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BOOK REVIEWS

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Garnett, J. B. and Marshall, D. E. *Harmonic measure* (Cambridge University Press, 2005), 588 pp, 0 521 47018 8 (hardback), £60.

The concept of harmonic measure is intimately connected with the boundary behaviour of conformal mappings. If ϕ maps the unit disk $D = \{z : |z| < 1\}$ conformally onto a Jordan domain Ω with extension to $\Gamma = \partial\Omega$, and z is a point of Ω with $z = \phi(w)$, then, given any Borel set E in Γ, the *harmonic measure* of E relative to Γ, with basepoint z, is

$$\omega(z, E, \Omega) = \omega(w, \phi^{-1}(E), D) = \int \frac{1 - |w|^2}{|\mathrm{e}^{\mathrm{i}\theta} - w|^2} \frac{\mathrm{d}\theta}{2\pi},$$

the integral taken over $\phi^{-1}(E)$.

It is natural to seek to compare harmonic measure with arc length on Γ and this is indeed possible if Γ is sufficiently smooth (e.g. piecewise analytic). But with the upsurge in interest in recent years in domains with fractal boundary, such as the von Koch snowflake, considerable attention has been devoted to comparing ω with Hausdorff h-measure Λ_h for various functions $h(t)$. Of particular importance in this connection is the celebrated result of Makarov that there is an absolute constant $C > 0$ such that for *any* simply connected domain Ω we have ω absolutely continuous with respect to Λ_h if

$$h(t) = t \exp\left[C\left(\log\frac{1}{t}\log\log\log\frac{1}{t}\right)^{1/2}\right] \tag{1}$$

but that there is another constant $c > 0$ such that there is a Jordan domain Ω with ω singular with respect to Λ_h if

$$h(t) = t \exp\left[c\left(\log\frac{1}{t}\log\log\log\frac{1}{t}\right)^{1/2}\right]. \tag{2}$$

In the present book, long awaited, all this is set forth in magisterial but eminently readable style. In order not to disturb the flow of the narrative, technical matters are given in 13 appendices. In particular, in Appendix J the nature of the above results (1) and (2) is explained in terms of the law of the iterated logarithm for Bloch functions. This, in turn, arises from the law of the iterated logarithm for real dyadic martingales with bounded jumps (the so-called Bloch martingales).

Most of the subjects of current interest in the theory of conformal mappings are presented clearly and cogently. These include the following.

(a) The results of Carleson, Jones and Wolff on harmonic measures in domains of infinite connectivity (complements of certain Cantor sets).

(b) The work of Carleman, Beurling and Ahlfors on estimating harmonic measure in terms of crosscuts, leading to Ahlfors's solution of the Denjoy conjecture.

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(c) The Hayman–Wu theorem that (with the previous notation)

$$\text{length}[\phi^{-1}(L \cap \Omega)] \leqslant \text{const.},$$

for any simply connected domain Ω and straight line L.

(d) The famous (or infamous) Brennan conjecture and the 'dandelion' construction.

In addition to all this, many other topics are considered: Bloch functions, BMO, extremal length, Schwarzian derivatives.

This book will surely be the standard reference for this topic for the foreseeable future. Indeed, one might almost say that it is the 'last word', except that such a carefully considered and well-presented book is bound to stimulate much further research in the subject.

The authors deserve our congratulations and our thanks.

J. M. ANDERSON

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Hirota, R. *The direct method in soliton theory* (translated from the Japanese and edited by A. Nagai, J. Nimmo and C. Gilson) (Cambridge University Press, 2004), xi + 200 pp., 0 521 83660 3 (hardback), £40.

The first recorded observation of a solitary wave was by John Scott Russell on the Union Canal near Edinburgh in 1834. (The solitary wave was recreated in 1995 on the canal's Scott Russell Aqueduct near Heriot-Watt University; a photograph of this event can be seen at www.ma.hw.ac.uk/solitons.) In 1895 Korteweg and de Vries showed that the solitary wave is described mathematically by a solution of the nonlinear evolution equation now known as the Korteweg–de Vries (KdV) equation. This equation, which describes shallow-water waves, is one of the simplest evolution equations in which the effects of nonlinearity and dispersion are balanced. With the advent of computers, nonlinear evolution equations could be solved numerically. In 1965 Zabusky and Kruskal carried out a numerical experiment on the KdV equation. They discovered that, when two or more solitary waves interact, each appeared to emerge from the nonlinear interaction intact and undistorted. They coined the word 'soliton' to denote this extraordinary type of solitary wave. In order to prove rigorously that solitons are not destroyed after their interaction, it was necessary to find exact solutions describing the interaction. This was achieved in 1967 by Gardner, Greene, Kruskal and Miura. They discovered the inverse scattering transform (IST) method that can be used to solve the initial-value problem for a restricted class of evolution equations. In the 1970s Hirota developed an ingenious method that is geared to finding multi-soliton solutions to evolution equations directly. Although the method is less general than the IST method in that it does not solve initial-value problems, it has the advantage of being applicable to a wider class of equations and is more straightforward. The method is now known as 'the direct method' or, outside Japan, 'Hirota's method'.

There are numerous books on various aspects of solitons. Although some describe Hirota's method, there has not been an introductory English-language book devoted to it. Apparently, when some of the western practitioners of the method found out that Hirota had written an introductory text in Japanese in 1992, they felt it should be translated into English. The result is this book. The translators admit that they have made minor changes to the original, but state that all such changes have the blessing of Professor Hirota.

In Chapter 1 the role of nonlinearity and dispersion in the formation of a solitary wave is discussed briefly. There follows a fascinating discussion of the motivation and chain of thought that led Hirota to the first key essential of the original direct method: the use of an appropriate dependent-variable transformation in order to convert a nonlinear evolution equation into a system of bilinear differential equations with respect to the new dependent variables. At first

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