

SURFACE BRIGHTNESS RADII AND DISTANCES OF CEPHEIDS
AND THE PERIOD-RADIUS RELATIONSHIP

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INTRODUCTION

Recently Gieren (1984) has derived accurate radii and distances of a sample of short-period classical Cepheids using the surface brightness (SB) method introduced by Barnes & Evans (1976). The results indicated that the period-radius (P-R) relationship obtained from SB radii might possess a slope close to the value of 0.82 defined by the Cepheids in clusters and associations (Ferne, 1983) and in conflict with the values obtained from Baade-Wesselink radii and from theoretical models (see Ferne, 1983). Since this finding would lend considerable support to the presently accepted absolute magnitudes of the cluster Cepheids, it was decided to obtain SB radii and distances of well-observed long-period Cepheids in order to strengthen the P-R relationship obtained from the SB technique.

NEW SB RADII AND DISTANCES OF LONG-PERIOD CEPHEIDS

Coulson et al. (1984) have published extensive contemporaneous UB_V(RI) and photoelectric radial velocity data of the 6 Cepheids given in Table 1 which have been used for the present analysis. These stars span a period range from 10 to 17 days. Truncated Fourier series of appropriate order were fitted to the velocity curves which were then integrated to obtain the displacement curves, using a projection factor of 1.31 (Parsons 1972).

Table 1 Radii and Distances of Long-Period Cepheids

Cepheid	P(days)	R(R _☉)	d(pc)	<V-R> _{OJ}	s
AQ Car	9.77	64.1±4.0	2971±186	0.589	-0.378±0.008
XX Cen	10.95	55.4±3.9	1388± 96	0.589	-0.383±0.012
XY Car	12.44	80.0±5.9	2830±211	0.643	-0.387±0.012
TT Aql	13.75	91.3±5.2	1101± 63	0.685	-0.391±0.012
XX Car	15.71	92.0±3.4	3868±145	0.616	-0.371±0.012
XZ Car	16.65	108.7±6.3	2714±156	0.683	-0.406±0.015

The Johnson (V-R) color curves of the stars were obtained using the transformation equation

$$(V-R)_J = 0.587 (V-R)_c + 0.413 (V-I)_c + 0.03 \quad (1)$$

given by Cousins (1981) whose validity has been checked by Gieren (1984). The reddening-free $(V-R)_0$ colors of the Cepheids on the Johnson system were obtained adopting $E(V-R) = 0.9 E(B-V)$ and using the $E(B-V)$ values derived by Coulson et al. (1984). The unreddened apparent V magnitudes were calculated with the A_V values listed by the same authors. Surface brightness radii and distances of the stars were then calculated using the set of equations

$$F_V = 4.2207 - 0.1 V_0 - 0.5 \log \phi \quad (2)$$

$$F_V = 3.956 - 0.363 (V-R)_0 \quad (3)$$

$$D_0 + \Delta D = 10^{-3} r \phi \quad (4)$$

The symbols have their usual meaning (see Gieren 1984). Equation (3) is the most recent calibration of the surface brightness parameter in terms of $(V-R)_0$ given by Barnes (1980).

Table 1 contains the radii and distances and their standard deviations which were obtained from the new data.

THE PERIOD-RADIUS RELATIONSHIP

The new radii of Table 1 for the long-period Cepheids were combined with the SB radii of 13 stars given by Gieren (1984) to construct a period-radius relationship. The radius value of V496 Aql (Gieren, 1984) was omitted for reasons discussed in that paper. At the short-period end, the star EU Tau ($P = 2.1$) was added whose SB radius is $18.4 R_\odot$ (Gieren; in preparation). This leaves a total of 20 Cepheids in the period range of 2 to 17 days. A least squares fit to the data yields the period-radius relationship

$$\log R = 0.786 \log P + 1.040 \quad (5)$$

$$\pm 0.047 \quad \pm 0.039 \quad (\text{s.d.})$$

which is shown in Figure 1 (solid line). This relationship is not compatible with the ones obtained from Baade-Wesselink radii, theoretical models or mixed-mode pulsation, but agrees within the errors of the coefficients with the P-R relation obtained from the cluster and association Cepheids which is, according to Fernie (1983)

$$\log R = 0.824 \log P + 1.042 \quad (6)$$

$$\pm 0.020 \quad \pm 0.020$$

Turning to the distances of the present long-period Cepheids, it is found that they are almost exactly on the scale of the distances given in the catalog of Fernie & Hube (1968); the mean distance ratio is 1.01 ± 0.04 (s.d.). Adding the other 13 Cepheids discussed by Gieren (1984) (omitting again V496 Aql), the mean distance ratio (in the sense $d(\text{SB})/d(\text{FH})$) is 0.983 ± 0.023 (s.d.). In Figure 2, the distance ratio of the Cepheids is plotted against the pulsation period, and

Figure 1. P-R relationship obtained from surface brightness radii (solid line). Broken line is the relationship obtained from cluster Cepheids.

