HOW TO QUANTIFY RIPPLE

H. Geib, C. Kühne, E. Morgenbrod

Carl Zeiss, Oberkochen

1. Introduction

Every manufacturer of large mirrors is familiar with the fact that the polishing process may not only lead to large-area surface errors, such as astigmatism or spherical aberration, but also to short-period errors. In particular, this applies to aspherical surfaces because they require polishing tools of small dimensions, as compared with the mirror diameter. These errors are not entirely irregular and may therefore not be treated as statistical errors; nor are they sufficiently regular to be described in terms of amplitudes of ZERNIKE polynomials as it is now ussually done in the case of large-area aberrations (F.FRANZA e. a. 1977). Unfortunately this still is true if very high radial and tangential orders are involved, e.g.in D. ANDERSON e.a.1982. The reason is quite obvious: The description by means of ZERNIKE polynomials is based on the assumption of a two-dimensional regularity which simply does not exist in practice.

We therefore omitted the two-dimensional aspect altogether when trying to find a method of making these small-area errors - which are called "ripple" in the following - accessible to quantitative description. The objection that this approach is incomplete and lacks mathematical strictness is outweighed by the higher degree of clarity and the easy comparability of the measuring results. Furhtermore, our work is to be considered only as a first attempt of dealing with the ripple problem with numerical methods.

2. Definition

First of all, the meaning of the term "ripple" is to be defind. Let us take a look at a Foucault test photograph Fig. 1 for this purpose: A regular zonal fine structure is clearly noticeable but the spatial frequency does not evenly cover the mirror, neither in the radial nor in the tangential direction. Also, it is neither constant nor does it satisfy a sufficiently simple relationship

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between the period length and the radius. However, it is characterized by the mainly rotational-symmetrical distribution of the ripple structure which is due to the polishing method. Unfortunately a Foucault test photograph only indicates the period length of the fine structure. The quantitative determination of the amplitudes is virtually impossible. This can only be achieved with the aid of interferograms, as shown in Fig. 2. However, it has to be accepted that in interferograms the tangential fringes of the ripple structure are not very pronounced. But if the evaluation is confined to one dimension - as it was in our case - the central radial fringe is sufficient for the determination of the ripple.

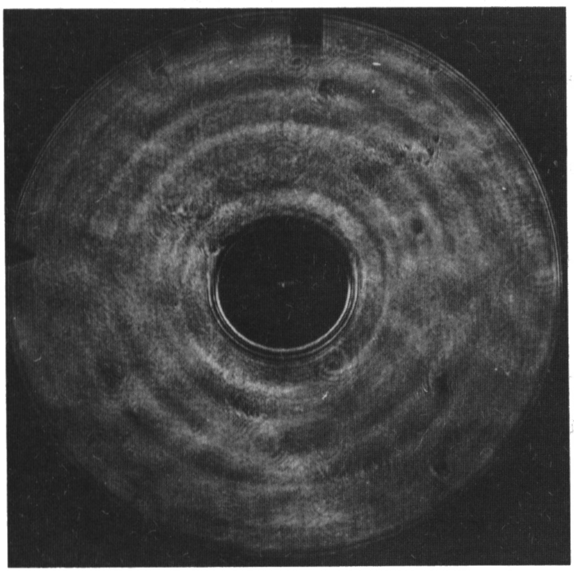

Fig. 1 Foucault Test Photograph, (3,5 m Primary MPIA)

Fig. 2 Interferogram, showing side interferences (1,2 m plano coudé mirror IRAK)

At the same time Fig. 2 shows a disturbance in the form of side interferences of extremely short period length. They practically always occur in laser interferometers (LUPI) induced by micro defects in interferometer elements as a result of the long coherence length of laser light. They considerably complicate the evaluation process. Although it is well known that these interferences must not be interpreted as mirror errors, it is rather difficult to separate them in the analysis without making any further theoretical assumptions. We therefore did not develop a special theory for the determination of these interfluences. They are not deducted from the results even if this leads to a deterioration of the numerical result - a perfectly justifiable approach, generally. In extreme cases - of which an example will be given - the influence can be directly identified and permits to be excluded in the evaluation of the measured result.

