Some Criticisms of Robert Simson by Sir T. L. Heath.

By Professor GEORGE A. GIBSON.

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Sir T. L. Heath's translation of *The Thirteen Books of Euclid's Elements,* with its Introduction and Commentary, is not merely a worthy tribute to the lasting merits of Euclid's work, but is at the same time a most valuable history of elementary geometry; the language in which he describes the character of Camerer's edition of Euclid's first six books is even more applicable to his own: " No words of praise would be too warm for this veritable encyclopaedia of information." There may, however, be room for difference of opinion on matters of detail, and I propose in this note to call attention to one or two passages in which I think he is in error in his criticism of Simson, whose edition of Euclid formed the basis of so many English text-books and kept alive the traditions of Greek geometry in this country long after Euclid's *Elements* had disappeared as a text-book on the Continent.

On page 111 of the first of Heath's three volumes we read, " Simson says in the Preface to some editions (*e.g.*, the tenth, of 1799) that the translation is much amended by the friendly assistance of a learned gentleman." Now Simson died in 1768 and only two editions were issued under his own supervision, the first, in Latin and in English, in 1756, and the second, in English, in 1762. Later editions cannot be trusted in matters of detail, and it is apparently by trusting to a later edition that an unwarranted charge of altering Euclid's text in the eighth proposition of Book IV. is made against Simson. In the note on Euclid VI., 8 (Vol. II., p. 211), it is said that Simson " assumes a particular case of VI., 21, which might well be proved here, as Euclid proves it, with somewhat more detail." This omission is of a kind that is quite inconsistent with Simson's attitude to Euclid and as a matter of fact the allegation, though made also by Todhunter in the same connection, is quite unfounded; both in the first and in the second edition this particular passage is given as fully as in Heath's own translation. The treatment that Simson's text has received at the hands of publishers and editors resembles in many ways that to which Euclid's was subjected; unfortunately the alterations in Euclid's text cannot be traced with the facility that is possible in Simson's.

In another case Simson is alleged to have spoilt a demonstration completely. In Heath's note on the 7th proposition of Book XI. (Vol. III., p. 286) the following passage occurs: "But, whatever be the value of the proposition as it is, Simson seems to have spoilt it completely. He leaves out the construction of a plane through EGF, which, as Euclid says, must cut the plane containing the parallels in a straight line; and, instead, he says 'in the plane ABCD in which the parallels are draw the straight line EHF from E to F.' Now, although we can easily draw a straight line from E to F, to claim that we can draw it *in the plane in which the parallels are* is surely to assume the very result that is to be proved. All that we could properly say is that the straight line joining E to F is in *some* plane which contains the parallels; we do not know that there is no more than *one* such plane, or that the parallels determine a plane *uniquely,* without some such argument as Euclid gives."

If Euclid's enunciation of XI., 7, were not before us it might be thought from this passage that it was something like this, "to show that through two parallel straight lines one and only one plane can be drawn." In Heath's translation the enunciation of Euclid is, "If two straight lines be parallel and points be taken at random on each of them, the straight line joining the points is in the same plane with the parallel straight lines," and the proof is as follows:—"Let AB, CD, be two parallel straight lines, and let points E, F, be taken at random on them respectively; I say that the straight line joining the points E, F is in the same plane with the parallel straight lines. For suppose it is not, but, if possible, let it be in a more elevated plane as EGF and let a plane be drawn through EGF, it will then make, as section in the plane of reference, a straight line. Let it make it, as EF; therefore the two straight lines will include an area: which is impossible. Therefore the straight line joined from E to F is not in a plane more elevated; therefore the straight line joined from E to F is in the plane through the parallel straight lines AB, CD."

In this proof only two planes are mentioned, and the plane of the parallels is one of them; Euclid's proof consists in showing that EF is the line of section in that plane, made by the plane through EGF. Now what is Euclid's supposition? Is it not that the line joining two points in a plane *do not lie in the plane* and is not this supposition a flat contradiction of Euclid's first Postulate? In reply to Simson's objection that in the proof of the 3rd proposition of Book XI. it is twice assumed that the straight line drawn from one point to another in a plane is in that plane, Heath argues that " in Prop. 3 there is nothing about a plane in which two parallel straight lines are; therefore there is no assumption of the result of Prop. 7. What is assumed is that given two points in *a plane* they can be joined by a straight line in the plane: a legitimate assumption."

