VLBI Astrometry

J. E. REYNOLDS
Australia Telescope National Facility, CSIRO

Abstract.
VLBI is at present the most accurate technique for measuring radiosource positions and the only method capable of high precision for a reasonable number of sources. The applications of VLBI astrometry in stellar, Galactic and extra-Galactic regimes are reviewed. In particular, substantial progress has been made in the last few years towards a global reference frame of extragalactic radiosource positions. The status of this frame, and of the link to the optical reference frames is also described.

Key words: Astrometry – VLBI – Phase-Referencing

1. Introduction

Astrometry, the precise measurement of the angular position and distance of celestial objects, is one of the oldest and most fundamental branches of astronomy. It was, after all, careful angular measurements of the planets in the 16th and 17th centuries that laid the observational basis for Kepler's laws of planetary motion and ultimately Newton's laws.

Very Long Baseline Interferometry, or VLBI, is at present the most accurate technique for measuring celestial angular positions at any wavelength, and the only method capable of ~milliarcsec (~5 nRad) precision for large numbers of objects. Historically, VLBI differs from conventional "phased" or "connected element" radio interferometry in the use of independent frequency and time standards at each antenna, and in the storage of the uncorrelated signal for subsequent correlation at a central site. The advent of phased VLBI arrays such as *MERLIN* has blurred this distinction somewhat, so that VLBI in the present article will be taken to mean any radio interferometry over baselines of ~30km or more.

2. Radio Astrometry with Interferometers

The principle of measuring angular positions with a two-element radio interferometer is simply that of measuring the geometric time delay, or path difference, in the reception of a celestial radio signal at the two interferometer elements as a function of time (hour-angle). This time delay τ and its rate of change $\dot{\tau}$ may be written as;

$$\tau = \mathbf{B} \cdot \hat{\mathbf{s}} = (X * \cos H - Y * \sin H) * \cos\delta + Z * \sin\delta, \quad (1)$$

$$\dot{\tau} = (-X * \sin H - Y * \cos H) * \cos\delta * \Omega, \quad (2)$$

where $\hat{\mathbf{s}}$ is the unit vector in the radio source direction, $\mathbf{B}$ = (X,Y,Z) is the interferometer baseline in a conventional (right-handed) geocentric coordinate system, H is the hour-angle of the source in the same system, δ is the source declination and Ω is the rotational angular velocity of the earth. Both expressions have a sinusoidal dependence on hour-angle, with the phase of the sinusoid giving the absolute right-ascension of the source and its amplitude the absolute source declination, provided the baseline is known and is not identically north-south. (In practice, it is necessary to interleave observations of several sources, in order to determine the baseline

J.G. Robertson and W.J. Tango (eds.), Very High Angular Resolution Imaging, 437–444.

components adequately). A conventional phased array or "connected element interferometer" (CEI) does not measure this time delay directly however, but measures a complex correlated amplitude given by;

$$S_{\text{correlated}} = S_{\text{total}} * f_{\text{visibility}} * e^{i\phi}, \; e^{i\phi} = e^{2\pi i * \nu\tau + i\phi_0} \tag{3}$$

where $S_{\text{total}} * f_{\text{visibility}}$ is the correlated flux of the source at the (~monochromatic) observing frequency ν on the interferometer baseline and the phase term ϕ_0 combines the contributions from instrumental calibration, atmospheric and ionospheric path differences as well as source structure effects. The so-called "phase delay"

$$\tau_p = \frac{\phi - \phi_0}{2\pi\nu} \tag{4}$$

defines an observable that can be used to determine absolute source positions (e.g. with Equation 1 above) provided that the phase-offset term ϕ_0 is stable or measurable to sufficient accuracy. In this case the uncertainty in a measurement of τ_p could approach the limit due to thermal noise alone, given by $\epsilon(\tau_p) \approx 1/(2\pi\nu * \text{SNR})$, where SNR is the signal to (thermal) noise ratio of the measurement. The corresponding uncertainty in source angular positions would then scale inversely with baseline length B according to $\epsilon(\theta) \approx c\epsilon(\tau_p)/B$. In practice, however, the final precision is dominated by systematic phase errors contained in ϕ_0 . Indeed, if the uncertainty in ϕ_0 approaches or exceeds ~1 radian, successive measurements of τ_p contain ambiguities of $1/\nu$ and cannot be "connected", rendering them useless for astrometric purposes. Normally the instrumental terms in ϕ_0 can be eliminated with suitable instrumentation (e.g. phase-stabilised cables or radio links and calibration tone injection) while source structure effects can be minimised by selecting sources of suitably small angular size, or by computing structure correction factors from radio images of the source. Variable propagation delays through the terrestrial ionosphere can be corrected by observing simultaneously at two widely-spaced frequencies (e.g. Shapiro 1976) but the variable properties of the troposphere pose a more fundamental problem and generally limit the accuracy of absolute source positions from phased-arrays or CEIs to ~ 0.03 arcsec (e.g. Wade & Johnston 1976, Johnston *et al.* 1985).