3. Measurement of the Interference Fringe

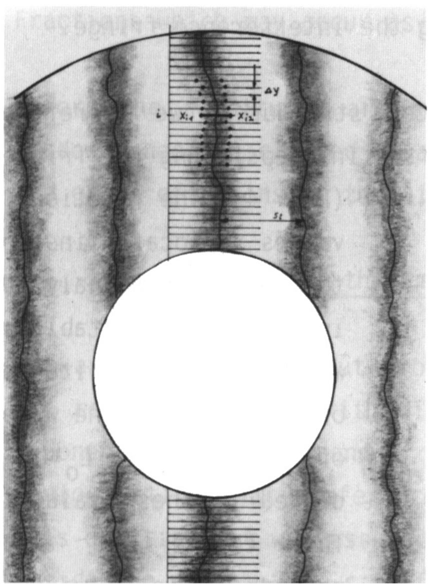

Fig. 3 Measurement of interference fringes

The measurement is performed according to the method shown in Fig. 3. The central interference fringe is photometered on the left and right side in equidistant steps of i = 1,2,3,...N and the coordinates $x_{i,1}$ and $x_{i,2}$ belonging to an arbitrary but fixed degree of density are measured. N is a sufficiently large number, approx. 400. Any bore which the mirror may have is skipped and not counted. The two x-values are averaged to x_i and the sequence of these mean values represents the ripple structure of the fringe.

$$x_i = \frac{1}{2}(x_{i,1} + x_{i,2}) \qquad i = 1, 2, \ldots N$$

To get clear of the scale factor of the coordinate measurement, the mean fringe spacing s is measured. Distance s is either 1 or 2 in nanometers, depending on whether the interferogram was obtained in a single or double application. Division by s gives x_i in nanometers. The linear quantity of Δy is obtained from the ratio of the actual mirror diameter and its image diameter.

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4. The Evaluation Philosophy

The obvious approach for us was to apply a Fourier analysis to the x_i values measured and to calculate a power spectrum. Information is thus obtained on the amplitudes in the fringes in the form of a function of their spatial frequency. Several tests have shown, however, that this function is of little informative value. The primary reason being that generally a fixed spatial frequency only extends over short distances along the interference fringe.

To be able to also evaluate short but pronounced structures, we therefore defined a "window", 2 L_0 in length, which is shifted in steps along the fringe (Fig. 4). The functional values x_i located inside this window are analyzed in order to establish which are the amplitudes of a sine or cosine wave of period length L_0 included in these values. The choice of two periods for each window lenght is a compromise between the following two considerations:

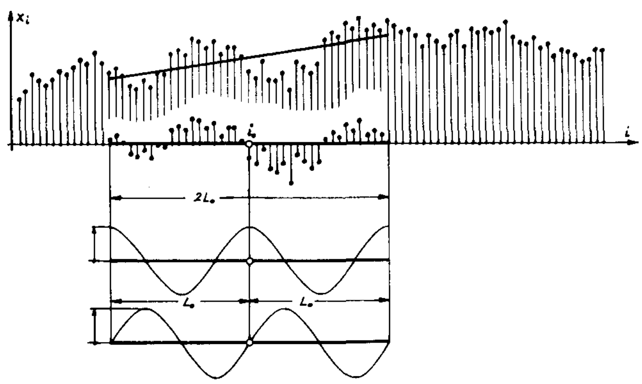

Fig. 4 Travelling window

- The more periods are included in the window, the more the ripple amplitudes of this period length are damped if they do not completely cover the length of the window. This would lead to an inadequate assessment of the ripple.

