THE FORMATION OF PLANETS AROUND STARS OF VARIOUS MASSES AND THE ORIGIN AND THE EVOLUTION OF CIRCUMSTELLAR DUST CLOUDS

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ABSTRACT. The formation of planets is investigated both in a gaseous nebula and after the nebula has been blown away. The capture of planetesimals by a protoplanet is investigated by taking into account the growth of planetesimals. The time of planet formation is determined as a function of distance from the central star. The formation time of Neptune is found to be 3.9×10^9 yr, a value shorter than the age of the solar system. The region where planets form within the stellar lifetime is determined, and it is found that only stars of mass less than several solar masses can have planets. The dust clouds around α Lyrae and β Pictoris are far outside the planet-forming regions. A model for the circumstellar dust cloud is proposed which can explain the basic properties of the β Pic disk.

1. INTRODUCTION

Formation processes of planets around the Sun have been investigated by many authors both in a gaseous nebula (Cameron and Pine 1973; Hayashi et al. 1977, 1985) and in the gas-free state (Safronov 1969; Matsui and Mizutani 1977, 1978; Hartmann 1978; Greenberg et al. 1978, 1984; Cox and Lewis 1980). Many astronomers believe that some fraction of solar-type stars is accompanied by planets. Such interest has been enhanced by the discoveries of dust clouds around α Lyrae (Aumann et al. 1984) and β Pictoris (Smith and Terrile 1984). However, few works have been done on the formation of extrasolar planets.

The composition and even the existence of solid materials in the gaseous nebula are determined by the temperature. The stellar luminosity influencing the temperature is very sensitive to the stellar mass M_*. The stellar lifetime is also sensitive to M_*. Therefore, the region where planets form within the stellar lifetime is rather sensitive to M_*. In Sections 2 and 3 we shall investigate the planet formation around stars of various masses and we shall determine the regions where planets may form within the stellar lifetime and the mass range of the star which may possess planets. In Section 4 we

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shall investigate the origin and evolution of the circumstellar dust
cloud and we shall compare the results with the observations.

2.   PROCESSES OF PLANET FORMATION

With the limited space available for this article we are obliged to
describe only the scenario of planet formation. The details will be
published elsewhere.

2.1. Structure of the Gaseous Nebula

By analogy with the solar nebula (Hayashi 1981), we assume that the
distribution of the surface density (mass per unit area) σ of the
gaseous disk is given in units of gcm^{-2} as

$$\sigma(r) = 1.7 \times 10^3 \ f_\sigma \ (M_*/M_\odot)^{5/6} r_{AU}^{-3/2}, \qquad (1)$$

where r_{AU} is the distance from the central star in AU, and f_σ is an
arbitrary parameter. The power 5/6 for M_* is obtained if one assumes
a sort of homology. The ambiguity in the M_*-dependence may be dis-
solved by investigating over the allowable range of f_σ . When

$$f_\sigma > \ 33.2 \ (M_*/M_\odot)^{-1/3}(T/100K)^{1/2}, \qquad (2)$$

T being the temperature of the nebula, the gaseous disk is gravita-
tionally unstable (Toomre 1964; Goldreich and Lynden-Bell 1965a,b),
and gaseous objects much heavier than Jupiter would form. We restrict
ourselves to the cases where solid planets may form. Of course the
formation of Jovian planets by capturing gas thereafter is not
excluded. Then, it would be sufficient to consider the cases of
$f_\sigma \lesssim \ 10$.
    Due to the gas pressure, the angular velocity Ω_g of the circular
motion of the gaseous nebula deviates slightly from the Keplerian
velocity $\Omega_K = (GM_*/r^3)^{1/2}$, G being the gravitational constant, as

$$\Omega_g^2 = \Omega_K^2 \ (1-2\eta \ ), \qquad (3)$$

where

$$\eta = - \ (P/2\rho \ v_K^2) \ \frac{dlogP}{dlogr} \ , \qquad (4)$$

ρ and P being the mean density and pressure, respectively, in the
nebula, and $v_K = (GM_*/r)^{1/2}$ being the circular Keplerian speed. This
small deviation plays an important role in the migration of
planetesimals as shown in Section 2.4.