This limit arises from the shape of the so-called structure function of the atmospheric phase, which characterises the RMS phase difference along two parallel paths through the troposphere as a function of their horizontal separation or baseline length (e.g. Thompson *et al.* 1986, Ch. 13 and references therein). Empirically this RMS phase difference is found to increase nearly linearly with increasing separation such that the astrometric precision of a CEI increases only slowly with increasing baseline. That is, the geometrical advantage of the longer baseline is largely offset by the increasing error in ϕ_0 as the tropospheric paths to each antenna become progressively decorrelated. Moreover, at separations of typically a few x 10km, the RMS phase difference reaches ~1 radian, at which point the phase delay cannot be unambiguously "connected" between observations of two sources separated by a large angle on the sky, as the phase differentials for the two widely separated sources are essentially uncorrelated. Thus absolute astrometry using phase delay becomes impossible over baselines longer than ~few x 10km, although differential astrometry between sources of sufficiently small angular separation is still possible; a technique known in VLBI as phase-referencing.

For absolute VLBI astrometry, the phase delay is discarded in favour of a more robust though less precise observable, the group delay. The significance of the group delay may be seen in the following 1st-order expansion of the interferometer phase;

$$\phi(\nu,t) = \phi(\nu_0,t_0) + (\nu-\nu_0)\frac{\partial\phi}{\partial\nu}(\nu_0,t_0) + (t-t_0)\frac{\partial\phi}{\partial t}(\nu_0,t_0) \quad (5)$$

$$= \phi(\nu_0,t_0) + 2\pi(\nu-\nu_0)\tau_g + 2\pi(t-t_0)\dot{\tau_g}\nu_0 \quad (6)$$

$$\tau_g = 1/2\pi * \frac{\partial\phi}{\partial\nu}(\nu_0,t_0) \quad \dot{\tau_g} = 1/2\pi\nu_0 * \frac{\partial\phi}{\partial t}(\nu_0,t_0) \quad (7)$$

where τ_g and $\dot{\tau_g}$ are the group delay and delay rate respectively, and ν_0 and t_0 are appropriate centre values for the expansion. It is straightforward to show that having removed instrumental effects (calibration) and dispersive effects (dual-frequency observations), the phase delay and group delay are equivalent estimators of the geometric delay, with two important differences. Firstly, the use of group delay essentially eliminates the problem of ambiguities which afflict the phase delay. Secondly, the uncertainty in the group delay due to thermal noise is given by $\epsilon(\tau_g) \approx 1/(2\pi\Delta\nu_{\rm RMS} * {\rm SNR})$, which is larger than that for the phase delay by a factor of $\nu/\Delta\nu_{\rm RMS}$ where ν is the observing frequency and $\Delta\nu_{\rm RMS}$ is the RMS bandwidth of the receiving system ($= \Delta\nu/\sqrt{12}$ for a rectangular bandpass of width $\Delta\nu$). The actual loss of accuracy is not as great as this ratio implies, as systematic effects, especially from the troposphere, are usually the limiting factor. It is also worth noting that for VLBI baselines (longer than 30~100km) the phase errors at each antenna arising from atmospheric effects are generally completely uncorrelated, so that the resulting uncertainty in source positions does indeed scale inversely with baseline length (i.e. $\epsilon(\tau_g) \sim 1/{\rm B}$).