- If there is only one period per window length, every single irregularity which just covers the window length will be fully included in the evaluation. Single irregularities, however, are not to be regarded as ripple.

The period number 2, in our opinion, makes sufficient allowance for both arguments.

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The scanning of the interference fringes with windows of 2 L_0 in length is performed by changing the centre position i_0 step by step. With one scan run completed, L_0 is changed and the procedure is repeated. We chose one quarter of the diameter to be the initial length of L_0 which means that the largest window is as long as the radius. The reduction factor of L_0 is 0.8. Reduction is continued until the number of reference points on L_0 is less than 4. L_0 is always chosen so that L_0 is an integral multiple of the step width Δh. Fractions which may occur as a result of the steps are rounded off.

Before the amplitudes are numerically determined, the x_i values inside the window range are cleared of an inherent linear regression, i.e. the mean value and grade of x_i inside the window are zeroed.

5. Determination of the Amplitudes

Our analysis started out from the question whether there are differences in the results obtained with different trigonometric functions, sine, cosine or exponential functions and if so, whether they are of a systematic nature which would justify that preference is given to one of these functions. To decide this question, it was assumed that the oscillation required of a ripple period inside the window 2 L_0 satisfies the equations

$$y_k = a \cdot \cos 2\pi \frac{\Delta h}{L_0} k + b \cdot \sin 2\pi \frac{\Delta h}{L_0} k \tag{1}$$

$$L_0 = n \cdot k \tag{2}$$

with Δh and L_0 being given and a and b being required. a and b are determined according to the Gaussian fit

$$Q = \sum_{k=-n}^{n-1} \{x_{i_0+k} - y_k\}^2 \Rightarrow \text{Minimum} \tag{3}$$

If it is further assumed that a ripple of period length L exists in the fringe and has a constant amplitude C along the fringe, the following applies to the measured values

$$x_k = C \cdot \cos(2\pi \frac{\Delta h}{L} k + \varphi) + x_k' \tag{4}$$

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Here φ is an arbitrary phase position in which i_0 is also to be included. x_k' contains all remaining portions of which can not be expressed in terms of this oscillation.

If a and b are now determined through differentiation of (3), and assuming that the summation residua of the remainders x_k' are sufficiently small, the following applies approximately:

$$a\cdot\sum_k \cos^2\frac{2\pi}{n}k + b\cdot\sum_k \sin\frac{2\pi}{n}k\cdot\cos\frac{2\pi}{n}k = C\cdot\sum_k \cos\left(\frac{2\pi}{n}\cdot\frac{L_0}{L}k+\varphi\right)\cdot\cos\frac{2\pi}{n}k$$

(5)

$$a\cdot\sum_k \sin\frac{2\pi}{n}k\cdot\cos\frac{2\pi}{n}k + b\cdot\sum \sin^2\frac{2\pi}{n}k = C\cdot\sum_k \cos\left(\frac{2\pi}{n}\cdot\frac{L_0}{L}k+\varphi\right)\cdot\sin\frac{2\pi}{n}k$$

The summation covers k = -n to k = (n-1), i.e. 2n values. If we restrict ourselves here to the case that L differs from L_0 only very slightly, and if we hold that

$$\frac{L_0}{L} = 1-\delta L \qquad |\delta L| \ll 1 \tag{6}$$

then (5) can be evaluated from δ L as follows:

$$n\,a = C\left[\sum_k \cos\left(\frac{2\pi}{n}k+\varphi\right)\cdot\cos\frac{2\pi}{n}k - \frac{2\pi}{n}\delta L\sum_k k\cdot\sin\left(\frac{2\pi}{n}k+\varphi\right)\cdot\cos\frac{2\pi}{n}k\right]$$

(7)

$$n\,b = C\left[\sum_k \cos\left(\frac{2\pi}{n}k+\varphi\right)\cdot\sin\frac{2\pi}{n}k - \frac{2\pi}{n}\delta L\sum_k k\sin\left(\frac{2\pi}{n}k+\varphi\right)\cdot\sin\frac{2\pi}{n}k\right]$$

