i.e., every point in the equatorial circle goes over into itself as the operation W obviously allows.

For the second case we have to put

$$V(z) = z \quad i.e., \ (1/z) = z$$

$$\therefore \qquad x^2 - y^2 + 2xyi = 1$$

$$\therefore \qquad xy = 0 \text{ and } x^2 - y^2 = 1$$

with the solutions

$$(1)\ x = 0 \qquad y = \pm i$$
$$(2)\ y = 0 \qquad x = \pm 1.$$

The 1° solution on application of W is easily shown to be false, while the 2° solution, corresponding to K and K′, satisfies the conditions.

Experimental Introduction to the Study of Magnetism.

By Professor C. G. Knott, D.Sc., F.R.S.E.

The aim of the scientific teacher is to teach the pupil how to think along scientific lines. By a suitable presentation of the facts of experience he should lead the mind of the learner to form almost intuitively the scientific law or generalisation which embraces them all. We may of course start with the law or formula, and develope it mathematically into all its ramifications. But that reduces itself to mere analytical skill. If carried out faithfully in the elementary teaching of science, it would tend to give the learner an erroneous conception of the whole method of scientific investigation and the meaning of scientific law. On the other hand, if we simply present a series of curious experiments without distinct intellectual linkage, we place the learner in the position somewhat of the old lady who, after reading through the dictionary, remarked that "it was vera interestin' readin', but a wee disconnecked."

In what follows I propose to indicate a course on magnetism which seems to lead naturally to a scientific grasp of the fundamental principles of the science. At the outset I may state briefly what seem to be the faults of the courses usually given in books, and presumably in schools and colleges.

First, from the very outset, a magnet is regarded as made up of two parts. Now there is a great advantage in certain problems of

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a more advanced character so to regard the magnet. But, to introduce the idea of *separate* poles at the very beginning, and to appeal to electrostatics for an analogy, is (unless the comparison be most carefully qualified) to suggest to the mind *separable* poles, and to raise up a needless barrier to the proper appreciation of an electromagnetic shell when the time comes—and it ought soon to come—for the discussion of iron-less magnets. A magnet is a whole, and should be treated as such.

Secondly, the conception of *poles* as *points* where magnetism is concentrated is made far too much of. Unless the statements are safeguarded by qualifications, inconsistency, if not confusion, is almost certain to appear sooner or later. For example, on page 4 of Mr Poyser's *Magnetism and Electricity*, a diagram is given showing the neutral line and magnetic axis of a bar magnet. The axis is defined as "the line joining the poles;" the poles are defined as "two points where the attractive power is greatest." Yet if we glance at the experimental figure on page 3, we shall see that the greatest "attractive power" is at the corners, which of course do not lie on the axis. The only true experimental definition of a pole is that *region*, not necessarily and not usually a *point*, towards which the lines of force converge. By using the word in this broad sense, we find no difficulty in at once appreciating the magnetic character of, say, a galvanometer coil, which certainly cannot for a moment be regarded as having point poles.

Thirdly, the important truth that the metal part of a bar magnet is not the whole magnet is not emphasised. The metal part is the nucleus; and the associated magnetic field is as much part of the magnet as the steel or iron. The conception of the magnetic field should be introduced as soon as possible.

Fourthly, the physical quantity by which we measure the strength or power of a given magnet is introduced as a product of two quantities, neither of which can be measured. Almost all books define magnetic moment as the product of the strength of the magnetic pole into the distance separating the poles; the books that do not so define it make no mention of it. And yet magnetic moment is *the* quantity with which we have to deal. It is the basis of our whole system of magnetic and electromagnetic measurements. It has long seemed to me that experiments should be so chosen as to bring out this property directly, and impress it upon the mind. What I now proceed to give is essentially a series of lecture notes, which requires amplification at the hands of the teacher. To make the course complete in itself, I must include many of the familiar experiments; but the manner in which these are arranged and discussed in connection with new, or at least unfamiliar, experiments is sufficiently original to merit, I think, some little attention. I assume the reader's ability to supply all the usual definitions of the terms used.

I. The orientation of a horizontal magnet freely suspended; in fact, the well-known property of the mariner's compass.

Mark the ends; always the same end which points north.

