and Spitzer 1956; Spitzer 1963; Mestel 1968; Nakano and Tademaru 1973; Nakano 1976; Silk 1976a). For a "magnetically strong" fragment with the flux near the virial theorem limit, this time reduces to $(5\alpha/4\pi G)(n_i/n_n) \simeq 2 \times 10^{14}(n_i/n_n)$ years. Flux-loss becomes "rapid" if this time falls well below the time-scale of the relevant dynamical process - free-fall or angular momentum transfer. The crucial questions are (1) the rate of magnetic braking, and (2) the level to which n_i/n_n falls. It is clear that flux-freezing remains an excellent approximation to much higher densities than in the first estimate (Mestel and Spitzer 1956), because of the higher collision cross-section (Osterbrock 1961), cosmic ray ionization (Hayakawa, Nishimura and Takayanagi 1961), radioactive ionization (Cameron 1962), and charge-transfer between molecular ions and metallic atoms (Oppenheimer and Dalgarno 1974). Nakano (1976) finds the ion density flattens off to $\simeq 10^{-1}/cm^3$ when recombination on grains is included (without grain recombination n_i/n_n would be larger by a factor $\simeq 10^2$). The flux-loss rate then equals the free-fall rate when $n_n \simeq 10^{10}$; however, if the fragment contracts in approximate centrifugal balance, rapid flux-loss would occur at somewhat lower densities.

It should be noted that at $10^{10}/cm^3$ a fragment is becoming optically thick (e.g. Low and Lynden-Bell 1976), and compressional heating at the free-fall rate would increase the temperature quasi-adiabatically: after a further density increase by a factor of $\simeq 10^3$, collisional ionization would begin to dominate. Thus it seems that if grain recombination occurs, and provided there is no local source of ionization, there is a narrow gap between the density at which flux-loss becomes rapid, and that at which thermal ionization takes over. However, there is another factor which merits detailed study. The processes of field reconnection and dynamical flux destruction may themselves yield locally generated cosmic rays. When flares occur on the sun we see a fraction of the magnetic energy released converted into high-speed particles. If similarly no more than one part in 10^3 of magnetic energy destroyed during star formation goes into sub-cosmic ray particles, it could make a significant contribution to the ion density when $n_n \simeq 10^9 - 10^{10}$, and so extend still further the epoch over which flux destruction is restricted to the dynamical rate.

With so much theoretical uncertainty, one welcomes all the evidence that observation can provide on field-strengths in regions of different density. I would caution against the argument that rapid flux-loss must have occurred during the star formation phase, because of the comparative weakness of stellar magnetic fields. It is true that even the strongest

fields observed in the magnetic Ap stars, when plausibly extrapolated inwards, yield magnetic fluxes much below the virial theorem limit. However, the relevant question is what would happen to a proto-star which managed to reach the pre-main sequence phase with a flux near this limit. We have already invoked dynamical instabilities to destroy excess flux during star formation, but emphasising that their time-scale is not shorter than the time of dynamical evolution of the cloud. But an opaque proto-star is by definition in approximate pressure equilibrium, and it contracts at a rate determined by heat loss - i.e. in the much longer Hayashi or Kelvin-Helmholtz time-scales rather than the free-fall time. Any instabilities will now have a much more leisurely time in which to develop. In particular, secular instabilities, dependent essentially on heat exchange, may cause flux-tubes to float to the surface in a time no longer than the overall contraction time - the "magnetic buoyancy" effect. It is even possible (cf. Levine 1974) that this destruction of a huge reservoir of magnetic energy may be connected with the T Tauri phenomenon, as suggested by Lynden-Bell (this volume).

The argument should not be taken to imply that the "fossil" theory of stellar magnetism must be discarded. The origin of the "weak" fields observed in the strongly magnetic stars and surmised to be present deep down in other stars of both early and late type (including perhaps the sun) remains a matter for debate. They could still be a relic of the interstellar field from which the stars formed; but perhaps, more plausibly, they could be slowly decaying relics of fields built up by dynamo action during the Hayashi phase, after the interstellar field has all been destroyed (cf. Mestel 1976).

Even if flux-loss does become rapid during star formation, this need not imply that magnetic effects can be subsequently neglected: the relic of the galactic field can still be important for angular momentum transport, albeit in a longer time-scale (cf. Section 6). Bisnovatyi-Kogan et al (1973) have explored the possibility that the turbulent velocities in clouds (or in proto-stars within clouds) could generate a magnetic field by the SKR dynamo mechanism. Their motivation is again to resolve the angular momentum problem; however, I suspect that in demanding that the braking should occur in a time short compared with the free-fall time, they are placing very severe constraints on the energetics of the dynamo process. If and when a star reaches the Hayashi phase, then a dynamo-built field is probable, and has available the much more leisurely thermal contraction time in which to remove more angular momentum, e.g. by coupling with a stellar wind.