3. Bandwidth Synthesis (BWS)

Examination of Equation 6 shows that the frequency (offset) $\nu - \nu_0$ and group delay τ_g are Fourier conjugates, in the sense that a Fourier transform of the complex interferometer amplitude (Equation 3) as a function of frequency yields a function in the domain of τ_g, which in this sense is often called the "lag" domain. In particular, the Fourier transform amplitude of the receiver bandpass function is called the lag or delay resolution function, and is the normalised response produced by an unresolved source with delay $\tau_g = 0$ (ignoring spectral-index effects). The peak in the function obtained by Fourier transforming the complex interferometer amplitude is in fact the best estimate of τ_g for a particular observation. Similarly, the Fourier transform of the complex interferometer amplitude as a function of time yields a function in the domain of fringe-rate (fringe-frequency) whose peak is the best estimate of the product $\dot{\tau_g}\nu_0$. Usually, these two transforms in frequency and time are performed together, producing a 2-D function in lag vs fringe-rate whose peak defines the best estimates of τ_g and $\dot{\tau_g}\nu_0$.

Given the constraints on total recorded bandwidth (bit rate) in VLBI systems, astrometric VLBI observations using group delay employ a technique known as "bandwidth synthesis" or BWS (Rogers 1976) in which the available recorded bandwidth is distributed across the largest possible range of sky frequencies. In early VLBI astrometric experiments this was accomplished by switching a single recording channel sequentially among a number of widely-spaced sky frequencies. Since the early 1980's, most astrometry and geodesy VLBI experiments have been made with the Mk-III VLBI recording system (Rogers *et al.* 1983), recording 14 simultaneous 2MHz channels (usually 6 channels at 2GHz and 8 channels at 8GHz), distributed in frequency to obtain the sharpest main peak in the lag resolution function (largest $\Delta\nu_{\rm RMS}$) while ensuring that the secondary peaks ("sidelobes") are neither too large nor too close to the main peak to cause ambiguity. Figure 1. shows the lag resolution function for the most common frequency set at X-band (8GHz), which spans 362MHz and has an RMS bandwidth $\Delta\nu_{\rm RMS}$ = 140MHz (FWHM in lag of $\sim$ 2.5ns). The multiple peaks in the function result from the finite sampling across the frequency spectrum and their separation (100ns) is determined by the minimum spacing (10MHz) between frequency channels. The ambiguities in τ_g caused by these "grating"-type responses are not normally troublesome, unless the *a priori* source or antenna coordinates are particularly poorly known.

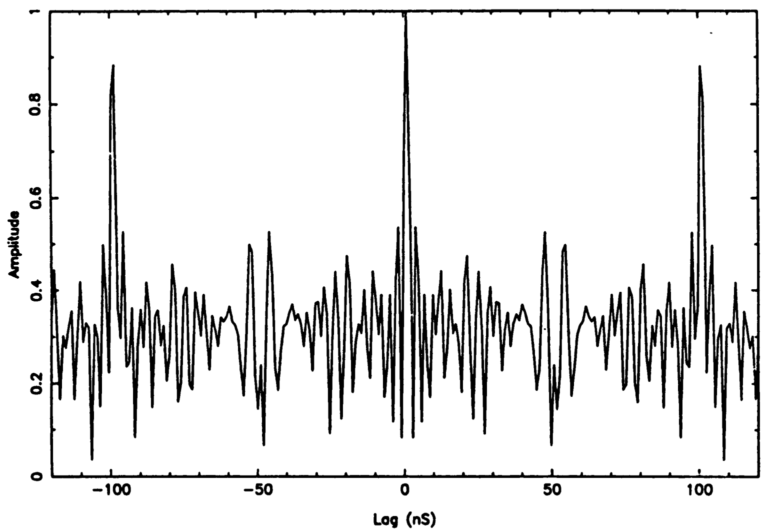

Fig. 1. The lag resolution function of the most commonly used BWS frequency set at 8.4GHz, using 8 frequency channels.

4. Data Reduction

Whilst it is possible to use Equations 1&2 to determine a position from observations of a single source, in practice a number of other unknowns need to be determined simultaneously. Hence typical BWS astrometry/geodesy experiments comprise repeated short (100 $\sim$ 200s) scans, switching between a number of sources as quickly as antenna drive rates will allow, for periods of up to ~ 24 hours or longer. Fringe-fitting extracts group delay, delay rate and phase delay estimates for each source scan. The group delay and, optionally, delay rate values are then modelled by least-squares minimisation to solve for a number of possible parameters, including;

- Source coordinates
- Station coordinates (and motion)
- Station clock errors and drifts
- Tropospheric delays
- Ionospheric (dispersive) delays
- Earth orientation (polar motion, dUT)

The two major software packages used for this type of least squares analysis are *CALC/SOLVE/GLOBL* (Ma *et al.* 1986), maintained at USNO and the Goddard Space Flight Center (GSFC), and *MODEST* (Sovers, 1991), formerly *MASTERFIT*, maintained at the Jet Propulsion Laboratory (JPL). Both packages can accept large databases ($N_{data} \sim 10^6$) comprising many combined experiments spanning a number of years, in order to solve for parameters "globally".