2.2. Formation and Fragmentation of a Dust Layer

The dust grains sink gradually to the central plane of the nebula with
a time   scale   of   about 10^3 times   the   Keplerian   period

$t_K = 2\pi (r^3/GM_*)^{1/2}$ (Safronov 1969; Weidenschilling 1980; Nakagawa et al. 1981). By sedimentation of dust there appears around the central plane a thin dense layer mainly composed of dust. This layer fragments into planetesimals due to the gravitational instability when the density reaches 0.17 times the Roche's density (Sekiya 1983). The mean mass of planetesimals thus formed is given by

$$m_0 = 4\pi^4 r^6 \sigma_s^3/(1.057M_*)^2, \tag{5}$$

where $\sigma_s = \zeta\sigma$ is the surface density of solid material, ζ being the solid mass fraction in the initial gaseous nebula. We take $\zeta = 0.0042$ when the solid part is composed of rocky materials and $\zeta = 0.018$ when it is composed of icy and rocky materials.

2.3. Kinetic States and Growth of Planetesimals

The mass spectrum of planetesimals changes with time due to collisional growth. However, except for the concluding stage of planet formation, the dominant portion of the population has a considerably smaller size than the largest bodies (Greenberg et al. 1978, 1984; Nakagawa et al. 1983). Therefore, we shall describe the planetesimals with their representative mass m without going into details of the spectrum.

The orbit of the planetesimal deviates from a circle and declines from the central plane to some extent. The mean amounts of these deviations (random motion) can be estimated in the following way.

The random motion of planetesimals is enhanced by mutual gravitational scattering with the time scale (Chandrasekhar 1942)

$$t_E = v^3/(6.49\,\pi\,G^2 m^2 n\ln\Lambda), \tag{6}$$

where v and n are, respectively, the random velocity and number density of planetesimals, and $\ln\Lambda$ is a factor coming from the integration with the impact parameter and takes a value of about 10.

The direct collision of planetesimals is highly dissipative (Fujiwara and Tsukamoto 1980). The random motion decays by the direct collision with the time scale

$$t_c = (4\pi a^2 nv)^{-1}(1+2\theta)^{-1}, \tag{7}$$

where θ is the so-called Safronov number given by

$$\theta = Gm/(av^2). \tag{8}$$

and $a = (3m/4\pi\rho_s)^{1/3}$ is the radius of the mean planetesimal, ρ_s being the density of solid matter. We take $\rho_s = 3$ g cm^{-3} when it is composed of rocky materials and $\rho_s = 1$ g cm^{-3} for icy and rocky materials.

The gas moves circularly nearly with the Keplerian speed. Therefore, the random motion of planetesimals is also dissipated by the gas drag force which may be written as

$$\mathbf{f} = -\frac{1}{2} C_D \pi \ a^2 \rho \ v \ \mathbf{v}. \tag{9}$$

Because the gravity of the planetesimal excerts some influence upon the fluid motion around it, the coefficient C_D may be written for the subsonic flow as

$$C_D = C_1 + C_2 \ (Gm/ac_s^2)^2, \tag{10}$$

where c_s is the sound speed in the gas. We find that the results of numerical simulations on the fluid motion around a solid body by Takeda et al. (1985) can be well represented by equation (10) with $C_1 = 2.5$ and $C_2 = 0.7$. The dissipation time of the random motion by the gas drag force is given by

$$t_g = 2m/(\pi C_D a^2 \rho v). \tag{11}$$

The mean eccentricity e and inclination i of the orbits of planetesimals are related to the random velocity v and the Keplerian speed v_K as

