Dynamical Evolution of Dense Stellar Systems
Proceedings IAU Symposium No. 246, 2007
E. Vesperini, M. Giersz & A. Sills, eds.
© 2008 International Astronomical Union
doi:10.1017/S1743921308015688

Evolution of Compact Binary Populations in Globular Clusters: a Boltzmann Study

Sambaran Banerjee and Pranab Ghosh

Tata Institute of Fundamental Research, Mumbai 400005, India
email:sambaran@tifr.res.in, pranab@tifr.res.in

Abstract. We explore a Boltzmann scheme for studying the evolution of compact binary populations in globular clusters. We include processes of compact binary formation by tidal capture and exchange encounters, binary destruction by exchange and dissociation mechanisms and binary hardening by encounters, gravitational radiation and magnetic braking, as also the orbital evolution during mass transfer, following Roche lobe contact. From the evolution of compact-binary population, we investigate the dependence of the model number of X-ray binaries N_{XB} on two essential cluster properties, namely, the star-star and star-binary encounter-rate parameters Γ and γ (Verbunt parameters). We find that the values of N_{XB} and their expected scaling with the Verbunt parameters are in good agreement with results from recent X-ray observations of Galactic globular clusters.

Keywords. stellar dynamics, scattering, binaries: close, X-rays: binaries, globular clusters: general

1. Introduction

In this era of high-resolution X-ray observations with *Chandra* and *XMM-Newton*, studies of compact binaries in globular clusters (henceforth GC) have reached an unprecedented level of richness and detail, so that such observational studies can be compared with results obtained from theoretical modeling of binary dynamics in globular clusters (see Hut *et al.* (1992) for a review). In this study, we introduce a method for studying the evolution of compact binaries in GCs wherein we use a Boltzmann equation to trace the time evolution of such populations. We emphasize that the formalism we describe is not a Fokker-Planck description but the original Boltzmann one, which in principle is capable of handling both the *combined* small effects of a large number of frequent, weak, distant encounters *and* the *individual* large effects of a small number of rare, strong, close encounters.

The dynamical properties of a GC core with mean density ρ, velocity dispersion v_c and core radius r_c can be described by the two quantities $\Gamma \equiv (\rho^2/v_c)r_c^3$ and $\gamma \equiv \rho/v_c$ as pointed out by Verbunt (2002), which we shall refer to as *Verbunt parameters* hereafter. Γ is a measure the total two-body encounter rate within a GC core and γ measures the rate of encounter of a *single* binary with the surrounding stars (Verbunt 2002). If the GC core is assumed to be virialized ($v_c \propto \rho^{1/2} r_c$), the specification of these two quantities uniquely determines ρ, r_c and v_c.

A dynamically formed compact binary between a non-degenerate star and a compact star may in general be detached and becomes an X-ray binary (henceforth XB) after the non-degenerate companion fills its Roche-lobe through evolution of the binary. Evolution of such *pre X-ray binaries* (henceforth PXB) is governed by orbital angular momentum loss and stellar evolution of the companion. In this study, we focus on the evolution of compact binary population in both the XB and PXB phase using our Boltzmann scheme,

https://doi.org/10.1017/S1743921308015688 Published online by Cambridge University Press

with particular attention to (a) period distribution of XBs and (b) number of XBs, which we relate to observations.

2. Model of compact binary population evolution in globular clusters

We consider a binary population described by a number distribution $n(a, t)$, where a is the binary separation, interacting with a *unevolving uniform* background of stars representing the core of a globular cluster consisting equal mass stars of $m + f = 0.6M_{\odot}$, a fraction k_X of compact stars of $m_X = 1.4M_{\odot}$ and a fraction k_b of primordial binary fraction. $n(a, t)$ is defined such that $n(a, t)da$ is the total number of compact binaries in the core within the radius interval a to $a + da$.

2.1. *A Boltzmann evolutionary scheme*

We explore a Boltzmann evolutionary scheme, wherein the evolution of $n(a, t)$ is described by the *collisional Boltzmann equation* (Spitzer 1987):

$$\frac{\partial n}{\partial t} = R(a) - nD(a) - \frac{\partial n}{\partial a} f(a), \tag{2.1}$$

where $R(a)$ is the total formation rate with the GC core, per unit a, of compact binaries with radius a, $D(a)$ is the destruction rate *per binary* of compact binaries of radius a and $f(a) \equiv da/dt$ is the total orbital evolution rate of the compact binaries (see Sec. 2.4). Eqn. (2.1) is the governing evolution equation a of compact binary population in an unevolving GC core (see Banerjee & Ghosh (2007) for derivation).

