

WILKINSON, J. H., *The Algebraic Eigenvalue Problem* (Clarendon Press, Oxford, 1965), 662 pp., 110s.

The algebraic eigenvalue problem is the determination of those values of λ (eigenvalues) for which the set of n homogeneous linear equations in n unknowns $(A - \lambda I)x = 0$ has a non-trivial solution. Corresponding to any eigenvalue λ , the set of equations has at least one non-trivial solution x (eigenvector). This deceptively simple problem has been known for many years, but only in the last decade have accurate and practical methods been produced for its solution. These methods are necessarily used in conjunction with a high-speed computer and in the present volume the algorithms are presented in a form which facilitates translation into ALGOL and related computer languages. There is no person more experienced or more able to describe the modern techniques which now exist for solving this problem than Dr Wilkinson, and no praise is too high for the present volume.

The first chapter is devoted to giving a brief account of classical matrix theory, showing in particular, how the eigensystem of a matrix is related to its various canonical forms. This is followed by a chapter entitled Perturbation Theory which is concerned with the eigenvalues and associated eigenvectors of the matrix $A + \varepsilon B$ where $|a_{ij}| < 1$, $|b_{ij}| < 1$ and ε can be arbitrarily small. The perturbation theory is mainly based on Gerschgorin's theorems and employs the Jordan canonical form. Chapter three contains an error analysis for both fixed-point and floating-point computation of the basic arithmetic operations involved in the various techniques for solving the eigenvalue problem.

Chapter four is concerned with the important topic of solution of linear equations. Error analyses are given for the method of Gaussian elimination and for triangularisation methods including those of Cholesky, Householder and Givens. Iterative methods are not included. This is followed by a chapter dealing with Hermitian matrices where the eigenvalues are all real and the eigenproblem is comparatively simpler. The methods of Jacobi, Givens and Householder are described, the last two reducing the original matrix to tridiagonal form. Methods for computing eigenvalues of tridiagonal matrices are given.

In chapter six we return to the general matrix A , and consider the reduction of A to Hessenberg form using similarity transformations. The latter half of the chapter considers the further reduction to tridiagonal form. The methods described include the important method of Lanczos. The eigenvalues of matrices of condensed form are considered in Chapter seven, and iterative methods (including Laguerre's method) of finding them discussed.

Chapter eight is concerned with the LR and QR algorithms. The first of these, developed by Rutishauser, gives a reduction of a general matrix to triangular form by means of non-unitary transformations. The QR algorithm developed by Francis is based on unitary transformations. The final chapter is concerned with iterative methods which are primarily concerned with the determination of an eigenvector. This includes Aitken's acceleration technique.

The present volume will prove invaluable to engineers, physicists, and applied mathematicians; in fact to anyone who is faced with the problem of numerically determining an eigenvalue of a large system. In addition, the numerical analyst who wishes to do research in the matrix field will derive great benefit from a study of this work in conjunction with L. Fox's, *An Introduction to Numerical Linear Algebra* (Oxford, 1964), and A. S. Householder's, *The Theory of Matrices in Numerical Analysis* (Blaisdell, 1964).

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