$$e \simeq i \simeq v/v_K. \tag{12}$$

The number density of planetesimals is given by

$$n = \sigma_s/(2mri). \tag{13}$$

The random velocity changes with time according to

$$v^{-2} \frac{d}{dt} v^2 = t_E^{-1} - t_c^{-1} - t_g^{-1}. \tag{14}$$

Because the direct collision is mostly coalescent as confirmed below, the representative mass of planetesimals grows with time according to

$$\frac{1}{m} \frac{dm}{dt} = 1/t_c \ . \tag{15}$$

In the gaseous nebula with $f_\sigma \gtrsim 1$ we find $t_c > t_g$, and then the relaxation of the random velocity is faster than the collisional growth. Therefore, the steady state is approximately given by

$$t_E^{-1} = t_c^{-1} + t_g^{-1}. \tag{16}$$

In the gas-free state with $t_g = \infty$, we find with the help of equations (6) to (8) that the steady state for equations (14) and (15) is given by

$$0.6/t_E = 1/t_c. \tag{17}$$

For the random velocity determined by equation (16) or (17) we find $\theta \gtrsim 0.4$ irrespective of the representative mass m. This means that the gravity of planetesimals is important in their collision.

Because the collision is highly inelastic, the energy of relative
motion is efficiently dissipated, and then they mostly coalesce by
their gravity.

The growth of planetesimals is obtained by integrating equation
(15) with the random velocity determined by equation (16) or (17) and
with the initial mass m_0 given by equation (5). The representative
mass m soon becomes insensitive to m_0.

2.4. Growth of the Protoplanet

So far we have investigated the growth of the representative mass of
planetesimals. However, the mass of planetesimals spreads to a con-
siderable extent by random coalescence. We call the most massive
object in some specified region a protoplanet. While planetesimals
grow by mutual sticking, they are gradually captured by the
protoplanet.

Let us consider a protoplanet of mass $m \gtrsim 10^2 m$. Because the
excitation time of its random velocity by collisions with plane-
tesimals is greater than the age of the protoplanetary system, its
random velocity is not greater than the random velocity of the mean
planetesimal (Nakano 1986). Therefore, the collision cross section of
the protoplanet with a planetesimal is much greater than the geomet-
rical cross section, and then the protoplanet captures the plane-
tesimals whose orbital semimajor axis is close to its own in a time
scale considerably smaller than the age of the system (Nakano 1986).

However, the protoplanet can hardly capture the planetesimals
with considerably different orbital semimajor axis. Such plane-
tesimals are captured only after their semimajor axis has approached
that of the protoplanet. The orbit of a planetesimal varies by mutual
gravitational scattering and by the gas drag force. The orbital
semimajor axis of the mean planetesimal changes by the mutual scatter-
ing by a factor of 2.72 (the base of natural logarithm) in a diffusion
time (Hayashi et al. 1977)

$$t_d = t_E \, (v_K/v)^2 . \qquad (18)$$

The gas motion is nearly completely circular, parallel to the central
plane, and slightly deviates from the Keplerian speed as shown by
equation (3). Therefore, the circular motion as well as the random
motion of planetesimals are dissipated by the gas drag force. Assum-
ing that the variation of the orbit of a planetesimal in a Keplerian
period is small and $\eta \ll e \simeq i \ll 1$, we have investigated the work of
the gas drag force on a planetesimal, and found that the total energy
E, and then the semimajor axis, of the planetesimal decrease by a
factor 2.72 in a flow time

$$t_E = |E/\dot{E}| \simeq t_g / (2\eta + \frac{5}{8} e^2 + i^2), \qquad (19)$$

where t_g is the time scale given by equation (11) averaged over the
Keplerian period. Equation (19) agrees well with the flow time given
by Hayashi et al. (1977).

