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Introduction

Despite the fact that what Kant has to say about mathematics is scattered throughout many texts, and is sometimes very cryptic, Kant's philosophy of mathematics has long been a fertile research area. It has emerged from work in this area that the development of Kant's Critical philosophy, his metaphysical and epistemological doctrines, and his conception of the method and systematicity of philosophy were clearly informed by his serious reflection on mathematics, logic and the exact sciences. Indeed, Kant uses mathematical truths – most famously that $7 + 5 = 12$ – as his primary examples of synthetic *a priori* judgments, the question of the possibility of which frames his entire project in the *Critique of Pure Reason*. Accordingly, the task of explaining Kant's philosophy of mathematics has largely centred on understanding his conception of mathematics as a domain of demonstrable synthetic *a priori* truths. More recently, however, Kant scholars have begun to consider the broader implications of this work for the Critical philosophy itself by examining the role of Kant's reflections on mathematics in his philosophy more generally.

While Kant's explanation of the synthetic apriority of mathematical truth hinges on his theory of mathematical concept construction, these potentially narrow observations about mathematics are deeply connected to many more far-reaching philosophical commitments. Ultimately, then, Kant's account of the philosophy of mathematics must be understood relative to a variety of aspects of the Critical philosophy. Accordingly, classic debates about the role of Kantian intuition in mathematical reasoning will be augmented by research into Kant's understanding of mathematical practice and the applicability of mathematics in scientific understanding and perceptual experience; the general definability of mathematical concepts; the relation between mathematical thought and our concepts of space, time, magnitude and purposiveness; and the specific role that mathematics plays in motivating Kant's unique account of our representational capacities and his defense of the transcendental ideality of the objects of experience. We hope that these papers will show how Kant's appeal to mathematics as a model of synthetic *a priori* cognition sheds light on our cognitive faculties in general.

Thomas Land's contribution to the volume explores precisely this question. In his paper, "Spatial Representation, Magnitude and Two Stems of Cognition", Land uses Kant's philosophy of mathematics to shed light on the distinction between sensibility and understanding, and in particular on how the two faculties cooperate to produce sensible representations. Land shows how Kant's account of mathematical construction can serve as a model for understanding the notion of the productive synthesis of imagination, which is supposed to involve both a sensible and a spontaneous aspect, and which underlies the empirical apprehension of appearances. More specifically, he argues that the representation of determinate spaces in intuition depends on an empirical version of the kind of pure synthesis involved in geometrical construction. This gives precise sense to the idea that both the concepts of mathematics and the categories are concepts of the form of appearances. Land's result has significant implications for the debate over conceptualist vs. non-conceptualist readings of Kant: it suggests that intuitions, although distinctively sensible, are also dependent on spontaneity, and thus on concepts – in particular, the concept of magnitude, which is the subject matter of mathematics.

Like Land, Daniel Smyth shows how mathematical considerations – in this case, about infinity – underlie Kant's distinction between the faculties of sensibility and understanding, and therefore also underlie the related distinction between intuition and concept. Against the common view that Kant simply presupposes his distinction among kinds of representations, Smyth contends in his "Infinity and Givenness" that the Metaphysical Exposition of the Concept of Space contains the first stage of an argument in support of Kant's distinction between conceptual and intuitive representation. According to Smyth, the argument begins with a functional conception of sensibility as object-giving, and turns on the infinitary structure of space – its continuity and open-endedness – which cannot be accounted for by the understanding. While the connection between intuition and infinity is familiar, Smyth turns the standard view on its head, claiming that the singularity of intuition is a *consequence* of the argument of the Exposition, rather than a presupposition: the Exposition fills out the minimal functional conception of intuition with which Kant begins the first *Critique*. In this way, Smyth argues that mathematical considerations underlie the fundamental distinction between the faculties of sensibility and understanding that Kant takes to be one of his most significant innovations.

An examination of mathematical concepts and definitions, and, in particular, of the precise role that such representations play in justifying mathematical judgments, is crucial for understanding Kant's philosophy of mathematics. The papers by Callanan and Heis provide careful analyses of these fundamental notions. Even in his pre-Critical work, Kant recognized that the method of mathematics gave it, its concepts, and its definitions, a special status. In his paper "Kant on the Acquisition of Geometrical Concepts", John Callanan argues for an as yet underappreciated distinction between mathematical and other knowledge claims. Kant importantly distinguishes between the acquisition conditions of concepts and the justification conditions for the use of those concepts. Callanan argues, though, that geometrical construction is both a form of concept acquisition *and* is sufficient to justify the use of those concepts. This peculiarity of mathematical judgments, according to Callanan, explains Kant's claim that mathematical judgments are 'combined with consciousness of their necessity'. In this way, the paper points to a new way of accounting for the modal phenomenology of mathematical judgments.