6. WEAKLY MAGNETIC, ROTATING SYSTEMS

Once a fragment has become magnetically "weak", the Lorentz force cannot prevent substantial flattening along the rotation axis; and likewise no significant angular momentum redistribution can occur within a free-fall time. A field of highly supersonic turbulence could provide temporary support, but again the system will flatten as the turbulence decays. McCrea (1960, and this volume) has argued that the very process

by which the turbulence deays - mutual collision and adherence of
"floccules" moving randomly and with supersonic speeds - acts broadly
like a gross turbulent viscosity, yielding a slowly rotating, condensed
core containing most of the mass, surrounded by a Keplerian disk con-
taining very little mass but most of the angular momentum - a proto-
solar system. Other authors(e.g. Ebert 1964; Cameron 1972) assume that
the cloud flattens into a disk-like structure, without much angular
momentum redistribution. The mass is concentrated towards the centre
via the same gross gravitational instability that yields the flat
rotation curve in disk-like galaxies (Mestel 1963; Hunter 1963). The
problem then reduces to the evolution of such a "stellisk".

    Ruzmaikina and Safranov (1976) have studied thermally-driven
processes, analogous to the Eddington-Sweet circulation and the
Goldreich-Schubert instabilities in stable stellar zones; they conclude
that their effect on the angular momentum distribution is small. If
the cloud retains a weak magnetic field with a component in the plane
of the disk, then the twisting of the field by the non-uniform rotation
will generate magnetic torques which will transport angular momentum and
so also mass, since the gas will move so as to maintain near centrifugal
balance (Mestel 1960). This may however also be less important than
dynamical transport: the disk may be spontaneously unstable towards
modes that transfer angular momentum outwards. In the galactic context,
Lynden-Bell and Kalnajs (1972) associate the development of a gross
spiral pattern with the tendency of a disk to develop internal gravi-
tational torques. The resulting evolution time is long compared with a
rotation period or a free-fall time, justifying the quasi-static
assumption. Most workers explicitly or implicitly assume that disks
will also be unstable on a smaller scale, so that a turbulent viscosity
is continually maintained at the expense of the gravitational energy.
The modes considered must be non-axisymmetric in order that azimuthal
pressure gradients and gravitational forces break the constraint of
detailed angular momentum conservation, so that Rayleigh's stability
criterion is not violated. A self-gravitating locality of a thin disk
is certainly gravitationally unstable; what remains unclear is whether
a Keplerian disk, with negligible self-gravitation, is nevertheless
unstable (e.g. von Weizsäcker 1944, 1947). Attempts to prove instability
have not carried conviction: Stewart (private communication) points out
formal gaps in the analysis of all papers on this problem, including his
own (1975, 1976). And even if turbulence does develop in a Keplerian
disk, it is doubtful (ter Haar 1950) if much angular momentum could be
transported from the central mass before the disk dissipates. This
would seem to imply that the process must involve a substantial outflow
of mass as well as angular momentum.

    This scenario of turbulent or magnetic redistribution of disk
angular momentum is currently being applied largely to X-ray sources.
Lynden-Bell and Pringle (1974), who develop in mathematical detail the
evolution of a viscous disk, suggest that the luminosity of a T Tauri
star is powered by the dissipation of a disk surrounding a star of
solar mass. A more radical application is due to Gold (1972, 1974),

who pictures a cold accumulation of matter, yielding first a cold
degenerate central body of mass similar to Jupiter's. Jupiter has a
metallic structure which strongly resists compression, so that the
steady accumulation of more mass (again via viscous angular momentum
transport) is accompanied initially by only a slight generation of heat,
which is in any case rapidly conducted away. However, once the central
mass has become of stellar order, the metallic structure breaks down,
and the cold planet contracts towards a black dwarf state. Further
mass accumulation will be accompanied by much more compressional heat
generation, so that the black dwarf becomes a white dwarf of finite
temperature: ultimately the hydrogen will ignite and the star will
evolve from a degenerate into a non-degenerate structure, with a
normal main sequence radius.