2.2. *Compact binary formation processes*

A compact binary can be formed by (a) tidal capture (tc) and (b) exchange encounter (ex1) as discussed below. If $r_{tc}(a)$ and $r_{ex1}(a)$ represents the rates of these processes respectively, then

$$R(a) = r_{tc}(a) + r_{ex1}(a), \tag{2.2}$$

where a is the radius of the compact binary so formed.

In tidal capture formation, a compact star, during its close passage by an ordinary star, loses its kinetic energy by raising non-radial oscillations on the later by its tidal force so that they become bound, provided the first periastron separation r_p is shorter than a critical value (Fabian *et al.* 1975). After getting bound, the binary is usually highly eccentric, and circularizes within several periastron passages to the radius $a \approx 2r_p$. We consider a simplified analytical approach involving the *impulsive approximation* (Spitzer 1987) which assumes that all the energy is deposited on the stellar surface instantly during the first periastron passage. It can be shown that (Banerjee & Ghosh 2007), for a Maxwellian velocity distribution, the rate function is nearly uniform in a for small a and falls off fairly sharply from about $a \approx 7R_{\odot}$, as shown in Fig. 1 (left panel).

Compact binaries can also be formed by exchange encounter between a compact star and a primordial non-compact stellar binary. During a close encounter between the compact star and the stellar binary, the compact star being generally heavier, preferentially replaces one of the binary members to form a PXB. We use the well-known Heggie, Hut & McMillan (1996) exchange cross section to estimate the (Maxwellian averaged) ex1 exchange rate as a function of binary radius a. For primordial binaries, we take the widely-used radius distribution $f_b(a) \propto 1/a$ (*i.e.*, a uniform distribution in $\ln a$). In this case, the ex1 rate will be constant with a (see Banerjee & Ghosh (2007) for details) as shown in Fig. 1 (left panel).

https://doi.org/10.1017/S1743921308015688 Published online by Cambridge University Press

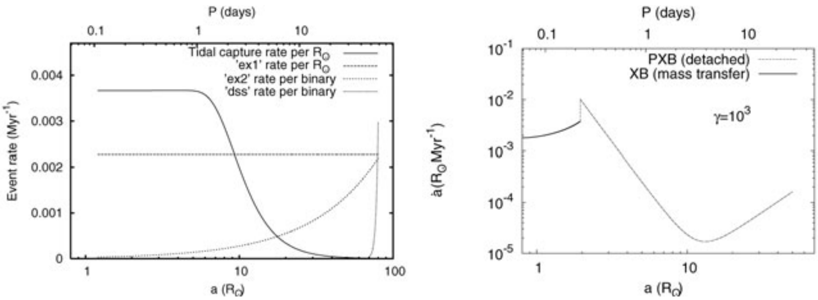

Figure 1. *Left:* A comparison of the dynamical rates. 'ex1' and 'ex2' rates have been multiplied by a factor of 50 and 60 respectively to make them visible in the same plot, while 'dss' rate have to be multiplied by a factor of $\sim 10^9$. *Right:* Hardening rate $\dot{a}$ of a compact binary as a function of the orbital radius a. For $a < 2R_\odot$ (solid line), mass transfer occurs representing the XB phase. Along abscissa, both orbital radius a and orbital period P scales are shown.

2.3. *Compact binary destruction processes*

A compact binary can be destroyed primarily by two processes, *viz.*(a) exchange encounter (ex2) and (b) dissociation (dss). Accordingly, the total destruction rate is:

$$D(a) = r_{ex2}(a) + r_{dss}(a) \tag{2.3}$$

In an exchange encounter (ex2) of a PXB with a compact star, the latter can replace the low-mass companion of the binary, forming a double compact-star binary. This, in effect, destroys the binary as an X-ray source as accretion is not possible in such a system, and it is essentially impossible for one of the compact stars in such a system to be exchanged again with an ordinary star in a subsequent exchange encounter, since $m_f = 0.6M_\odot$ is much lighter than $m_X = 1.4M_\odot$. As before, we estimate the Maxwellian averaged ex2 rate using the exchange cross section formula of Heggie, Hut & McMillan (1996), which is proportional to a as demonstrated in Fig. 1 (left panel).