5. Limitations

**Ionosphere:* The excess group delay due to charged particles in the ionosphere undergoes large diurnal fluctuations of up to .1~1nS / sin El (3~30cm/ sin El) at 8GHz and, uncorrected, constitutes the largest single source of error in BWS astrometry observations. The use of dual, simultaneous, widely-separated observing frequencies (e.g. S/X bands) reduces the effect to generally negligible levels.

**Troposphere:* Variable path lengths through the troposphere consist of a large but slowly varying "dry" component of ~200cm/ sin El and a smaller but more rapidly varying component due to water vapour of order ~30cm/ sin El (e.g. Thompson *et al.* 1986, Eqs. 13.13, 13.19). It is possible to estimate dry tropospheric effects from surface meteorological data at the antenna sites but the erratic and variable nature of the wet component is more of a difficulty, and represents the dominant source of error in most BWS VLBI experiments. Current practice is to employ a "self-calibration" procedure which estimates the zenith tropospheric delay at each antenna as a piecewise-linear function of time by adopting an *a priori* model or "mapping function" describing the elevation dependence of tropospheric delay. Several such mapping functions exist (Sovers, 1991), for example the CfA-2.2 model of Davis *et al.* (1985). The self-calibration interval is typically ~few hours, with final residual errors of few mm ~ 1cm.

**Source Structure:* The QSOs and AGN comprising most VLBI astrometry observing lists tend to be dominated, at least on intercontinental baselines of ~ 10^4km, by a single high surface-brightness "core", without detectable proper motion (e.g. Bartel *et al.* 1986). At the level of a few milliarcseconds, however, a number of sources show time-dependent motion or excessively high post-fit residuals (e.g. Jacobs 1993), arising from internal structure and motions. From the point of view of the construction of a rigid reference frame, an obvious expedient is to omit such sources from the programme, but it is possible that at the ~few x 0.1milliarcsec level many or perhaps a majority of sources exhibit some form of structure. In so far as individual sources are concerned, full VLBI imaging at the appropriate resolution and frequency can be used to compute structure factors allowing astrometric observations to be corrected to some reference position within the source image (i.e.

an identifiable "core"). Observations of this type are likely to be undertaken in the future with the VLBA.

6. Future Developments

**Wider Bandwidths:* The advent of the Mk-IV VLBI recording system (Whitney *et al.* 1991) with a 4~8-fold improvement in recorded bandwidth over Mk-III, and the introduction of wideband (~1GHz) receiver systems promises to push BWS astrometry/geodesy limits down to a few x 0.1milliarcsec and a few mm in source and station coordinates respectively, as well as allowing the inclusion of many additional weaker sources in astrometric catalogues.

**VLBA:* As a dedicated VLBI instrument the VLBA may be expected to make a considerable impact in astrometry, not only in conventional BWS astrometry experiments, but in differential (phase-referencing) measurements and imaging source structure, for both of which the VLBA is well equipped.

**Orbiting VLBI:* The extension of VLBI baselines into space, beginning with the *VSOP* and *RadioAstron* missions scheduled for launch in 1995, has the potential to push absolute VLBI astrometry below the 0.1milliarcsec level, provided the additional complexities of orbit determination can be solved to sufficient precision. It is likely that the early OVLBI missions will be confined to astrometry of the differential (phase-referencing) type.

7. VLBI Global Reference Frame

The construction of a quasi-inertial extragalactic reference frame comprising a rigid, uniformly dense grid of ~400 extragalactic source positions of milliarcsec precision (Johnston *et al.* 1988) is well underway (Russell *et al.* 1993). While progress in the southern sky ($\delta < -40^o$) was initially slow owing to the scarcity of suitable southern hemisphere facilities (e.g. Harvey *et al.* 1992), rapid progress has been made over the last few years (Russell *et al.* 1992, Reynolds *et al.* 1993) to match the more extensive observations in the north (Fey *et al.* 1992 and references therein). The target of 1milliarcsec precision has already been reached for the great majority of "northern" sources ($\delta > -40^o$).