Gold argues further that his picture can account for the rotation
rates of stars of different mass. An element of matter can reach the
central body only when its angular momentum has been reduced to the
value appropriate to the orbital velocity at the equator. When the
white dwarf expands by a factor of 30 or more to become a normal star,
the angular velocity decreases by 10^{-3}, and the resulting main sequence
star would necessarily be a slow rotator. Mass subsequently accreted
by the same frictional process will have the specific angular momentum
appropriate to the new radius, so that the angular velocity should
increase with mass.

While some of these ideas have been foreshadowed (e.g. von
Weizsäcker 1944, 1947), the picture with a degenerate central body is
new and merits quantifying. Emphasis on the rate of compressional
heating may not be relevant, as hydrogen fusion will begin even at zero
temperature if the density is sufficiently high ("pycnonuclear reactions").
There is the danger that in evolving from a fully degenerate to a non-
degenerate state, the star will reach temperatures (near 10^8 ^0K) at which
the thermonuclear fusion rate is so fast that the star will be destroyed
(Mestel 1952). In the Gold picture the Hayashi phase never occurs, so
some other evolutionary epoch must be associated with the T Tauri
phenomenon: a synthesis of the Gold and Lynden-Bell and Pringle pictures
may be possible.

7.   THE LIMIT OF FRAGMENTATION AND THE INITIAL MASS SPECTRUM

An essential part of Hoyle's scheme (1953) is that successive
fragmentation ceases when the last fragments are sufficiently opaque for
the heat of compression to be trapped rather than radiated away. Note
that this criterion necessarily involves a characteristic time: more
heat is generated during the free-fall of a slowly rotating fragment
than during the agglomeration of mass at the centre of a disk via slow
angular momentum transport. It is thus not clear that the simplest
case - a non-rotating, non-magnetic cloud - will necessarily yield a
lower bound to the mass-spectrum, even though the centrifugal and
magnetic terms in the virial theorem act against gravity. However, it
is certainly valuable to see what answer the simplest case yields to the

minimum mass problem. A recent discussion by Rees (1976) puts the
essence of the problem succinctly. A freely-falling fragment of mass
M and radius r has the possibility of further fragmentation if it can
radiate at something like the rate $(GM^2/r)(G\rho)^{\frac{1}{2}}$. Now a cloud becomes
opaque when its radiation rate per unit area approaches that of a black
body: thus the condition that an opaque body should still be able to
fragment implies essentially a lower limit on its temperature T. In
standard notation the maximum radiation rate is $fac\pi r^2 T^4$, where f ($\leq$1)
depends on the details of the physics and on the geometry of the
fragment. Further fragmentation of an opaque body ceases to be possible
when this maximum rate is inadequate to get rid of the compressional
heat generated. Rees shows that this yields a limiting Jeans mass
which can be written as $M_F \simeq M_{ch}\mu^{-9/4}f^{-\frac{1}{2}}(kT/m_p c^2)^{\frac{1}{4}} \simeq 10^{-3}(T/10)^{\frac{1}{4}}f^{-\frac{1}{2}}\mu^{-9/4}M_{ch}$
where $M_{ch} = (\hbar c/Gm_p^2)^{3/2}m_p$ is approximately the Chandrasekhar mass.
What immediately appears is the remarkable insensitivity of this final
fragment mass to the temperature - a variation from 10 to 10^4 increases
M_F only by about 6. There is little scope for variation in μ; to
increase M_F above the very small value predicted above one must look
for factors substantially reducing the efficiency factor f. Note that
the density at which fragmentation ceases varies strongly with
temperature ($\propto T^{5/2}$): the higher the temperature the higher is the
rate of compressional heat generation that the fragment can radiate
away and so the smaller the radius which it can reach.

    Rees's treatment has the merit of showing that a mass M_{ch} of
stellar order automatically appears. However, the factor f hides a
lot of physics: both f and T depend on the track followed by contracting
and fragmenting clouds in the T-ρ plane, up to the point when they first
become opaque, and so on their composition and environment. Low and
Lynden-Bell (1976) study the details of the radiation transfer through
an opaque spherical Jeans mass. They confirm that M_F in a dark molecular
cloud is small - $0.007M_{\odot}$ - and is insensitive to changes in the opacity.
The minimum is reached at optical depth $\tau \simeq 3$ (justifying Rees's
assumption of a small variation of T through the fragment) and at
densities $\simeq 5 \times 10^{11}/cm^3$. Silk (1976a), using slightly different
parameters, finds $M_F \simeq 0.01M_{\odot}$.