Let $\Delta r = c_m r$ be the mean migration distance of planetesimals, i.e., the mean variation of the orbital semimajor axis suffered by the planetesimal before it is captured by the protoplanet. Then the migration rate of the planetesimal is given by

$$(c_m^2 t_d)^{-1} + (c_m t_f)^{-1}. \tag{20}$$

As long as a protoplanet much more massive than the mean planetesimal exists, equation (20) gives the capture rate of the planetesimal. Because $\Delta r \simeq (r_{n+1} - r_n)/2$, a half of the orbital distance of the resultant adjacent planets, we take

$$c_m = \frac{1}{2} \ln (r_{n+1}/r_n). \tag{21}$$

In the solar system r_{n+1}/r_n lies between 1.3 and 2.0. We shall use $c_m = 0.20$ corresponding to the mean value $< r_{n+1}/r_n > \simeq 1.5$.

Both t_d and t_f are functions of the position r and the mean mass m of planetesimals, and m is a function of time t through equation (15). Therefore, the migration rate given by equation (20) is a function of r and t. We can consider that the planet forms when most of the planetesimals in a specified region have been captured by the protoplanet, i.e., at a time t satisfying

$$t^{-1} \simeq (c_m^2 t_d)^{-1} + (c_m t_f)^{-1}. \tag{22}$$

In the gas-free state we take $t_f = \infty$. Equation (22) determines the time of planet formation t_{pf} as a function of r.

It is to be noticed that equation (22) determines t_{pf} only when an object much more massive than the mean planetesimal already exists. As seen in the numerical simulations (e.g., Greenberg et al. 1978, 1984), some objects of mass greater than $10^2 m$ already exist when the representative mass becomes about $10 m_0$. Therefore, we consider that the planet forms at the time t when both of the following two conditions are satisfied for the first time: (i) the migration rate given by equation (20) exceeds t^{-1}, and (ii) the mean mass m is greater than $10 m_0$. In most cases condition (i) actually determines the epoch of planet formation.

3. NUMERICAL RESULTS ON PLANET FORMATION

According to the theory on the structure of Jupiter (Hubbard et al. 1980), its core mass amounts to 20 to 30 times the Earth mass M_E. Therefore, $f_\sigma \simeq 2$ seems more suitable to the solar system than $f_\sigma = 1$. Figure 1a shows the time of planet formation t_{pf} as a function of r for $M_* = 1M_\odot$, $f_\sigma = 2$, and $c_m = 0.20$. The dashed line represents t_{pf} in the gaseous nebula and solid line in the gas-free state. The step on the dashed line corresponds to the state of $T = 170K$. We have assumed that the planetesimals are composed of rocky materials in the inner region, and icy and rocky materials in the outer region. For the gas-free state we have assumed that such transition occurs at

T = 100K due to the lower sublimation temperature at lower pressure (Gaustad 1963). Because the gaseous nebula must have been blown away in the early stage of the solar evolution, probably at $t \simeq$ several x 10^7 yr (Sekiya et al. 1980), the planets in the inner region must have formed in the gaseous nebula and those in the outer region in the gas-free state. The Earth formed at $t \simeq 1.7 \times 10^6$ yr in the gaseous nebula, and Neptune at $t \simeq 3.9 \times 10^9$ yr, within the age of the solar system, even in the gas-free state. This is in remarkable contrast with the previous theory which derived 1×10^{11} yr as the formation time of Neptune (Safronov 1969). Within the lifetime of the Sun 1×10^{10} yr planets form only at $r \lesssim$ 46AU.

Figure 1b shows the time of planet formation t_{pf} for $M_* = 2M_\odot$, $f_\sigma = 2$, and $c_m = 0.20$. Planets form only at $r \lesssim$ 10AU within the stellar lifetime 1.3×10^9 yr.

