any clear revelations.) The second graph, showing $f(x/y)$ against $\log(x/y)$, has a remarkably straight portion, though I can conceive of no reason why it should be so. It is worth mentioning that the proportion of squares did not seem to change significantly when a smooth hardboard surface was replaced by its reverse rough side. If anything, there was a slight tendency for the unusual events (square side up when x large) to occur, though this has not been worth subjecting to any statistical tests.

Clearly there is a related problem, that of the die which, though cubical, has its centre of mass not coincident with its geometrical centre: can a theoretical probability be calculated for each face when the position of the centre of mass is known?

It would be interesting to hear whether any reader can throw light on these problems.

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Correspondence

Surjections and all that

DEAR EDITOR,

There are two errors in Note 63.29 (December 1979). The definition of b_k is quite wrong, and the lemma is falsely stated.

Gerrish defines b_k as the number of functions $S \to T$ (where S is an m-set and T an n-set) whose ranges exclude at least k elements of T, and states that

$$b_k = \binom{n}{k}(n-k)^m.$$

This is in fact wrong, as we can easily see by considering b_1. Let y be one definite element of T. There are $(n-1)^m$ functions $S \to T$ whose ranges do not contain y. The author appears to have summed this over all $y \in T$ to get $b_1 = n(n-1)^m$; but in doing so he has miscounted. For example, those functions whose range is $T \setminus \{y', y''\}$ are counted twice, and so on.

Note that if the author had omitted the words "at least" from his definition of b_1, and if X_y were the number of functions whose range is exactly $T \setminus \{y\}$, then this number (X say) is the same for all $y \in T$, and nX is indeed the number of functions $S \to T$ whose ranges exclude exactly one element.

The number of functions $S \to T$ whose ranges exclude *at least one* element of T is $n^m - e_{mn}$, in the author's notation. Possibly a false analogy with this result is what misled him.

More care is also needed in the statement of the lemma which precedes the author's second proof. This should read:

If

$$F(n) = \sum_{j=0}^{n} \binom{n}{j} G(j) \tag{1}$$

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for $n = 1, 2, \ldots, H$, *then*

$$\mathrm{G}(n) = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \mathrm{F}(k) \tag{2}$$

for all these n.

But the truth of (1) for a single n by no means implies the truth of (2) for that n.

Yours sincerely,

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Chemical correspondence continued

DEAR DOUGLAS,

I was most interested to read about your ingenious use of splines to solve Dr Morris's problem in *Gazette* No. 426 (December 1979). An alternative method is as follows.

Let $\mathrm{N}(t)$ be the number of distributions with total t. Then $\mathrm{N}(t)$ is the coefficient of x^t in

$$(x^{-I} + x^{-I+1} + \ldots + x^I)^n.$$

(To see this, use Dr Morris's description of the problem on p. 234 and imagine multiplying out n brackets, each being $(x^{-I} + x^{-I+1} + \ldots + x^I)$.) Therefore $\mathrm{N}(t)$ is the coefficient of x^{t+nI} in

$$\begin{aligned}
&(1 + x + \ldots + x^{2I})^n \\
&= (1 - x^J)^n (1 - x)^{-n}, \quad \text{where } J = 2I + 1 \text{ as in your article,} \\
&= \left\{1 - \binom{n}{1} x^J + \binom{n}{2} x^{2J} - \ldots + (-1)^n \binom{n}{n} x^{nJ}\right\} \\
&\quad \times \left\{1 + \binom{n}{1} x + \binom{n+1}{2} x^2 + \binom{n+2}{3} x^3 + \ldots\right\}.
\end{aligned}$$

Let q be the quotient and r the remainder when $t + nI$ is divided by J, so that

$$t + nI = qJ + r, \quad \text{where } 0 \leqslant r < J.$$

Then

$$\mathrm{N}(t) = \binom{n + t + nI - 1}{t + nI} - \binom{n}{1}\binom{n + t + nI - J - 1}{t + nI - J} + \binom{n}{2}\binom{n + t + nI - 2J - 1}{t + nI - 2J} - \ldots + (-1)^q \binom{n}{q}\binom{n + r - 1}{r}.$$

It is straightforward to check that this agrees with your formula at the foot of p. 237. Note that both our methods use inequalities somewhere. They are explicit in your $(x)_+^{[n-1]}$ and in my definition of q and r.

Yours sincerely,

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