In his contribution to the volume, "Kant (vs. Leibniz, Wolff and Lambert) on Definitions in Geometry," Jeremy Heis spells out Kant's theory of real definitions by contrasting it to those of three of his predecessors. While Leibniz, Wolff, Lambert and Kant all share the view that geometrical definitions are real definitions, they nevertheless disagree about what constitutes a real definition, and so about what accounts for the status and reality of geometrical definitions of, for example, circle and parallel lines. Heis argues that Kant rejects definitions that Leibniz, Wolff and Lambert each accept because of the uniquely stringent requirements his philosophy of geometry imposes on real definitions. In particular, Heis shows that Kant's conception of mathematics as rational cognition from the construction of concepts, and his claim that concepts (including mathematical concepts) "rest on functions", explains his disagreement with his predecessors over real definitions.

The considerations adduced by Heis are standardly thought to show that for Kant, *only* mathematical concepts admit of real definition. But by means of a detailed analysis of Kant's treatment of definitions, Tyke Nunez challenges this widely accepted view to show that some philosophical concepts do, in fact, admit of definition. In his "Definitions of Kant's Categories," Nunez argues that for Kant, the schemata of the categories ensure the reality of their definitions. Because these definitions do not provide a rule for the construction of their objects, they will not be genetic like those of the concepts of mathematics. Nonetheless, in both cases, the reality of the definitions rests on the claim that they are concepts of the form of objects of possible experience. In this respect, mathematics and philosophy turn out to differ less than we might have expected.

The broad themes of construction and definition are brought together in Katherine Dunlop's paper "Arbitrary Combination and the Use of Signs in Mathematics". She uncovers a role for the perceptibility of mathematical symbols in Kant's Prize Essay, and compares it to the role that sensibility plays for Wolff and Leibniz, revealing a continuity between the views of Kant's predecessors and his own pre-Critical views on the role of signs in mathematics. Dunlop argues that this role for sensibility secures the objective reference of mathematical concepts, but it cannot address the question of their universal applicability. That problem is addressed in the first *Critique*, not by appeal to the constructibility of the concepts, as recent commentators have argued, but by the claim that pure intuition is the form of empirical intuition. Again, as in the case of the papers by Land and Nunez, we see how mathematics is implicated in Kant's doctrine of the form of appearances.

Daniel Sutherland's paper reveals the significance of the notion of mathematical construction beyond Kant's philosophy of mathematics by focusing on its centrality to Kant's *Metaphysical Foundations of Natural Science*. He shows how Kant's account of the possibility of mathematical cognition and his explanation of the applicability of mathematics in the first *Critique* provides a grounding for and shapes Kant's account of the possibility of

Newtonian mathematical physics in the *Metaphysical Foundations*. Because mathematical cognition rests on construction, the mathematizability of physical concepts – above all, motion – depends on their constructibility. He explains how motion and composition of motion are constructed in the Phoronomy in a way that accommodates eighteenth-century treatments of instantaneous velocity without running counter to Kant's claims that motion is an intensive magnitude and is apprehended in an instant. At the same time, Kant's argument in the Phoronomy reveals features of construction that are also important for our understanding of the cognition underlying *pure* mathematics.

In her "Kant on Conic Sections," Alison Laywine illustrates the general theme of this volume by arguing that the four passages where Kant discusses conic sections directly "mirror and illuminate Kant's evolving views on central aspects of his philosophy" from 1763 to 1790. Like Land and Smyth, she takes up the question of the relations among the faculties, in this case, of reason, imagination and judgment, and investigates them in the context of her treatment of Kant's discussion of conic sections. Kant cites Apollonius' theory of conic sections as exhibiting a high degree of systematic unity. We might naturally think that this kind of unity is best brought about by an algebraic treatment of conic sections. But this gives rise to a puzzle: the texts in which Kant discusses Apollonius' conic sections reveal his commitment to *geometrical* construction in the theory of conic sections. How does Kant see geometrical construction as yielding systematic unity in any way comparable to algebraic techniques? What seems to be required here is something analogous to the role the Schematism plays in explaining how we arrive at universal conclusions from particular images, but in this case at the level of reason rather than understanding. Laywine suggests that perhaps the notion of reflective judgment from the third *Critique* could fulfill this function. As she points out, it should then come as no surprise that the section on the critique of teleological judgment begins with a discussion of mathematical purposiveness.

This topic of purposiveness is taken up by Courtney Fugate in his paper "'With a Philosophical Eye': The Role of Mathematical Beauty in Kant's Intellectual Development", again while attending to the task of illuminating the relations between the faculties. Fugate traces the historical and metaphysical roots of Kant's notion of mathematical purposiveness with the aim of showing, like Laywine, that it has greater significance for Kant than just serving to illustrate the different types of purposiveness. That significance lies in the role of reflections on mathematical purposiveness in Kant's discovery of the synthetic *a priori*, and, in particular, in what Fugate calls an inexplicable "purposiveness between our own *a priori* faculties of intuition and understanding".

It should be clear from the foregoing that Kant was struck by the special nature of mathematics: its certainty, its evidence, the nature of mathematical proof and the remarkable fit between mathematics and the natural world, all of which informed his more general philosophical views. The papers in the current volume examine these issues in new ways. It is our hope that these papers inspire still more work on the broad implications of Kant's philosophy of mathematics.

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