We have also investigated the formation of planets around stars of $M_*/M_\odot$ = 0.5, 0.7, 3, 5, and 10. Figure 2 has been compiled from all the results. The solid line shows the position where t_{pf} is equal to the stellar lifetime for $f_\sigma = 2$. The dashed lines show the positions of constant temperature. The fraction of matter which is in the solid state becomes very small at $T \gtrsim$ 1300K (Grossman 1972). Therefore, planets can form only in the region surrounded by the solid line and the dashed line with T = 1300K. Therefore, only stars of mass less than several solar masses can have planets. The dot-dashed line shows the position where t_{pf} is equal to the stellar lifetime for $f_\sigma = 10$. The planet-forming region in this case is slightly larger
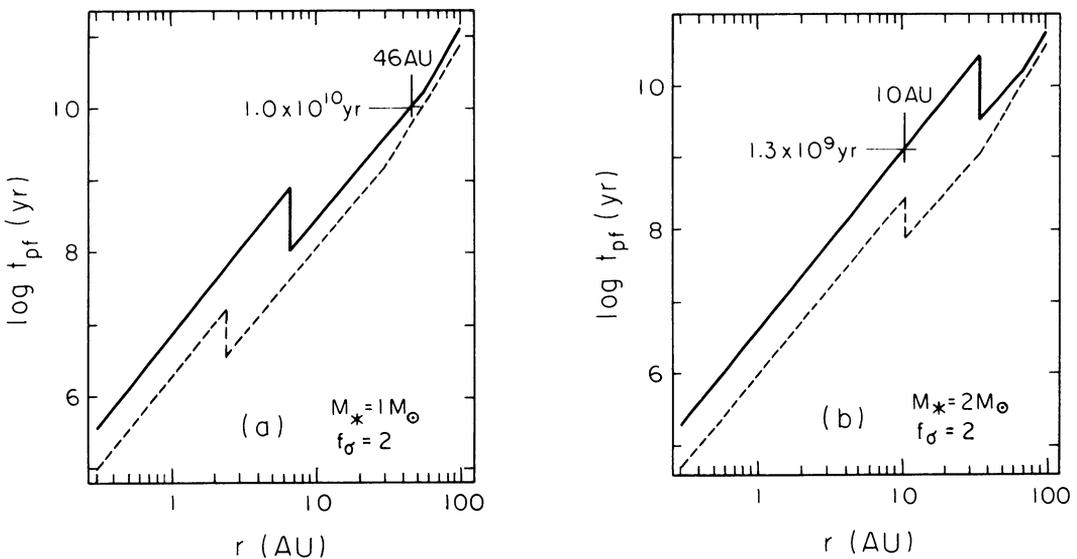

Fig.1. The planet formation time t_{pf} as a function of the distance r from the central star of M_*=1$M_\odot$ (a) and 2$M_\odot$ (b) for $f_\sigma = 2$ and $c_m = 0.20$. The dashed line represents t_{pf} in the gaseous nebula and the solid line in the gas-free state.
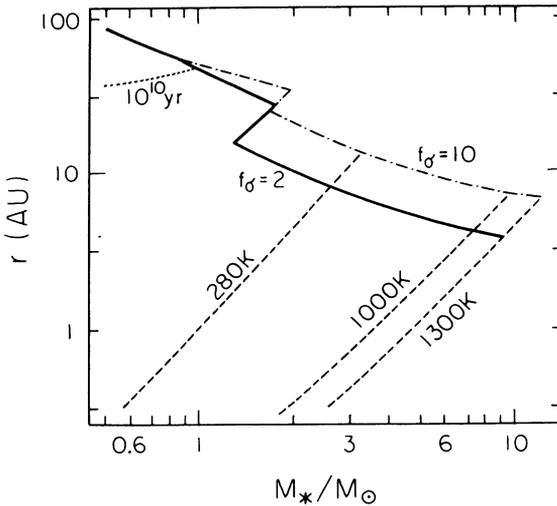
Fig.2.   The planet-forming region as a function of the stellar mass M_*. Planets form within the stellar life-time inside the solid line for f_σ = 2 and inside the dot-dashed line for f_σ = 10. Planets hardly form in the region with T $\gtrsim$ 1300K.

than in the case of f_σ = 2. Only stars of $M_* \lesssim 3M_\odot$ can have a planet on which the temperature is nearly equal to that of the Earth (280K) and living things can exist at least with respect to the temperature.