Thus we get

$$a = C\cdot\cos\varphi\left[1-\frac{\pi}{n}\delta L\left\{\cot an\frac{2\pi}{n}+2\tan\varphi\right\}\right]$$

(8)

$$b = C\cdot\sin\varphi\left[1-\frac{\pi}{n}\delta L\cdot\cot an\frac{2\pi}{n}\right]$$

The amplitudes a and b of the cosine or sine function can be combined to give the amplitude c of the exponential function

$$c = C\left[1-\frac{\pi}{n}\delta L\left\{\cot an\frac{2\pi}{n}+\sin 2\varphi\right\}\right] \tag{9}$$

The result can be interpreted as follows:

- for a ripple of period length L_0, i.e. $\delta L = 0$, a and b follow faithfully the phase position φ of the ripple oscillation. As we can expect, c = C is also a constant over the whole length of the fringe, as each integer numbered periodic section can be precisely represented by the exponential function.

- for a ripple with a slight deviation $\delta L \neq 0$ from the period length L_0 certain deviations arise in determining C, which are proportionate to δL. The deviations consist partly of an element (cotan $2\pi/n$) which depends purely on the point number n, i.e. on the length L_0 itself. This element is constant in the amplitudes a, b and c. With a and c however other terms appear which are dependent on the phase φ . The sine amplitude b is not affected by φ.

- the factor (π/n)cotan$(2\pi/n)$ is zero for the minimum point number (n = 4) and for large n develops asymptotically and unvaryingly into 1/2. For a small δL the deviation of the amplitude thus remains small and is negligible. In other words, ripple periods in the close vicinity of L_0 are still sufficiently well determined by the amplitudes.

The amplitudes thus incorporate a certain bandwidth. And this accords satisfactorily with the fact that the period lengths of the ripple structures are subject to marked fluctuations. So we have also selected the interval between successive L_0 (with a factor of 0.8) in such a way that each period length can be deemed as a deviation δL from the foregoing one. This guarantees a certain constancy of transition between various period lengths and ensures that no ripple period is suppressed in the analysis.

6. Testing the Evaluation

To test the method and determine the best way to represent it, we examined numerically a ripple structure of known size and period length. The COSINE test, as we called it, consisted of three overlapping cosine functions of the amplitudes $\pm$ 10 nm and the period lengths L = 64, L =20, and L = 8. Fig. 5 shows the input data of the test.

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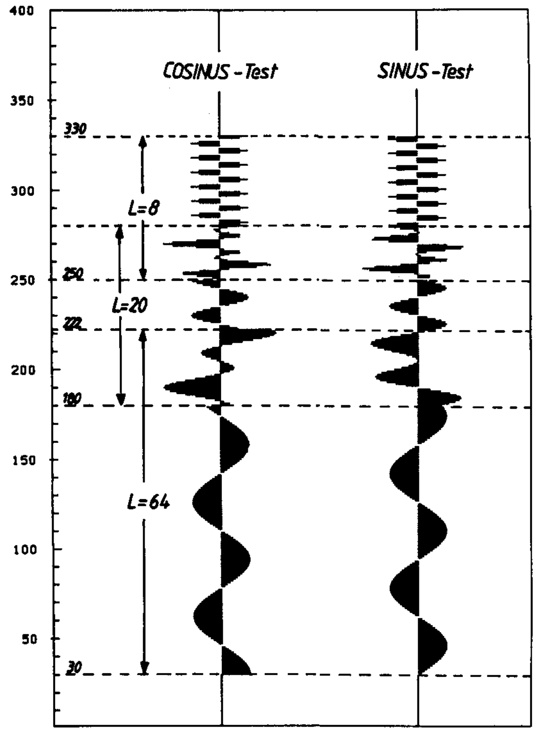

Fig. 5 Input Data of COSINE- and SINE-Test

In a second test, the SINE test, the cosine functions of Fig. 5 were substituted by sine functions. In each of these tests the amplitudes a, b and c were determined according to equations (8) and (9).