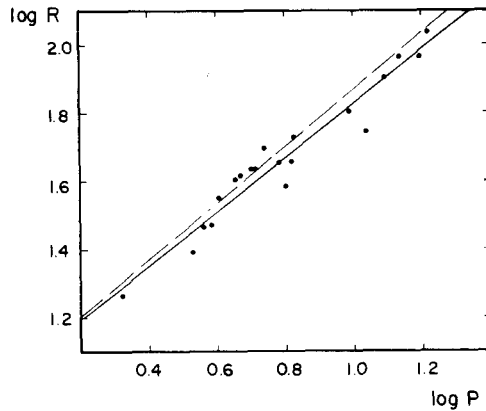
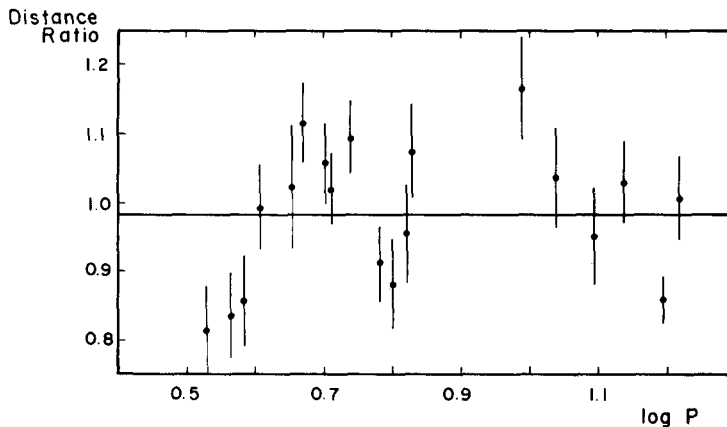


Figure 2. Distance ratio against period, for 19 Cepheids with accurate surface brightness distances.



clearly no period dependence is visible in these data. This means that for pulsation periods up to 17 days, the distances of Cepheids derived with the SB technique, and therefore their absolute magnitudes, are in almost perfect agreement with the scale of the period-luminosity relation for classical Cepheids. This finding increases confidence, on the other hand, in the correctness of the radii derived from the same method.

THE SLOPE OF THE SURFACE BRIGHTNESS - $(V-R)_J$ RELATION

Barnes (1980) has suggested that the slope of -0.363 of the $F_V - (V-R)_J$ relation (3) may not be applicable when dealing with the redder, longer-period Cepheids whose intrinsic $(V-R)_J$ colors exceed ~ 0.6 . Since several of the present stars have intrinsic $(V-R)_J$ colors close to 0.7 (see Table 1), it was desirable to derive their slopes in order to check if significant deviations from the value used in (3) do occur. For this purpose, the method devised by Thompson (1975) was used which permits to calculate the variations of the visual surface brightness S_V during the pulsation cycle from the known displacements r and a known mean radius R according to

$$S_V = V + 5 \log (1 + r/R) + \text{const} \quad (7)$$

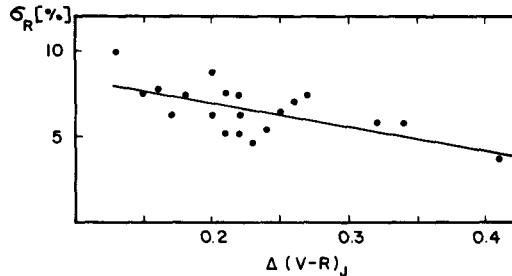
For each of the present long-period Cepheids, S_V was determined in this way as a function of phase and a least squares fit yielded the slope of the $S_V - (V-R)_J$ relation. An excellent linear relationship between S_V and $(V-R)_J$ was found to hold for each of the stars. From this, the slope s of the $F_V - (V-R)_J$ is obtained by multiplication with $-1/10$. The values obtained for s and their standard deviations are given in Table 1. The values range from -0.371 (XX Car) to -0.406 (XZ Car) and the average slope defined by the present 6 Cepheids is -0.386 ± 0.005 . This value may be compatible with the value of -0.363 ± 0.011 quoted by Barnes (1980) for the shorter period Cepheids, but the present data suggest that for the redder Cepheids the slope of the $F_V - (V-R)_J$ relation might become increasingly more negative. However, more long-period Cepheids need to be studied to confirm this trend.

CONCLUSIONS

The present period-radius relation from SB radii of classical Cepheids agrees within the errors with the one derived from 27 cluster Cepheids (Ferne 1983), but its slightly smaller slope could mean an alleviation of the discrepancy with theoretical radii. The present results imply that the radii derived from the cluster Cepheids, as well as from the SB technique are basically correct, implying that Baade-Wesselink radii are definitively too small for periods larger than 10^d . This is supported by the finding that SB distances, independently of period, agree very well with the scale of the P-L-C relation.

The present work confirms that the SB method is able to yield standard deviations of radii and distances in the order of 5%. Figure 3 shows that the σ decrease slightly with increasing amplitude of the Cepheid's $(V-R)$ color curve. This means that for the large-amplitude, long-period, distant Cepheids the most accurate distances can be obtained, a welcome result for galactic structure investigations.

Figure 3. Standard deviation of surface brightness radii against the (V-R) amplitude of Cepheids.



ACKNOWLEDGMENTS

My thanks are due to I.M. Coulson and J.A.R. Caldwell who obtained a large fraction of the observations used in this paper, to the directors of the South African Astronomical Observatory and of the European Southern Observatory for observing time at these institutions, and to Colciencias for research grants Nos. 20004-1-35-81 and 20004-1-39-82.

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