4.   A MODEL FOR CIRCUMSTELLAR DUST CLOUDS

Alpha Lyrae has a dust cloud emitting thermal radiation of about 85K from a region of radius around 85AU (Aumann et al. 1984). From its luminosity $L_* = 58L_\odot$ the mass of α Lyr is estimated to be between 2.5 and 2.9$M_\odot$. Smith and Terrile (1984) found that β Pictoris is accom-panied by a dust disk nearly edge-on to us whose radius is at least 400AU. The mean inclination of the orbits of dust in this disk is less that 5° at r $\simeq$ 300AU, and the distribution of the surface bright-ness on the central plane is approximated by the power law $r^{-4.3}$ at the wavelength $\lambda \simeq 0.89\mu$ m. The mass and the luminosity of β Pic are about 2$M_\odot$ and 10$L_\odot$, respectively. These dust clouds are far outside the planet-forming regions obtained in Section 3. We shall investigate the origin and the present status of these clouds.

4.1. Regions of Planetesimal Formation

Because planetesimals form by sedimentation of dust in the gaseous nebula, the outer boundary of the planetesimal-forming region depends on the time when the gaseous nebula is blown away. If the gaseous nebula around a star of M_* = 2$M_\odot$ survives up to the end of the Hayashi phase $t_{EHP} \simeq 2 \times 10^6$yr, the outer boundary is around 530AU as long as the gaseous nebula extends at least out to this radius. Similarly for M_* = 3$M_\odot$ with $t_{EHP} \simeq 5 \times 10^5$yr, the outer boundary is around 250AU. If the gaseous nebula is blown away before the dust sinks, the solid material is also blown away together with the gas.

4.2. Random Motion of Planetesimals

Because the gaseous nebula is blown away in the early stage of the stellar evolution, the planetesimals in the outer region spend most of their life in the gas-free state. Therefore, the time variation of their random velocity v is described by equation (14) with $t_g = \infty$. Because v is initially very small and the available time is too short to attain the equilibrium given by equation (17), t_E is much smaller than t_c. In addition, because t_c is much greater than the stellar lifetime, the mass of the planetesimal is nearly equal to the initial value m_0. Then, equation (14) is easily integrated as

$$i \simeq v/v_K = (13 \pi^2 \ln \Lambda)^{1/4} M_*^{-1/2} (m_0 r^2 \sigma_s t/t_k)^{1/4}$$

$$\propto \zeta f_\sigma M_*^{-1/24} r^{1/8} t^{1/4} . \qquad (23)$$

The last expression in equation (23) is for σ given by equation (1).