Fig.6, 7 and 8 show the results of the COSINE test, Fig. 9, 10 and 11 those of the SINE test. As we might expect, the amplitudes a and b of the cosine and sine functions fluctuate between positive and negative values and are phase displaced in relation to each other by about 90°. c is by definition always positive. One can also see clearly how on the one hand the three different period lengths are separated from each other, but on the other hand also overlap into the neighbouring L_o.

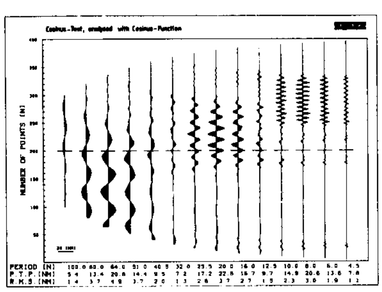

Fig. 6 COSINE-Test with cosine - function

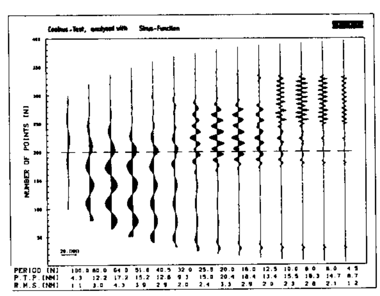

Fig. 7 COSINE-Test with sine-function

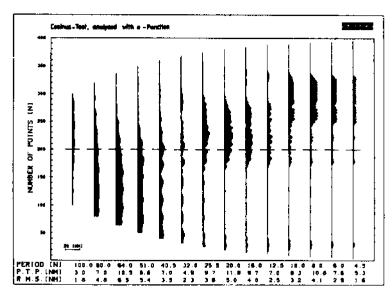

Fig. 8 COSINE-Test with e-function

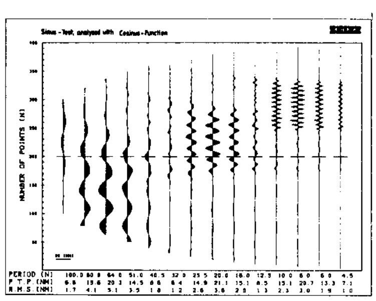

Fig. 9 SINE-Test with cosine-function

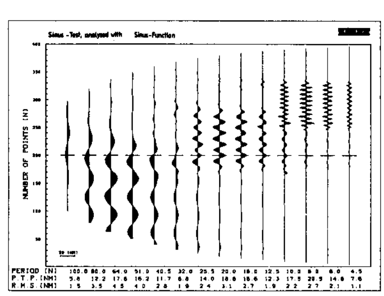

Fig. 10 SINE-Test with sine-function

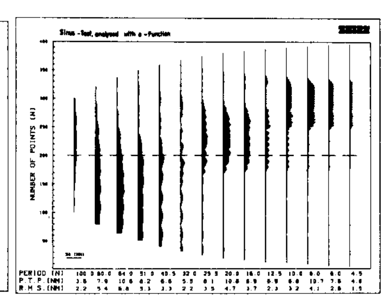

Fig.11 SINE-Test with e-function

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A closer comparison shows that the cosine amplitude between the various period lengths fades quickest. This fact becomes clearer if we illustrate the difference in the extremes of amplitude as a function of the period length (Fig. 12.)