Do the same for a second magnet; and establish the usual laws of apparent attraction and repulsion between the dissimilarly and similarly marked ends.

This principle enables us rapidly to mark magnets.

II. Study positions of a magnet in which it does not disturb the lie of a suspended needle; both being in the same plane.

Determination of certain critical configurations, in which the needle seems to lose its power of pointing north. Why? Because the magnet's influence counteracts the original influence.

Conclusion: the earth is a magnet.

Introduce the conception of a magnetic Field or region where a free magnet points in a definite direction. The behaviour of a magnet in a magnetic field demonstrates the existence of both magnet and field. Every magnet has a field associated with it.

III. Induction the fundamental action of the magnetic field.

Usual experiments with nails, iron filings, etc. The iron first magnetised, and then acted upon as if it were a magnet. Study of the configuration of a field near a magnet by the help of iron filings.

IV. Experimental tracing of lines of force by step by step motion of a small magnet. The combined field of force due to a magnet and the earth's horizontal field studied very easily by this means (see Glazebrook's *Practical Physics*).

To study the lines of force due to a magnet only, suspend the small exploring magnet over a loose sheet of paper, and render it astatic by a large magnet suitably adjusted. Lay the magnet whose field is to be studied on a marked position on the paper. Then by moving the sheet with magnet attached under the suspended magnet we may point by point trace out a line of force. For any new position the sheet must be adjusted, so that the one end of the suspended magnet lies exactly over the point which, in the immediately preceding position, lay under the other end.

The above might not be suitable for showing in a lecture. In this case it will suffice to show, by study of the behaviour of a suspended needle in the presence of any magnet, what are the directions of the field in the axial line (end-on), and in what might be called the equatorial plane (broadside-on) of the magnet.

V. Quantitative properties of magnetic fields associated with magnets.

Many modes of attack.

The following seems to have special merits :—

Suspend a needle in the earth's field, and note its position.

Set a magnet end on to east, and adjust till the needle is deflected through 45°. We see at once that the needle is under the action of two equal fields (the earth's and the magnet's); for they are at right angles to each other, and make equal angles with the needle on opposite sides of it. Measure the distance between the middle points of magnet and needle; take number of vibrations of the needle, say in five minutes.

Invert the magnet and adjust again till the deflection is 45°. Measure the distance as before; and take vibrations.

Place the magnet to the west, and make exactly the same adjustments for the two end-on positions of it; each time taking vibrations.

Take the mean of the four distances so obtained; and at this distance set the magnet on the north and south line, first to the north and then to the south. Take the vibrations in all positions; for only two will this be possible. For the other two the needle is nearly astatic.

Take the magnet right away; and count the vibrations of the needle when under the earth's influence only.

Let n_0 = number of vibrations per minute when earth's field (H) acts alone.

n_1 = mean of numbers of vibrations in the four east and west positions of the magnet.

n_2 = mean of numbers of vibrations in the two possible north and south positions, in which the field is obviously 2H.

Then it will be found that

$$n_0^2 : n_1^2 : n_2^2 = 1 : \sqrt{2} : 2.$$

May use two magnets, and get n_3^2 proportional to 3H.
Thus we find

1. Mutually perpendicular equal fields are compounded like forces.

Easy to extend this for all fields by showing that two magnets, each giving 45° when acting singly, will give when acting together (one to the east and one to the west) a deflection of 63½°, whose tangent is 2.

2. When a given magnet vibrates in a field, the square of the number of vibrations per second varies as the strength of the field.

This vibration law is of supreme importance. As in the case of the pendulum, the acting couple is proportional to the square of the number of vibrations; and thus we conclude that the couple acting on a magnet is proportional to the field, other things being the same.

VI. Comparison of Magnetic Moments. The simplest way to define the magnetic moment of a magnet is to say that it is proportional to the couple acting on it when it is set east and west in the earth's horizontal field.

We could then express all magnetic moments in terms of the magnetic moment of a chosen standard magnet.

The couples may be measured statically. More accurately, however, by small oscillations.