    One can hardly expect any fragmentation process to occur with
complete efficiency, and it would not be surprising if some masses,
larger than these embarrassingly low values, do crash through the
$\tau = 3$ stage without having broken up (e.g. because of chance inelastic
collisions between fragments). It is reasonable to ask for what mass
range an opaque body at these densities can behave like a proto-star:
i.e.,settle into hydrostatic equilibrium with a large internal
temperature gradient, and contract not at the free-fall rate, but at
the slower rate determined by heat flow down the temperature gradient.
It is known that a slight departure from hydrostatic balance will lead
to a collapse rate close to free-fall; thus for hydrostatic balance to
remain an excellent approximation, we require that the heat flow down
the temperature gradient be less than the compressional heat that
would be generated if the body were to contract at the free-fall rate.

This procedure yields masses $\simeq 0.2 M_{\odot}$ (Mestel and Spitzer 1956; Gaustad 1963). Proto-stars of smaller mass will contract Kelvin-Helmholtz-wise until they reach internal temperatures of $\simeq 2000^{0}$K, when dissociation and ionization of hydrogen leads to an effective $\gamma \simeq 1$ and a second collapse phase (Cameron 1962). Greater masses cannot retain enough heat to remain in hydrostatic equilibrium: they will collapse dynamically until dense and hot enough inside for gaseous rather than dust grain opacities to dominate.

Whether $0.007 M_{\odot}$ or $0.2 M_{\odot}$ is the typical mass to emerge from this simplest fragmentation model is clearly linked with the details of the dynamics of fragmentation. Gaustad (1963) and Spitzer (1968) argue that masses $< 0.2 M_{\odot}$ will achieve a high degree of symmetry, and so will be less liable to fragment permanently during the second stage of collapse (due to hydrogen ionization); whereas larger masses, which never achieve an early Kelvin-Helmholtz phase, will be more likely to continue their break-up. Silk (1976a) suggests that the mass spectrum may indeed peak near $0.2 M_{\odot}$, with special conditions being required to produce stars like the sun, let alone the minority of more massive stars.

One can hardly fail to agree with Rees that the stellar mass spectrum is likely to be controlled by the details of the dynamics of fragmentation. Rees's factor f depends on geometry: Silk (1976a) and Hutchins (1976) point out that Hoyle's argument applied to a disk-like body yields $\simeq 0.1 M_{\odot}$ rather than $0.007 M_{\odot}$. Smith and Wright (1975) have emphasized the effect of heat from other fragments on thermal evolution. Silk (1976b) points out that radiation from the first-forming opaque fragments raises the temperature of the remaining gas, and so inhibits further fragmentation. He is thus able to derive an initial mass spectrum; however, this will presumably be affected by the occasional recombination of fragments following inelastic collisions (Layzer 1963; Arny and Weissman 1973).

My own feeling is that the angular momentum problem is paramount, and that resolution of the mass spectrum problem must await answers to the questions posed in Section 4-6. If the magnetic field becomes weak before the proto-stellar phase, and the angular momentum problem has not been fully resolved during star formation, then one of the scenarios of Section 6 would be relevant. But if the field remains "strong" all the way to the pre-main-sequence phase, then not only can it resolve the angular momentum problem but it must increase the mass of the fragments forming at each stage. We recall that if $\underset{\sim}{B}$ and $\underset{\sim}{\Omega}$ are more-or-less parallel, then fragmentation can occur following flow down the field-lines. Thermal pressure is all that limits break-up, and the final masses will again be fixed by the opacity: one hazards a guess that the centrifugal and magnetic forces will not increase M_{F} by large factors. But if $\underset{\sim}{B}$ and $\underset{\sim}{\Omega}$ are more-or-less perpendicular, then we noted that in some cases there is no obvious way in which the cloud can flatten along the field. Contraction occurs because of loss of angular momentum, but most of the gravitational energy is converted into

magnetic and centrifugal energy rather than into the random kinetic
energy that is the basic requirement of fragmentation. Break-up into
mutually orbiting blobs can occur, but only following dynamical flux
destruction, and this requires a sizeable gravitational distortion of
the field. If the magnetic field remains "strong" all the way to the
proto-star phase, then the inhibition of fragmentation could lead to
final masses far above the $0.007M_\odot$ or $0.2M_\odot$ estimates discussed above.

A final remark on the build-up of massive O and B stars by
accretion of interstellar gas by stars of lower mass. A weakness of
the earlier work (McCrea 1953; Mestel 1954) is the implicit requirement
of a comparatively low angular momentum of the inflowing gas about the
star. If however one postulates that the star has some magnetic flux
with some field-lines emerging from the surface, this objection may be
removed. Inflowing gas of low density will be forced to corotate with
the star, the excess angular momentum being transferred inwards by the
magnetic stresses: this could simultaneously ensure that these stars
rotate rapidly.

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