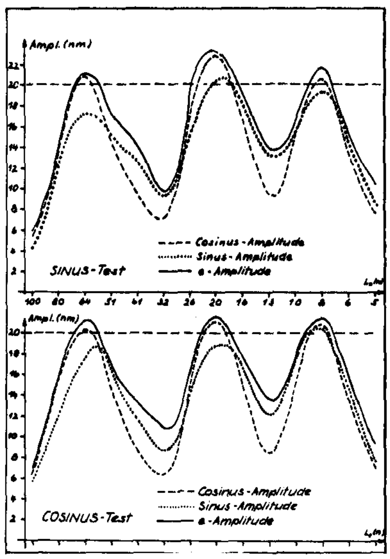

Fig. 12 Comparison of extremes of amplitudes

In both tests the sine amplitude proves to be inferior. It does not always reach the nominal input amplitude of ± 10 nm. This does not happen with the cosine amplitude which also shows much lower minimum values between the nominal period lengths. The exponential amplitude, which by definition is always larger than a or b, always attains the input amplitude but is inferior to the cosine amplitude in sharpness of definition. The causes of this different behaviour are not yet fully established. We suspect that it may be because the cosine amplitude can react more sensitively to inconstancy in the procedure than the sine amplitude, because it is jumping up and down between zero and one at the ends of the window.

The slight increase in amplitude in the nominal period lengths is clearly the result of the fact that the positive and negative extremes can arise from different elongation. Thus they are always affected by an unfortunate combination of other influences from the neighbouring period lengths.

As a result of this test, we chose the cosine amplitude as the value which should represent the ripple structure of an interferogram.

7. Quantitative Evaluation

To determine the quality of mirrors we normally use the standard deviation and the maximum peak-to valley deviation of the phase disturbances of a wavefront. In support of this procedure we have also analysed the difference between the extreme positive and negative elongations of the cosine amplitude a for each

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period length L_0. And the positive and negative extremes can definitely belong to various oscillations of a. We have further calculated the standard deviation of the elongations.