The moment of inertia of the magnets may be eliminated thus. Take three magnets of moments m_1 m_2 m_3, all unknown. Suspend all by a light frame at the end of a long thin wire. Let τ be the rotational moment brought into play in virtue of the torsion of the wire. Have all three magnets pointing similarly, and take vibrations. Invert each in turn, and take vibrations for each configuration. Then if n_0 n_1 n_2 n_3 are the vibration numbers, we get

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$$m_1 + m_2 + m_3 + \tau \propto n_0^2$$
$$-m_1 + m_2 + m_3 + \tau \propto n_1^2$$
$$+m_1 - m_2 + m_3 + \tau \propto n_2^2$$
$$+m_1 + m_2 - m_3 + \tau \propto n_3^2.$$

Hence

$$m_1 : m_2 : m_3 : \tau$$
$$= n_0^2 - n_1^2 : n_0^2 - n_2^2 : n_0^2 - n_3^2 : (n_1^2 + n_2^2 + n_3^2 - n_0^2)/2$$

In this way the significance of magnetic moment is made obvious.

I have made the experiment, and found it very satisfactory.

VII. By the familiar Gaussian experiments, we readily establish that in the axial and equatorial directions from a given magnet

$$\text{Field} \propto 1/(\text{Distance})^3$$

if the distance is not too small.

Also that the equatorial field is half the axial field at the same distance from the magnet's centre.

Also that at equal distances from different magnets, the fields are proportional to the magnetic moments as given in VI. Hence we may put

$$\text{Axial Field} = 2\text{K}/r^3$$
$$\text{Equatorial ,,} = \text{K}/r^3$$

where K is the magnetic moment measured in chosen units.

Note.—This law is exact for spherical magnets; and might be established by using electromagnets with spherical or short cylindrical iron cones.

It is best to use the earth's field as provisional unit.

VIII. Measurement of H and K in absolute units.

The couple vanishes when the magnet points north or south in the earth's field, and is at a maximum when it points east and west.

This maximum couple is proportional to the field and to the magnetic moment.

Define it as being measured by the product of the two—say KH.

We may suppose this measured statically in absolute units.

Thus HK = C a known quantity.

By deflection experiments, end-on, at distance r

$$\frac{2K}{r^3}/H = \tan\theta$$

or $$K/H = \tfrac{1}{2}r^3\tan\theta, \text{ a known quantity.}$$

Hence
$$\left.\begin{aligned} K^2 &= \tfrac{1}{2}Cr^3\tan\theta \\ H^2 &= \frac{2C}{r^3\tan\theta} \end{aligned}\right\}$$

and H and K are both found.

IX. Potential Energy of Magnet in Field.

For any deflection θ, it may be shown, by suitable combinations as in Section V., that the couple acting on the magnet K in field H is $HK\sin\theta$.

Hence work done in moving the magnet from θ to θ' is

$$\int_{\theta}^{\theta'} HK\sin\theta d\theta = -HK(\cos\theta' - \cos\theta)$$
$$= V' - V$$

where V′ and V are the potential energies in the positions θ' and θ. We may write

$$V = \text{constant} - HK\cos\theta.$$

Now in the earth's field there is no tendency to translation, so that the potential energy depends only on orientation.

This holds for all magnets in uniform fields ; and may be assumed to hold for the magnetic molecule in any field.

Putting then the constant = 0, we get

$$V = -HK\cos\theta.$$

This makes the potential energy vanish when the magnet is at right angles to the lines of force.

X. Mutual action of two small Magnets.

From the fact that magnetic fields are superposable, we infer that a small magnet may be decomposed into any three elements, just as a force is decomposed.

Let K, K′, be the magnetic moments of two magnets making

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direction cosines $\lambda\ \mu\ \nu$, $\lambda'\ \mu'\ \nu'$, with regard to any chosen system of axes, of which one (the x-axis) is the line joining their centres.

Let r be the distance between these centres.

Decompose K into $\quad$ $K\lambda$, $K\mu$, $K\nu$,
and K' „ $\quad$ $K'\lambda'$, $K'\mu'$, $K'\nu'$.

