ARE NUMERICAL SIMULATIONS RELIABLE? PROPOSED IMPROVEMENTS AND TESTS

F. R. BOUCHET
I.A.P. and CPT-Ecole Polytechnique

When one builds a code to simulate numerically a process, the first concern is the range of validity of the results. This can be accessed empirically, though the results can be misleading if the tests are too naïve. For particle-mesh codes simulating the gravitational clustering, an analytical theory has been proposed in Bouchet et al. 1985. It yields the numerical dispersion relation of the system in the linear regime,and thus describes how the linear growth rate is affected by the discretisation. The theoretical predictions are in agreement with the results of actual numerical experiments: both show that the results of standart particle-mesh codes should not be trusted at distances smaller than 6 to 8 grid-spacing Δx (depending on the detail of the algorithm).

Bouchet and Kandrup (1985) use this theory to first propose a prescription for building an "optimal" algorithm, which amounts to introduce such "errors" in the resolution of the Poisson equation that the inaccuracies arising from the other steps of the computation are compensated.

Second, we argue that to test a simulation, a natural quantity to consider is the fractional difference $\Delta=(F_T-F_C)/F_T$ between the true force F_T and the computed one F_C, for a given 1D density perturbation. In a true computation the force field is spatially rapidly varying (it's high frequency component is important), so it might be misleading to consider a two particles density perturbation, since the description of the high frequency part of the two-particle force is not important to achieve a good accuracy. It appears more appropriate to consider Δ for structures of typical length λ (e.g. a sinusoid) and ask: how large must be λ to achieve, say, a 5% accuracy. For plain particle-mesh (PM) codes, the answer is again $\lambda \geq 6-8\,\Delta x$. To increase the dynamical range, it has been proposed (P^3M algorithm) to add to the mesh force a short-range correction computed by summing the two-body interactions of the closest particles (nearer than r_s). This of course improves the short-range description, but outside r_s, the code is a plain PM one, and $\Delta_{P^3M} \equiv \Delta_{PM}$. For the code of Efsthathiou et al. (1985), a 5% accuracy requires $\lambda \geq 11\Delta x$ (and Δ can be as large as 19% for $\lambda = 2r_s$!) Since the errors are limited to some intermediate range, specific statistical properties as the 2-point correlation function might be unaffected, but this has to be empirically proved for any new studied property.

Bouchet F.R., Adam J.-J., Pellat R., 1985, A&A, 144, 413.
Bouchet F.R., Kandrup H., To appear in ApJ, dec 1, 1985.
Efstathiou G., Davis M., Frenk C., White S., 1985, ApJ Suppl, 57, 241.

J. Kormendy and G. R. Knapp (eds.), Dark Matter in the Universe, 279.