The project is a collaboration of several institutions, including NRL, USNO and NASA/DOSE (formerly CDP) in the USA, CRL in Japan, ATNF, AAO and the Universities of Tasmania and Western Sydney in Australia, the Hamburger Sternwarte of Germany and the Hartebeesthoek Radio Astronomical Observatory of South Africa. A parallel programme to measure ~50milliarcsec positions on the FK5 for all optical identifications in the sample is also underway.

An additional VLBI astrometric catalogue of significance is that maintained by JPL (Sovers *et al.* 1988, Jacobs 1993), again at milliarcsec precision, but generally restricted to ecliptic declinations for spacecraft navigation.

8. Differential Astrometry

As mentioned above, the phase delay can be retained as an astrometric observable in VLBI between pairs of objects separated by a sufficiently small angle. The term

https://doi.org/10.1017/S0074180900108149 Published online by Cambridge University Press

"isoplanatic angle" is applied to the maximum angular separation within which the RMS phase differential on an interferometer baseline is ~1radian or less. The value of the isoplanatic angle is a function of frequency and is $5^o \sim 10^o$ at 5GHz, falling to $\sim 0.5^o$ at 400MHz, as ionospheric effects become dominant.

The phase delay differential between sources within this angle can then be used to estimate their accurate angular separation. Equivalently, the observed interferometer phase of one object can be used to calibrate the phase of the second, allowing coherent integration over the entire period of the observation, a technique known as phase-referencing. This applies even when the second object is too weak to be detected within the coherence time (5~10min at 5GHz), thereby allowing detection of much weaker sources than is possible in conventional VLBI. The limiting precision of phase-referenced VLBI astrometry is generally dominated by residual systematic effects which scale with the angular separation of the sources, with typical results of the order of ~0.1milliarcsec/degree (~3 x 10^{-8}) or better. Applications of phase-referenced VLBI astrometry include;

**QSO Proper Motions:* Bartel *et al.* (1986) have used phase-referencing to place upper limits of ~ 20μarcsec/year on the relative motion between two QSO cores.

**Radio Stars:* The large increase in sensitivity allowed by phase-referencing has found a particularly useful application in the detection of active stars. Lestrade *et al.* (1990) were able to detect the stellar system Algol at a flux density of S_{5GHz} = 3mJy and measure its angular distance from the reference source to ~0.5milliarcsec. The measurement of precise radio positions of radio stars is currently the most promising means of providing a ~milliarcsec link or "frame-tie" between the optical frame of ~10^5 stellar positions and proper motions defined by the *HIPPARCOS* mission, and the extragalactic VLBI reference frame. At present the most accurate tie between the frames is based on radio star measurements from the VLA, and is limited in accuracy to about ~50milliarcsec (Walter 1993). A long-term programme of VLBI radio star measurements has been underway in the northern hemisphere for several years (Lestrade *et al.* 1992), while a more recent but active programme in the south is also making steady progress (White *et al.* 1990, Jauncey 1991). Of particular astrophysical interest is the accurate registration of the optical and radio images of the remnant SN1987A (Staveley-Smith *et al.* 1992) using VLBI measurements of two nearby radio stars (HD32918 and HD36705) to provide an accurate local orientation of the *HIPPARCOS* and VLBI reference frames.

**Pulsars:* Approximately ~50 pulsar proper motions have been measured to a precision of ~10milliarcsec/year or better using phase-referenced VLBI (see review by Bailes & Johnston, 1993), providing important information on pulsar formation and evolution. The millisecond pulsar PSR 1937+21 is of particular interest for astrometry as it has not only a ~milliarcsec phase-referenced VLBI position, but a timing position in the planetary ephemeris reference frame measured to similar accuracy. While at present this is the only pulsar having both accurate VLBI and timing positions, a bright millisecond pulsar found recently in the southern sky (Johnston *et al.* 1993) is a most promising candidate for a second, offering the possibility of a milliarcsecond tie between the extragalactic VLBI and planetary frames. Such a tie is of considerable interest in a number of areas including solar system dynamics and spacecraft navigation.