As the compact binaries in the GC core are hard, they can only be dissociated by the small number of stars that constitute the high-velocity tail of the Maxwellian distribution. Thus dissociation constitutes a negligible channel for compact binary destruction (see Banerjee & Ghosh (2007) and references therein for details).

2.4. *Compact binary hardening processes*

As explained in detail in Banerjee & Ghosh (2006) (henceforth B06), the processes that harden binaries are of two types, *viz.*, (a) those which operate in isolated binaries, and are therefore always operational, and (b) those which operate only when the binary is inside a globular cluster. In the former category are the orbital angular momentum loss by *gravitational radiation* and *magnetic braking*, and in the latter category is that of *collisional hardening*. Collisional hardening refers to the process of preferential hardening due to repeated encounter by the background stars, according to Heggie's law (Heggie 1975). As discussed in detail in BG06, collisional hardening, which is proportional to a, dominates at larger orbital radii, while gravitational radiation and magnetic braking, which increase steeply with decreasing a, dominate at smaller orbital radii. It is these processes that harden a compact binary from its PXB phase, up to the point of Roche lobe contact ($a_L \approx 2R_\odot$), whence it turns on as an X-ray binary (XB) – either a CV or a LMXB, depending on the nature of the degenerate accretor. A typical form of hardening rate as a function of a is shown in Fig. 1 (right panel). At the minimum at $a \sim 14R_\odot$, gravitational radiation hardening ($\dot{J}_{\rm orb}/J_{\rm orb} \sim a^{-4}$) takes over from collisional hardening. Magnetic braking, having a steeper dependence on a (Verbunt-Zwaan scaling,

https://doi.org/10.1017/S1743921308015688 Published online by Cambridge University Press

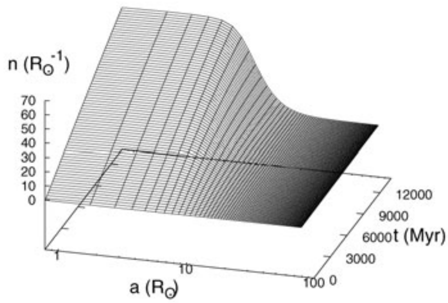

Figure 2. Three-dimensional surface $n(a,t)$ describing the model evolution of population-distribution function of compact binaries for GC parameters $\rho = 6.4 \times 10^4 \ M_\odot \ \mathrm{pc}^{-3}$, $r_c = 0.5$ pc, $v_c = 11.6$ km sec^{-1} (roughly corresponding to 47 Tuc).

$\dot{J}_{\rm orb}/J_{\rm orb} \sim a^{-5}$), dominates at still shorter binary radius, during the mass transfer, which is shown with the thick line in Fig. 1 (right panel). It is important to note that during mass-transfer, the hardening rate remains nearly constant with a.

3. Results

A typical result from our computed evolution of the compact-binary distribution function $n(a,t)$ is shown in Fig. 2. The distribution function is seen to evolve such that the compact binary population grows predominantly, with a nearly uniform distribution function at shorter radii ($a < 10R_\odot$, say), and a sharp falloff longward.

The overall shape of the distribution function results from (a) the predominance of tidal capture formation rate for shorter binary radii (see Fig. 1), (b) inflow of binaries shortwards due to hardening and (c) higher ex2 destruction rate at larger radii. The uniformity in the distribution function for about $a < 10R_\odot$ mainly results from that in the tidal capture rate (Fig. 1).

The total number of X-ray binaries N_{XB} in a GC at any given time can be computed directly by integrating $n(a,t)$ over the range $a_{pm} \leqslant a \leqslant a_L$ representing mass transfer, where a_{pm} is the value of a corresponding to the period minimum $P \approx 80$ min, and a_L is the value of a at the first Roche lobe contact and onset of mass transfer. We determine N_{XB} for a representative evolutionary time of ~ 8 Gyr to study its dependence on the Verbunt parameters Γ and γ. Fig. 3 (left panel) shows the computed surface $N_{XB}(\gamma, \Gamma)$. As discussed in details in Banerjee & Ghosh (2007), the falloff of N_{XB} towards increasing γ from the fold is a signature of the increasing compact binary destruction rate with γ. Thus, the value of $\gamma(\approx 3 \times 10^3)$ corresponding to the fold seems to be a good estimate of the threshold γ above which the destruction processes dominate. However, the falloff towards decreasing γ is only an artifact of the assumption of virialization in evaluating the cluster parameters over the grid (Banerjee & Ghosh 2007). As can be seen in Fig. 3, most of the observed GC with significant numbers of XBs (filled squares), lie close to the fold of the $N_{XB}(\Gamma, \gamma)$ surface, indicating that the computed N_{XB} approximately follows the observed ones.

