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On some applications of Melnikov's method to chaos and subharmonics

J.R. CHRISTIE

Chaotic behaviour in the sense of Smale of some dynamical systems can be established via the existence of a transverse homoclinic point, using the Smale-Birkhoff homoclinic theorem.

Melnikov's method is a technique for investigating the existence of transverse homoclinic points of certain classes of dynamical systems. In this thesis, we consider autonomous systems of ordinary differential equations which are subjected to time-periodic perturbations. We assume that the unperturbed system possesses either a homoclinic orbit or a heteroclinic cycle to apply Melnikov's method. This requirement has been quite a severe restriction for the application of Melnikov's method. We give several new applications of Melnikov's method in this thesis.

For the subharmonic Melnikov theory, we assume in addition that the unperturbed system has a continuous family of periodic orbits. We use the subharmonic Melnikov method to investigate the existence of resonant periodic orbits for the perturbed system. Apart from the periodic family requirement, the difficulty in calculating the subharmonic Melnikov function has been restrictive for the application of the subharmonic Melnikov method. We also give some new applications of the subharmonic Melnikov method in this thesis.

The homoclinic and heteroclinic Melnikov method and the subharmonic Melnikov method are reviewed in Chapter 1. In subsequent chapters, we give applications of these methods (or extensions of them) to problems from Physics, Biology and Chemistry.

In Chapter 2, we apply both the homoclinic Melnikov method and the subharmonic Melnikov method to examine the dynamics of a one-dimensional anharmonic oscillator under perturbation which has applications in optics and quantum mechanics.

For Chapter 3, we investigate the motion of a mass hanging from an overhead crane using both the heteroclinic Melnikov method and the subharmonic Melnikov method.

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A system modelling "the battle of the sexes" in evolutionary biology is considered in Chapter 4 and again we use both the heteroclinic Melnikov method and the subharmonic Melnikov method to examine the dynamics of a time-periodic perturbation of the system.

Lotka-Volterra systems have been used extensively in modelling population dynamics and in Chapter 5, we apply the heteroclinic Melnikov method for slowly varying systems to study the dynamics of a three-dimensional perturbed Lotka-Volterra system. In a particular case, we can reduce our system to a two-dimensional Lotka-Volterra system and this leads us to an application to the self-organisation of macromolecules.

Finally, in Chapter 6, we develop the subharmonic Melnikov theory for three-dimensional perturbed generalized Hamiltonian systems and we finish by giving applications to some fluid mechanics problems.

High Frequency Radar Division
Defence Science and Technology Organisation
PO Box 1500
Salisbury SA 5108
Australia
e-mail: john.christie@dsto.defence.gov.au