Now Simson's objection is that in Proposition 3 Euclid assumes that *the line which joins two points in a plane lies in the plane,* and that it is this assumption which Euclid in Proposition 7 supposes not to be true. In other words, Simson says that if E, F are two points in a plane it is not legitimate to suppose that the line joining them does not lie in the plane; if it be granted that the parallels are *in a plane* it would be according to Heath himself, " a legitimate assumption " that the line joining two points in that plane lies in the plane. The difficulty therefore must be that Euclid does not suppose the parallels to lie in a plane though how the line of section can be said to be in the same plane with the parallels unless we know beforehand that the parallels do lie in a plane is a mystery. I do not think this mystery is cleared up by the following observations:—"The subject-matter of Book I. and Book XI is quite different; in Book I. everything is in one plane, and when Euclid in defining parallels says they are straight lines *in the same plane, etc.,* he only does so because he must, in order to exclude non-intersecting straight lines which are *not* parallel." Where does Euclid remove the restriction that the lines must be in the same plane? He has given no new definition. If they are in a plane is it not legitimate to assume that the line joining two points on them lies in the plane?

Again, I see no ground for the suggestion that Euclid has in view the possibility that there may be more than one plane through two parallel lines. His proof has not the slightest analogy to the procedure by which the plane through three points is shown to be unique, since that proof would require that a line such as EF must lie in the plane of the parallels, and this is precisely what the argument in XI. 7 supposes not to be true.

Simson's notions of textual criticism were rudimentary; he seems to have held the delightfully simple view that Euclid's text as it originally stood was perfect, and when he discerned what he thought to be an imperfection he at once set it down to "Theon or some unskilful editor." Proceeding on this totally uncritical basis, he lays it down (note on Euclid I., 35) that "in the Elements no case of a proposition that requires a different demonstration ought to be omitted"; this may or may not be a sound canon for a school text-book, but it is obviously a very unsound criterion in textual criticism. Many of his additions are, I think, not merely useful, but necessary for a school text-book, but it is extremely unlikely, and it is impossible to prove that they can be assigned to Euclid. Heath usually notes the changes introduced by Simson and occasionally (*e.g.*, Vol. II., 211, 230) characterises Simson's remarks as "hypercritical." On the other hand, Heath goes to extreme lengths in defending the real Euclid—the Euclid of Heiberg's text—and, like Heiberg himself, has a special criterion by which to account for various omissions which Simson thought to be incompatible with the ideal he had set up as the perfect work of Euclid. "Euclid's method," he says (Vol. I., 246) "is to give one case only, for choice the most difficult, leaving the reader to supply the rest for himself. Where there was a real distinction between cases, sufficient to necessitate a substantial difference in the proof, the practice [of the great Greek geometers] was to give separate *enunciations* and proofs altogether, as we may see, *e.g.*, from the *Conics* and the *De Sectione rationis* of Apollonius."

Now there is no question that the multiplication of "cases" of propositions in the commentators was carried to a ridiculous extent, but I do not think that Simson's additions come within this category at all, nor is it by any means clear that Euclid's text justifies the above statement unless we make considerable reservations. An example in point is VI., 8, a proposition referred to above in another connection. In the note on I. 8 Heath says: "It is to be observed that in I. 8 Euclid is satisfied with proving the equality of the vertical angles and does not, as in I. 4, add that the triangles are equal and the remaining angles are equal respectively. The reason is no doubt . . that when once the vertical angles are proved equal the rest follows from I. 4 and there is no object in proving over again what has been proved already." The last sentence is in line with Heath's statement of Euclid's method, but it also expresses the grounds (in the main) on which Simson based his opinion that the demonstration was not Euclid's. It is surely going too far to say that Euclid's text "is really nothing more than a somewhat full citation of VI. 4."

Again, compare Euclid's proof of I. 26 with that of I. 8 in respect of detailed statement of inferences. Do the propositions III. 25, 33, 35, IV. 5 VI. 33 exemplify the dictum of "the one case, for choice the most difficult?" Had Euclid discussed the case of VI. 33 in which a multiple exceeds two right angles he might have thrown some light on his conception of an angle as a magnitude. It is curious how far enthusiasm for Euclid will carry some of his admirers. In his note on VI. 18 Simson objects that the proposition is proved only for quadrilaterals, and Heath says the objection is of no importance. I agree that the objection is in itself of no importance, but to Simson the question is whether the proposition could be as Euclid stated it. Now in his note on VI. 20 Heath writes: "The first part of the Porism, stating that the theorem is true of *quadrilaterals* would be superfluous but for the fact that technically, according to Book I., Def. 19, the term 'polygon' . . . used in the enunciation of the proposition is confined to rectilineal figures of *more than four sides* so that a quadrilateral seems to be excluded." It would seem therefore that Euclid's proof of VI. 18 does not extend to polygons in Euclid's meaning of the word, so that Simson's objection is not so unreasonable as it looks. The same sort of reluctance to admit any flaw in Euclid is seen in Heath's note on VI. 25; "the mistake of using triangle is not one of great importance." Simson, of course, attributes to "some unskilful hand" the blunder of "triangle" for "rectilineal figure," but Heath's explanation does not seem to me at all satisfactory, while the other defects, which Heath himself appears to admit, show that there is something wrong from the standpoint of a pure Euclidean proof.