Figure 3 shows the mean inclination of planetesimal's orbit for $M_* = 2M_\odot$ and $f_\sigma = 3$ at some epochs. Each line is labeled with the value of t in units of 10^8yr. The straight lines in the outer region come from equation (23). In the inner region the random velocity is nearly in a steady state and the growth of planetesimals is taken into account. The inner end of the curve is at the outer edge of the region where planets have already formed at each epoch. At $t = 1 \times 10^8$yr the mean inclination at r = 300AU is about 3° and is consistent with the thickness of the β Pic disk.
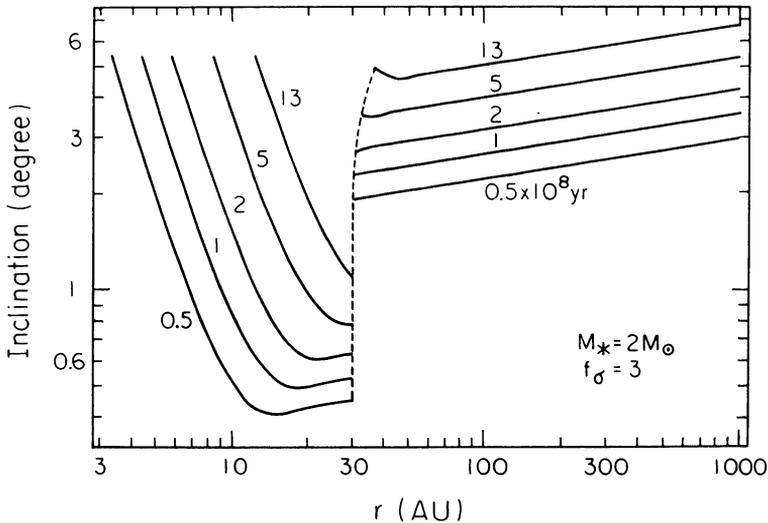

Fig.3. The mean orbital inclination of planetesimals as a function of the distance r from the star at some stages for $M_*=2M_\odot$ and $f_\sigma = 3$. Each line is labeled with the value of t in units of 10^8yr.

4.3. Collisions of Planetesimals

From equations (5), (7), (8) and (23) the number of collisions that each planetesimal has experienced up to t is given by

$$N_c(r,t) = \int_0^t t_c^{-1} dt = (3.17 M_*/\rho_s r^3)^{2/3} t/t_K$$
$$+ 2.22 (\ln\Lambda)^{-1/2} (3.17 M_*/\rho_s r^3)^{1/3} (t/t_K)^{1/2}. \quad (24)$$

Even at the end of the stellar life t = 1.3 x 10^9 yr for $M_* = 2M_\odot$, we have N_c = 0.19 and 0.048 at r = 100 and 200AU, respectively. Thus only a small fraction of the planetesimals undergo collisions in the outer region. When $N_c \ll 1$, the second term in equation (24) is dominant. Although the collision is mostly coalescent, some small particles are scattered. Because small particles are numerous, they collide with each other frequently. Collisions of small particles with negligible gravity must be mostly destructive. The circumstellar dust cloud is inferred to form by the collisions of such small particles.

4.4. The Surface Brightness of the Dust Disk

The radiation of the central star is scattered mainly by the small particles. The intensity F $\Delta\Omega$ of the scattered light from the dust along the light of sight into a solid angle $\Delta\Omega$ is given by

$$F \Delta\Omega = \frac{\Delta\Omega}{4\pi} A \varepsilon \int \sigma_s(r') (2i(r')r')^{-1} N_c(r',t)$$
$$f_s(r',t) L_*(t) (4\pi r'^2)^{-1} dx, \quad (25)$$

where A is the albedo of the dust, ε is the mass fraction of solid matter which becomes small particles due to collisions, r' is the distance from the star, f_s is the ratio of the cross section to mass of small particles, L_* is the stellar luminosity, and the integration is along the line of sight. We assume that the small particles have a size distribution n(a)da $\propto$ a$^{-3.5}$da, where a is the radius. The size distribution of the asteroids can be approximated by this law (Hughes 1982; Ishida et al. 1984). Both analytic and numerical investigations show that the size distribution of solid bodies under collisional fragmentation is also well represented by this law (Dohnanyi 1969; Hellyer 1970, 1971). For this size spectrum we have

$$f_s = (3/4\rho_s) (a_1 a_2)^{-1/2}, \quad (26)$$

where a_1 and a_2 are, respectively, the minimum and maximum radii. The minimum size is determined by the Poynting-Robertson effect as (Allen 1973)

$$a_1 = 3L_* t/(4\pi c^2 \rho_s r^2), \quad (27)$$

where c is the light speed.