To further clarify these trends, we display in Fig. 3 (right panel) Γ/N_{XB} vs. γ, for a *particular value of* Γ. It has been shown in BG06 that the toy model of these authors leads to the scaling that Γ/N_{XB} is a function of γ *alone* (*i.e.*$\Gamma/N_{XB} \sim g(\gamma)$), which is a monotonically increasing function of γ. The close bunching of the $\Gamma/N_{XB} - \gamma$ curves, as

https://doi.org/10.1017/S1743921308015688 Published online by Cambridge University Press

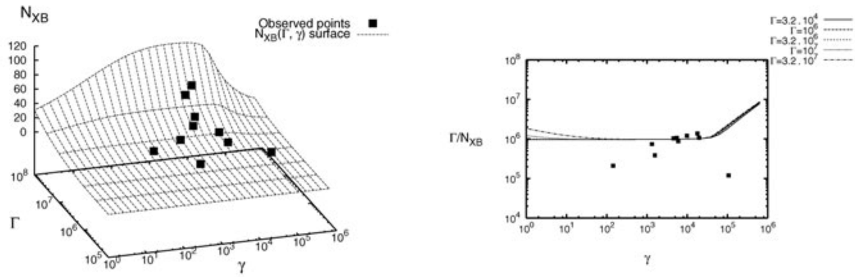

Figure 3. *Left:* Computed $N_{XB}(\Gamma, \gamma)$ surface. *Right:* Computed Γ/N_{XB} as a function of γ, showing scaling (see text). The computed curves for various values of Γ are closely bunched, as indicated. For both the figures, the overplotted filled squares are the positions of the Galactic globular clusters with significant numbers of X-ray sources from Pooley *et al.* (2003).

can be seen in Fig. 3, indicate that this scaling does carry over approximately to this more detailed study, thereby giving an indication of the basic ways in which the dynamical binary formation and destruction processes work. The above "universal" function $g(\gamma)$ of γ, except for a feature at low values of γ, is still a monotonically increasing one, reflecting the increasing strength of dynamical binary-destruction processes with increasing γ.

4. Concluding remarks

This work is an initiative of using Boltzmann equation to study the evolution of compact binaries in dense stellar systems. Not only we used simplified analytical models for dynamical formation and destruction of compact binaries, but also restricted ourselves only to systems like CVs and short period LMXBs, where the mass transfer occurs when the donor is in its main sequence, so that its stellar evolution is unimportant. To obtain a more realistic picture and consider other kinds of X-ray binaries, one should include more detailed treatment of tidal capture and consider the effects of stellar evolution in compact binary evolution. Such details can in principle be included in the Boltzmann scheme as the scheme itself is sufficiently generic. Such developments are in progress.

Acknowledgements

I am glad to thank the organizers of the "IAU Symposium 246" for having the opportunity to speak about this work during the event.

References

Banerjee, S. & Ghosh, P. 2006, *MNRAS*, 373, 1188. (BG06)
Banerjee, S. & Ghosh, P. 2007, accepted for publication in *ApJ*, arXiv:0708.1402.
Fabian, A. C., Pringle, J. E., & Rees, M. J. 1975, *MNRAS* (Short Communication), 172, 15P.
Heggie, D. 1975, *MNRAS*, 173, 729.
Heggie, D., Hut, P., & McMillan, S. L. W. 1996, *ApJ*, 467, 359.
Hut, P. *et. al.* 1992, *PASP*, 104, 981.
Pooley, D. *et al.* 2003, *ApJL*, 591, L131.
Portegies Zwart, S. F., Hut, P., McMillan, S. L. W., & Verbunt F. 1997, *A&A*, 328, 143.
Spitzer, L. Jr. 1987, *Dynamical Evolution of Globular Clusters*, Princeton Univ. Press.
Verbunt, F. 2002, in: *New horizons in globular cluster astronomy*, Proc. ASP conf. series.

https://doi.org/10.1017/S1743921308015688 Published online by Cambridge University Press