BOOK REVIEWS

There are a few typographical errors in the text, but they (and the corresponding corrections) are obvious.

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Modern general topology, by Jun-iti Nagata. viii+353 pages. (Bibliotheca Mathematica, Vol. VII.) North-Holland, Amsterdam; Noordhoff, Groningen; Interscience, New York. 1969. U.S. \$14.75.

This book rather sharply divides into two parts. The first of them, including the first three chapters, corresponds to a first course on General Topology. It contains an introduction to set theory and definitions of basic concepts of general topology such as topological space, open and closed sets, basis and neighbourhood basis, convergence of nets and filters, continuous mapping, subspace, product space, quotient space, inverse limit space, separation axioms, axioms of countability, connected, compact, paracompact, fully normal, metric spaces, together with numerous easy consequences of the definitions.

While both subject and methods of this first part are rather elementary, the second part, consisting of the last four chapters, gives account of a series of various special topics, a considerable part of which dates from the last ten or fifteen years and was never treated in a monography till now. Thus this second part perfectly justifies the title of the book.

Ch. IV begins with Tychonoff's theorem on the product of compact spaces, then deals with the theory of Stone-Čech's, Wallman's and, more generally, Shanin's compactifications. Kaplansky's theorem on the lattice of continuous real functions on a compact Hausdorff space is proved together with some corollaries, and the chapter ends with a short account on sequentially compact, countably compact, pseudo-compact, real-compact and k-spaces.

In Ch. V, we find various characterizations of paracompact spaces, among them A. H. Stone's theorem on the coincidence of paracompactness and full normality for T_2 -spaces, further the characterizations by the existence of a cushioned or a σ -cushioned open or a σ -closure-preserving open refinement for each open covering. As a corollary, we get the theorem on the paracompactness of closed continuous images of paracompact T_2 -spaces. Then various characterizations of countably paracompact normal spaces follow as well as basic facts on strongly paracompact spaces. Finally, we find the characterization of paracompact T_2 - (countably paracompact, normal) spaces by the property that the product of the given space with every compact T_2 -space (a closed interval) is normal.

Ch. VI deals with metrizable spaces and some generalizations of them. First of all, various characterizations of metrizable spaces are presented, among them the famous metrization theorems of Alexandroff and Urysohn, the author and Smirnov,

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Bing, Alexandroff, Arhangelski and others. Čech's theorem on the complete metrizability of topologically complete metrizable spaces follows. Then we find imbedding theorems such as Dowker's theorem on the imbedding of metrizable spaces in generalized Hilbert spaces as well as Kowalski's theorem on the imbedding in the product of star-spaces. Several conditions follow for a union or image of metrizable spaces to be metrizable, as well as a representation theorem for metrizable spaces as perfect images of subspaces of generalized Baire spaces.

In the next two sections of this chapter, the author presents basic facts on uniform and proximity spaces, the former concept being defined according to Tukey's method with the help of coverings. Completions of uniform spaces, compactifications of proximity spaces and the equivalence of a proximity with a totally bounded uniformity are presented. The chapter ends with a discussion of Morita's *P*-spaces; in particular, we find the characterization of normal *P*-spaces by the property that the product of the space with every metrizable space is normal.

In the last chapter, the author presents some important topologies for mapping spaces (topology of pointwise convergence, uniform convergence on given subsets, compact-open topology) together with Ascoli-Arzelà's theorem. Then we find a series of theorems on the existence of a mapping of a given type such that a space of a given type is mapped onto a space of another given type. One of them is Nagami's theorem on the characterization of paracompact T_2 -spaces by the property that they are perfect images of zero-dimensional paracompact T_2 -spaces; we also find some results on *M*-spaces, an important class of *P*-spaces. The next section gives account of the characterization of compact T_2 -spaces as inverse limit spaces of polyhedra, then presents Flachsmeyer's construction of Wallman's compactification as an inverse limit space. The purpose of the final section is to treat the principal results of Michael on continuous selections of many-valued mappings into normed linear spaces.

This summary of the content of the book shows that the interested reader will find an extremely rich collection of beautiful results on important questions of General Topology. In general, the treatment is clear and the proofs are well detailed. However, the reading of the text is not always easy owing to the fact that if the author refers to proposition A then, quite often, it is meant not Proposition A but a part of the proof of Proposition A, or a statement similar to Proposition A, or Proposition A together with a series of other propositions not referred to. There are also some unpleasant misprints.

The main text is completed by a great number of references to further literature, and a collection of exercises is added to each chapter. In connection with the latter, a hint on the proof would sometimes be helpful for an unroutined reader.

The book can be warmly recommended to anyone who is interested in getting an outline on the development of General Topology in the last decades.

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