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NONLINEAR THERMOMAGNETIC INSTABILITIES IN FERROMAGNETIC NANOFLUIDS

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A finite-amplitude instability has been analysed to discover the exact mechanisms leading to the appearance of various magnetoconvection patterns in a differentially heated vertical layer of a nonconducting ferrofluid placed in an external magnetic field perpendicular to the walls. The physical results have been obtained using a version of a weakly nonlinear analysis that is based on the disturbance amplitude expansion. This enables a low-dimensional reduction of a full nonlinear problem in supercritical regimes away from a bifurcation point. The details of the reduction are given in comparison with traditional small-parameter expansions (see [1]). It is also demonstrated that Squire's transformation can be introduced for higher-order nonlinear terms, thus reducing the full three-dimensional problem to its equivalent two-dimensional counterpart and enabling significant computational savings. The full three-dimensional instability patterns are subsequently recovered using the inverse transformations. The analysed instabilities are shown to occur as a result of supercritical bifurcations.

Three parametric regions are considered characterised by gravitational and magnetic Grashof numbers (Gr and $\mathrm{Gr_m}$ below) that quantify the effects of gravitational and magnetic buoyancy, respectively. At vanishing and small values of Gr, magnetoconvection arises caused by the dependence of the fluid's magnetisation on the local temperature. It takes the form of stationary vertical rolls resulting from a single instability mode as determined by a linear stability analysis. Consequently, a single-mode expansion is used to characterise the nonlinear instability patterns in this regime. The hierarchical systems of equations arising at various orders of disturbance amplitude are derived and solved leading to a low-dimensional Landau equation describing the temporal evolution of the disturbance amplitude.

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It is subsequently used to explore the perturbation dynamics. It was found that the instability occurs due to a supercritical pitchfork bifurcation so that the onset of convection is correctly determined by a linear stability analysis. The analysis also shows that the gravity-suppressing magnetoconvection (the amplitude of magnetoconvection decreases when $\mathrm{Gr_m}$ increases) and the computed values of the Nusselt number attain a maximum at $\mathrm{Gr} = 0$. In addition, to account for the spatial variation of disturbance amplitude, multiple spatial scales were used in conjunction with a multiple-time-scale expansion to derive the Ginzburg–Landau equation. The analysis of its solutions showed that the instability leading to the onset of magnetoconvective rolls is absolute so that the magnetoconvection is self-sustained. This is indeed confirmed by experimental observations reported in the literature: the detected magnetoconvection patterns are very robust once they appear.

At small values of $\mathrm{Gr_m}$ the thermogravitational instability occurs in the form of two vertically counter-propagating thermal waves. It is studied using a two-mode expansion. A pair of coupled Ginzburg–Landau equations are derived that model the spatio-temporal evolution of the two waves. Their analysis confirms that such a wave instability occurs as a result of a supercritical Hopf bifurcation and has a convective nature. Thus, such waves can only be observed in a finite-flow domain if a permanent source of perturbations of finite amplitude is available in the system, making experimental observations of thermogravitational waves much trickier. Yet if a permanent source of perturbations inducing one of the waves is available the counterpart wave will also be excited via the two-mode coupling.

Finally, the mixed regime corresponding to finite values of Gr and $\mathrm{Gr_m}$, where both vertically counter-propagating thermogravitational waves and stationary vertical thermomagnetic rolls are present forming an overlapping pattern, is analysed using a three-mode expansion. The coupled amplitude equations for the three disturbance amplitudes are derived and used to study the mode interaction and stability characteristics of the mixed magneto-thermogravitational convection. It is shown that while the two instabilities can co-exist they have a mutually suppressing effect on each other: the thermomagnetic rolls pass energy to the thermogravitational waves reducing the size of their equilibrium amplitude while the wave amplitudes increase above what would be observed for individual instabilities.

Reference

[1] P. Dey and S. A. Suslov, 'Thermomagnetic instabilities in a vertical layer of ferrofluid: nonlinear analysis away from a critical point', *Fluid Dyn. Res.* **48**(6) (2016), Article ID 061404.

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