It is at times amusing to compare the remarks of whole-hearted admirers of Euclid. The demonstration of VI. 24 as it appears in the Greek text is considered by Simson to have been made up by some unskilful editor out of two others; Heath suggests possible reasons for Euclid's procedure, and does not think "the proof unsystematic or unduly drawn out," while Heiberg, though disagreeing with Simson, says "it must be acknowledged that here Euclid has not quite maintained his usual standard of lucid arrangement" (*confitendum est, Euclidem hic nonnihil a solito ordine dilucido defecisse*).

Heath's general attitude to Simson is, I think, less cordial than that of most English admirers of Euclid, but there is one section of Simson's work to which he gives almost unqualified praise, namely, Book V., and the praise seems to me thoroughly deserved. One remark, however, strikes me as rather misplaced. In the note on V. 18 he speaks of Simson's proof as "intolerably long and difficult to follow unless it be put in the symbolical form." Undoubtedly it is long and difficult to follow, though as reproduced by Playfair in the symbolical form it is certainly neither long nor difficult; but the defect of Simson's proof is common to the whole of Euclid's presentation, and one would have expected a simpler presentation on Euclidean lines as a justification of such an epithet as "intolerable." Heath's notes on this proposition do not give any other general proof on Euclidean lines than that of Simson, Almost from the first appearance of Simson's edition of the Elements his treatment of Theon and "the unskilful editor" was a favourite topic for remarks of a more or less good-natured kind. It seems very hard for the enthusiastic Euclidean to maintain proper limits to his admiration. Simson gave reverence to a Euclid that was largely of his own creation, and Heath has, I think, succumbed to the temptation of reading modern conceptions into the language of Euclid and of accepting as certainty what is at most high probability. If, for example, Euclid fully realised what Heath holds to be essentially involved in the first and second Postulates it is hard to see why the phrase "two straight lines cannot enclose an area" should ever appear in his text at all, as it undoubtedly does in XI. 3 and XI. 7, and why the "common segment" and the "unequal circumferences" should disfigure XI. 1. Heath's suggestion that the proof in XI. 1, "can hardly be Euclid's" has really no more warrant than many similar suggestions of Simson. It is besides not easy to understand how such precise conceptions on the straight line as are attributed to Euclid are compatible with the absence of any workable definition or postulate of the plane.

Again, Heath says (Vol. I., 249) "it may be that Euclid himself was as well aware of the objections to the method [of superposition] as are his modern critics," and yet he states on page 225 that "the method can hardly be regarded as being, in Euclid, of only subordinate importance; on the contrary, it is fundamental. Nor, as a matter of fact, do we find in the ancient geometers any expression of doubt as to the legitimacy of the method." Surely this suggestion as to Euclid's views is a foolish extravagance; there is no evidence in favour of it—at least none is produced—and all the available evidence is against it.

Heath's volumes are full of most interesting matter, and there are various points on which discussions might be held, but I have said enough on the special matters that struck me in regard to Simson. In concluding this note, however, I would deal with one other point, namely, why was it that Euclid's *Elements* was retained as the text-book of geometry in this

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country so long after its disuse on the Continent? Other causes may have been at work, but I think the chief reason is to be found in the excellence of Simson's edition. Several English versions and adaptations preceded it, but there is no doubt at all that it was through Simson that Euclid's work became so familiar that "Euclid" and "geometry" were to many almost synonymous terms. It was perhaps fortunate that Simson's notions of textual criticism were so crude because the changes he made in the text were usually of a kind that contributed to an easier and a fuller understanding on the part of the ordinary schoolboy. In a sense Simson may be said to have been a pioneer; later editors while adhering in the main to Simson's text introduced such changes as the accumulated experience of the schoolroom suggested but, without that text to start from, the school editions would probably have been very different.

During the last years many books have been produced to take the place of the old "Euclid," but as yet there seems no agreement as to a recognised successor. For my own part I am strongly in favour of Euclid as the standard text-book of elementary geometry, but not of Euclid as presented in a literal translation. In the closing years of last century there were several quite satisfactory text-books that adhered to the order and the spirit of Euclid's geometry and that, with possible modifications, might be again brought into general use. Pedantic adherence to Euclid's text would be fatal to any school book, and it is as a school book that it can now be of general service. For the mathematical expert the problem of Euclid is quite different and the needs of the mathematician are amply provided for in Heath's translation and commentary.