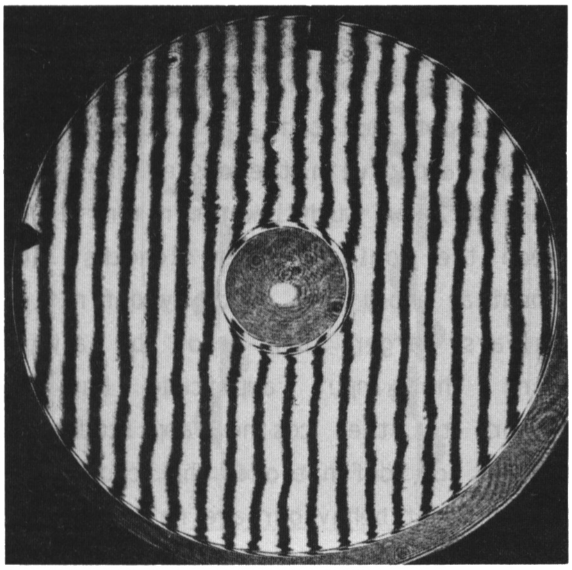

Fig. 13 3.5 m - Primary Mirror of MPIA, Interferogram

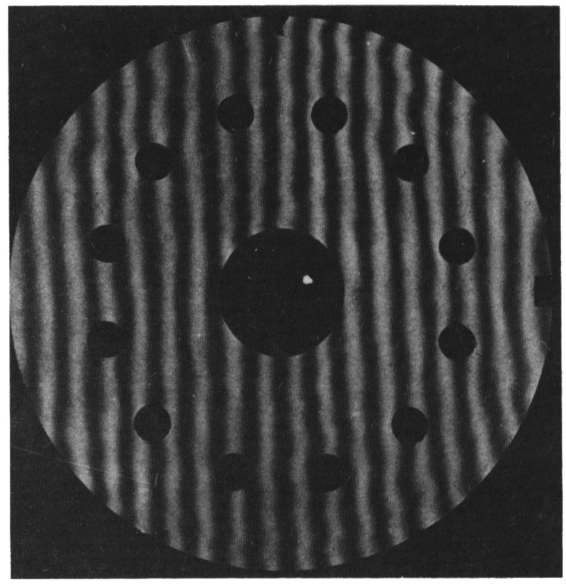

Fig. 15 1.1 m -RC Secondary Mirror of Iraq, Interferogram

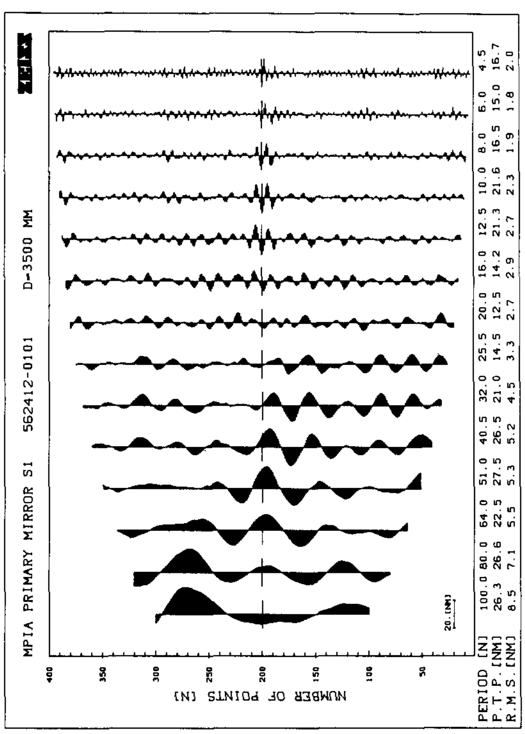

Fig. 14 3.5 m Primary Mirror of MPIA, Ripple

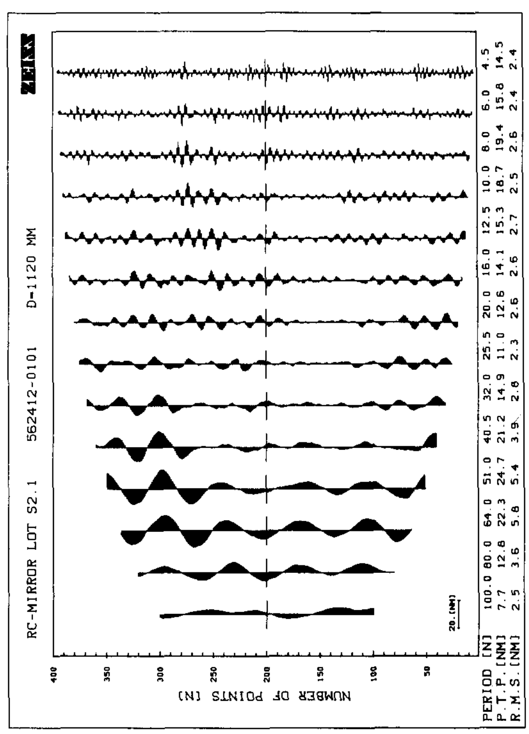

Fig. 16 1.1 m - RC-Secondary Mirror of Iraq, Ripple

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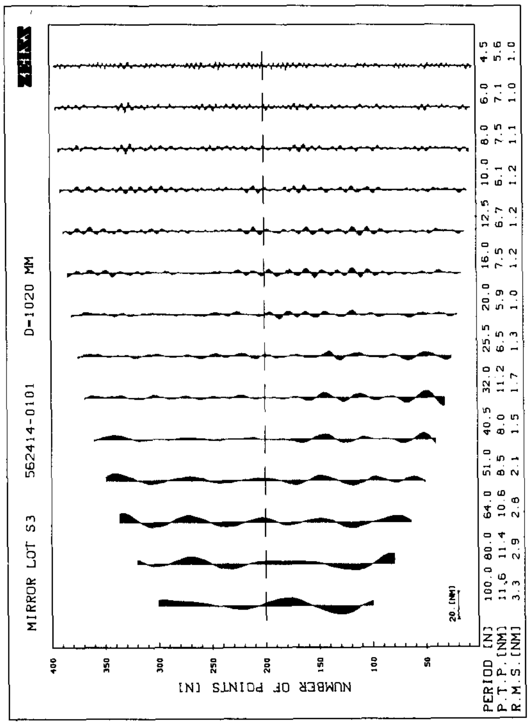