By application of IX. we readily find the potential energies of the different pairs of components. They are shown tabulated in the following scheme :—

TABLE OF POTENTIAL ENERGIES BETWEEN VARIOUS COMPONENTS.

	$K'\lambda'$	$K'\mu'$	$K'\mu'$
$K\lambda$	$-\frac{2KK'}{r^3}\lambda\lambda'$	0	0
$K\mu$	0	$+\frac{KK'}{r^3}\mu\mu'$	0
$K\nu$	0	0	$+\frac{KK'}{r^3}\nu\nu'$

Hence the potential energy of the two magnets is

$$W=\frac{KK'}{r^3}\{(\lambda\lambda'+\mu\mu'+\nu\nu')-3\lambda\lambda'\}$$

Or if θ, θ', are the angles made by K, K', with the line joining them, and ϕ is the angle between K, K',

$$W=\frac{KK'}{r^3}(\cos\phi-3\cos\theta\cos\theta').$$

If they lie in one plane

$$\phi=\theta'-\theta$$

and
$$W=\frac{KK'}{r^3}(\sin\theta\sin\theta'-2\cos\theta\cos\theta').$$

Apply this to study the field due to a given magnet; in other words, to find the direction and magnitude of the force at a point

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whose co-ordinates are r and θ referred to the centre and axis of the magnet.

Let K be the magnet of reference.

Let K′ be another small magnet placed at the point $(r\theta)$, and let K′ be placed perpendicular to the line of force so that W vanishes. This condition gives

$$\tan\theta'\tan\theta = 2.$$

Hence if β be the angle between r and the line of force

$$\tan\theta' = \cot\beta,$$

and hence $$\tan\theta = 2\tan\beta.$$

To find H the value of the force, let K′ lie along r; then $\theta' = 0$ and $$W = -K'2K\cos\theta/r^3.$$

But this may also be written

$$W = -K'H\cos\beta$$

$$= K'H/\sqrt{(1+\tfrac{1}{4}\tan^2\theta)}.$$

Hence $$H = \frac{K}{r^3}\sqrt{(3\cos^2\theta+1)}.$$

XI. Case of circular coil.

Following out the same method, let us consider the field due to a circular coil at any point in its axis, assuming Ampère's rule that such a circuit is equivalent to any coterminous magnetic shell, magnetised normally with strength equal to the current.

Take as shell the cone with vertex at P; and place at P the magnet K pointing along the axis of the coil.

The element at S is equivalent to a small magnet of moment ωi (ω the small area) and with axis $\perp^r$ to PS.

Hence the mutual potential energy of each element and K is

$$w = \frac{i\omega K}{PS^3}(\sin 90^\circ \sin\alpha - 2\cos 90^\circ \cos\alpha)$$

$$= \frac{i\omega K}{PS^3}\sin\alpha$$

where α is the semi-vertical angle at P.

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Hence for the whole circular strip through S

$$dW = 2\pi K i r \sin^2 a dr / r^3$$
$$= 2\pi K i \sin^2 a dr / r^2$$

Integrating from $r = R$ to $r = \infty$, we get for the potential energy of the circuit and the magnet

$$W = -\frac{2i\pi K \sin^2 a}{R} = -K\frac{2\pi i a^2}{R^3}$$

where a is the radius of the circle.

Hence the field at the point P is

$$i\frac{2\pi a^2}{R^3}.$$

At the centre this becomes

$$i\frac{2\pi}{a}.$$

In this last example, it is assumed that the equivalence of circuits and magnets has been established experimentally. For this purpose the usual experiments are amply sufficient.

The experimental treatment of the subject of magnetic induction has been greatly improved in these later days, thanks chiefly to such men as Ewing and Hopkinson, following up along the lines of Faraday and Maxwell.

Sixth Meeting, April 8, 1892.

Professor J. E. A. Steggall, M.A., President, in the Chair.

On a surface of the third order.

By R. E. Allardice, M.A.

In the second part of Professor Chrystal's *Algebra* (Exer. V., No. 4) the following exercise is given :—

If $2xyz - x^2 - y^2 - z^2 + 1 = 0$, and x, y, z are all real, then all, or none, of the quantities x, y, z lie between -1 and $+1$.

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