From these considerations the surface brightness of (energy flux from) the part of the edge-on disk, subtending a solid angle $\Delta\Omega$ to the observer, is given by

$$F_{edge}(r,t)\Delta\Omega = 1.75 \times 10^{-3} \Delta\Omega \; A \; \epsilon \; cG^{1/8} \; \rho_s^{-5/6}(\ln\Lambda)^{-3/4}$$
$$(L_*/a_2)^{1/2}M_*^{35/24}t^{-1/4}r^{-35/8}. \qquad (28)$$

It is to be noticed that F_{edge} is independent of σ_s. It decreases slowly with time as $t^{-1/4}$. The r exponent, $35/8 = 4.375$, is very close to 4.3, the exponent obtained from the observations of the β Pic disk. The surface brightness of the β Pic disk at r = 100AU is 16 mag/arcsec^2 at λ = 0.89μm (Smith and Terrile 1984). This brightness is obtained from equation (28) with the reasonable values of the parameters $M_* = 1.8M_\odot$, $L_* = 10L_\odot$, $t = 10^8$yr, A = 0.2, $\epsilon = 0.1$, and a_2 = 50cm.
The surface brightness of the disk seen face-on is given by

$$F_{face}(r,t) \Delta\Omega = 1.27 \times 10^{-2}\Delta\Omega \; A \; \epsilon \; cG^{1/4} \; \rho_s^{-5/6}$$
$$(L_*/a_2\ln\Lambda)^{1/2}M_*^{7/12} \; \sigma_s r^{-11/4}. \qquad (29)$$

For σ given by equation (1), F_{face} is proportional to $f_\sigma M_*^{17/12}r^{-17/4}$. The r exponent, $17/4 = 4.25$, is very close to that for the edge-on disk. This surface brightness changes with time only through L_*.

4.5. The Optical Thickness of the β Pic Disk

Beta Pictoris with a spectral type of A5V is fainter than an unobscured A5V star by 0.3 to 0.8 magnitudes. Because no reddening has been found in the spectrum of β Pic, most of the obscuration must come from the circumstellar dust cloud. Therefore, the optical thickness of the dust cloud between the star and us is between 0.3 and 0.7.
If the above model disk with $f_\sigma = 3$ which explains well the thickness and the surface brightness of the β Pic disk is seen strictly edge-on, the optical thickness of the parts r = 4 to 11AU, 11 to 16AU, 16 to 25AU, and r > 25AU is 0.42, 0.32, 0.14, and 0.12, respectively. Therefore, the total optical thickness is about 1.0. However, if the disk has an inclination of 1° to us, the region between 9 and 25AU, where the mean inclination of the orbit is less than 1° as seen in Figure 3, does not contribute to the optical thickness. Therefore, the total optical thickness is about 0.4, consistent with the observations.
Thus, the geometrical and optical thickness and the surface brightness distribution of the β Pic disk are explained with our model which has been constructed with some simple reasonable assumptions.

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TERRILE: In regard to the optical thickness of the Beta Pictoris disk,
the inner region of the disk will not contribute to the total optical
depth because material will have been cleared out by planet formation

NAKANO: You are right. With my model at $t = 1 \times 10^8$ yr, for example,
planets have already formed inside about 4 AU.

NORMAN: In your scheme, when do the planets form? In the molecular
disk era, the T-Tauri-phase, or later. What can observers look for,
for example, in the early stages of planetary formation even in
molecular disk e.g. sedimented dust disks - CO disks?

NAKANO: For $M_* = 1\ M_\odot$ with $f\sigma = 2$, for example, a planet forms at $t =
2 \times 10^6$ yr at $r = 1$ AU and at later stages in the outer region. Be-
cause the era of bipolar flow would continue for less than about
10^5 yr, the planets form in the T-Tauri and later phases. Because the
gaseous nebula has a very high surface density, it is opaque to most
of the molecular lines. In addition, the planet-forming region has a
very small surface area. Therefore the intensities of any molecular
lines from the disk are very weak.