Fig. 17 0.7 * 1.1 m Coude Mirror, of Iraq, Ripple

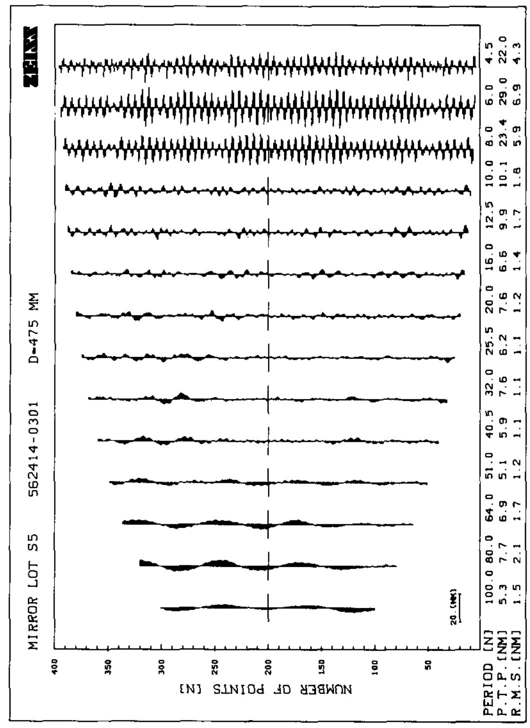

Fig. 18 0.5 m Coude Mirror, of Iraq, Ripple

Figs. 13 to 20 show the results of four examples. Figs. 13 and 14 show the 3.5 m primary mirror of the MPIA-telescope. Figs. 15 and 16 show the 1.2 m RC secondary mirror of the 3.5 m Iraq-telescope and Figs. 2 and 17 its 0.7 * 1.1 m Coude reflecting mirror.

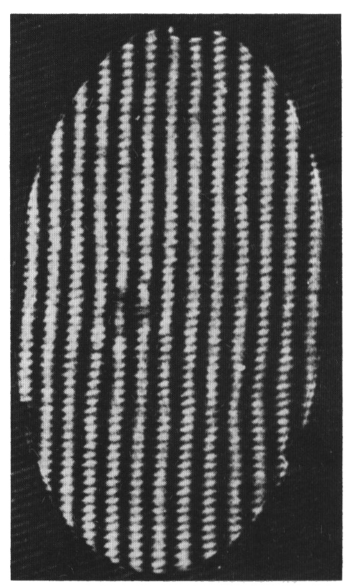

Fig. 19 0.5 m Coude-Mirror of Iraq Interferogram with Laser interferences

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Figs. 18 and 19 are particularly remarkable. They show a small plane mirror with a ripple structure which is not just an error of the mirror but is caused by interference from the laser light. This effect is reflected in the analysis by a marked rise in the amplitudes at the small period lengths. In this extreme case it is easy to establish a direct connection between the increased amplitudes and the laser interferences. However this is not always so straightforward. In other cases it remains difficult to distinguish between the mirror's own ripple and the laser interferences. The computer cannot make this distinction. Thus it will always require the eye of an experienced optical technician to make a definitive judgment on the quality of mirrors.

The method of evaluating ripple structure described here is relatively new. More research must be done for instance on the question of what p.t.p. or r.m.s. amplitudes are permissible and still allow the mirror to be ranked as good. We shall be trying to collect further data to enable us to make quantitative evaluations.

8. Acknowledgement

We are extremely grateful to Dr. R. N. Wilson of the ESO for his advice on our method of analysis and the valuable hints given us in the course of our discussions.

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