**Galactic Masers:* Expanding spherical shells around OH-IR stars (e.g. Moran 1993) and statistical parallax measurements of proper motions within Galactic maser complexes (e.g. Reid *et al.* 1988) have provided a number of independent distance measurements, including the Galactic centre. Extragalactic H_2O masers should in the near future be accessible with the extra resolution of OVLBI, giving independent estimates of the distance to a number of nearby galaxies.

**Gravitational Deflection of Light:* VLBI has been used to measure the solar gravitational deflection, confirming the General Relativistic estimate to $\sim 0.2\%$ (e.g. Robertson *et al.* 1991). More recently, gravitational deflection around Jupiter has also been measured directly (Lowe 1993).

References

Bailes, M. & Johnston, S.:1993, in *URSI Review of Radio Science*, ed. W.R.Stone (in press).

Bartel, N. *et al.*:1986, *Nature*, **319**, 733.

Davis, J.L. *et al.*:1985, *Radio Science*, **20**, 1593.

Fey, A.L. *et al.*:1992, *Astron. J.*, **104**, 891.

Harvey, B.R. *et al.*:1992, *Astron. J.*, **103**, 229.

Jacobs, C.S.:1993, in Proc. IAU Symp. 156, *Developments in Astrometry & Their Impacts on Astrophysics & Geodynamics*, Shanghai, 1992 (in press).

Jauncey, D.L.:1991, *Aust. J. Phys.*, **44**, 785.

Johnston, K.J. *et al.*:1985, *Astron. J.*, **90**, 1343.

Johnston, K.J. *et al.*:1988, in Proc. IAU Symp. 129, *The Impact of VLBI on Astrophysics and Geophysics*, eds. M.Reid & J.Moran, Reidel:Dordrecht, p317.

Johnston, S. *et al.*:1993, *Nature*, **361**, 613.

Lestrade, J.F. *et al.*:1990, *Astron. J.*, **99**, 1663.

Lestrade, J.F. *et al.*:1992, *Astron. & Astrophys.*, **258**, 112.

Lowe, S.T.:1993, in Proc. IAU Symp. 156, *Developments in Astrometry & and Their Impacts on Astrophysics & Geodynamics*, Shanghai, 1992 (in press).

Ma, C. *et al.*:1986, *Astron. J.*, **92**, 1020.

Moran, J.M.:1993 in Proc. Symp. on *Sub-Arcsecond Radio Astronomy*, Manchester, 1992 (in press).

Reid *et al.*:1988, *Astrophys. J.*, **330**, 809.

Reynolds, J.E. *et al.*:1993, *Astron, J.*, (to be submitted).

Robertson, D.S., Carter, W.E. & Dillinger, W.H.:1991, *Nature*, **349**, 768.

Rogers, A.E.E.:1976, in *Methods of Experimental Physics* Vol 12C, ed. M.L.Meeks, Academic Press:New York, p139.

Rogers, A.E.E. *et al.*:1983, *Science*, **219**, 51.

Russell, J.L. *et al.*:1992, *Astron. J.*, **103**, 2090.

Russell, J.L. *et al.*:1993, in Proc. Symp. on *Sub-Arcsecond Radio Astronomy*, Manchester, 1992 (in press).

Shapiro, I.I.:1976, in *Methods of Experimental Physics* Vol 12C, ed. M.L.Meeks, Academic Press:New York, p261.

Sovers, O.J.:1991, *JPL Publication 83-39, Rev. 4.*

Sovers, O.J. *et al.*:1988, *Astron J.*, **95**, 1647.

Staveley-Smith, L.S. *et al.*:1992, *Nature*, **355**, 147.

Thompson, A.R., Moran, J.M. & Swenson, G.W.: 1986, *Interferometry and Synthesis in Radio Astronomy*, Wiley:New York.

Wade, C.M. & Johnston, K.J.:1976, *Astron. J.*, **82**, 791.

Walter, H.:1993, in Proc. IAU Symp. 156, *Developments in Astrometry & Their Impacts on Astrophysics & Geodynamics*, Shanghai, 1992 (in press).

White, G.L. *et al.*:1990, *Astron. J.*, **99**, 405.

Whitney, A.R. *et al.*:1991, NOAA Technical Report NOS 137 NGS 49, Proc. of the AGU Chapman Conference on Geodetic VLBI: Monitoring Global Change, U.S. Department of Commerce, Rockville, Maryland.