

NEUTRON STAR STRUCTURE FROM PULSAR OBSERVATIONS*

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Abstract. We examine what inferences can be made regarding neutron star structure from observations of micro- and macrogitch behavior. After considering various theories it seems plausible that crust-quakes offer an explanation for the Crab microglitches, while corequakes can explain the Vela macrogitches. It is concluded that the Crab pulsar has a mass of less than $0.5 M_{\odot}$ and is $\sim 90\%$ superfluid neutrons while the Vela pulsar may possess a solid neutron core and have a mass of $\sim 0.7 M_{\odot}$ with a superfluid neutron abundance of $\sim 15\%$.

1. Introductory Remarks

Following the identification of pulsars as rotating neutron stars, interest in the calculation of the stellar structure of neutron stars has increased appreciably. As we have heard at this meeting from Bethe and Negele, the structure and composition of the outer portion of a neutron star ($\rho \lesssim 2 \times 10^{14} \text{ g cm}^{-3}$, say) is by now comparatively well understood theoretically; however, for densities which lie between $2 \times 10^{14} \text{ g cm}^{-3}$ and $10^{16} \text{ g cm}^{-3}$ there have been a number of different proposals for the stellar composition (Sawyer, 1972; Scalapino, 1972; Sawyer and Scalapino 1973; and the papers of Pandharipande and Canuto in this volume) with concomitant predictions for various aspects of stellar behavior. It may well be some time before there exists a theoretical 'consensus' on the behavior of neutron star matter in this density region, so that it is natural to consider to what extent *observations* of pulsar behavior provide confirmation of stellar structure calculations.

The clues which pulsar observation provide concerning neutron star structure are not exactly numerous. To be sure, the long term stability of pulsar signals strongly suggest that the outer crust of a pulsar is solid, in accord with theoretical predictions (Ruderman, 1969). Moreover, for one pulsar, that in the Crab nebula, energy balance considerations provide, in principle, a way of determining the stellar moment of inertia; however, as we shall see, this approach presently offers little more than an order of magnitude estimate of this quantity. The remaining current observational clues come from the Crab and Vela pulsars, both of which have been observed to speed up suddenly on more than one occasion (Richards *et al.*, 1969; Lohsen *et al.*, 1971; Pappalios *et al.*, 1970, 1971; Lohsen, 1972; Reichley and Downs 1969, 1971; Radhakrishnan and Manchester 1969) and both of which display a generally restless behavior between speedups (Boynton *et al.*, 1972; Reichley and Downs, 1970).

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We learn from these observations in three ways:

(i) Examination of the behavior of the pulse frequency in the period following a large speedup, or *macroglitch*. The spin-up of the outer crust, and those parts of the interior which are strongly coupled to it, acts as an external probe of the remaining interior matter (Baym *et al.*, 1969b).

The observation of macroscopic relaxation times (~ 1.2 yr for Vela, ~ 4 days for the Crab pulsar) for the transfer of the shift in angular velocity from the crust to the interior neutrons provides strong evidence for the presence of superfluid neutrons and protons inside these stars. Moreover, from the fraction of the frequency jump which relaxes we can estimate the stellar abundance of the neutron superfluid.

(ii) Consideration of the origin of macroglitches. To the extent that these result from processes inside the neutron star, an understanding of their origin, magnitude, and frequency may confirm existing stellar structure calculations or suggest possible inconsistencies therein.

(iii) Consideration of the origin of the restless behavior of the Crab pulsar. The resulting noise in the rotational frequency spectrum may be attributed to frequency microglitches (of either sign); again, an understanding of its origin provides a further test of the applicability of present stellar models to pulsar behavior.

The major portion of this talk will be devoted to the above three problems; in the concluding sections we examine other ways of deducing stellar structure from pulsar observation, and consider what future observations might be especially relevant to the determination of the structure of neutron stars.

2. After a Macroglitch

After the observations of the sudden Vela speedup in 1969 a two component description of the dynamics of a neutron star was proposed to explain quantitatively the two prominent features of the pulsar's postglitch behavior (Baym *et al.*, 1969b).

(a) the tendency for a substantial part of the sudden increase in Ω to relax in a roughly exponential way.

(b) the observation that immediately after the glitch the fractional jump in the slowing down rate is much greater than that in Ω ($\Delta\dot{\Omega}/\dot{\Omega} \gg \Delta\Omega/\Omega$).

The two component model assumes that all the conducting components of the star, which are presumably tied together by a uniform magnetic field, spin up together during a glitch – whatever the glitch origin. This is reasonable since the Alfvén velocities in the magnetosphere and within the star communicate any spin-up to all such conducting components in a time ($\sim 10^3$ s) too short to be observed. The moment of inertia of this fraction of the star, I_c , includes the electrons and nuclei of the crust, the electrons and protons of the core, possible charged hyperons in a superdense central core of a heavy neutron star, the magnetosphere, and finally any other component sufficiently strongly coupled to these that it shares the increased angular velocity of the glitch in a time too short to be resolved (a few days in Vela, a few hours in the Crab pulsar). The rest of the star, which is weakly enough coupled to the charged

components to respond to them on a longer time scale, has a moment of inertia I_n . This slowly responding component is the neutron superfluid. Because of the weak coupling, the crystal spin-up thus effectively acts as an external probe of the neutron superfluid; study of the superfluid response therefore provides a measure of both the strength of the coupling and of the stellar abundance of the superfluid.

The simplest version of the two component theory assumes that the neutron superfluid can be described by a single angular velocity, $\Omega_n \neq \Omega$, and that the coupling between the charged and superfluid components is given by:

$$I_c \dot{\Omega} = -\alpha - \frac{I_c}{\tau_c} (\Omega - \Omega_n), \quad (1)$$

$$I_n \dot{\Omega}_n = \frac{I_c}{\tau_c} (\Omega - \Omega_n). \quad (2)$$

(The external torque α , and τ_c also depend on Ω .) After a sudden initial jump $(\Delta\Omega)_0$ in Ω , the post-glitch behavior described by (1) and (2) is

$$\Omega(t) = \Omega_0(t) + (\Delta\Omega)_0 [Qe^{-t/\tau} + (1-Q)], \quad (3)$$

where $\Omega_0(t)$ is the extrapolated frequency in the absence of the glitch,

$$Q = \frac{I_n}{I} \left(1 - \frac{\Delta\Omega_n}{(\Delta\Omega)_0} \right), \quad (4)$$

and

$$\tau = \tau_c \frac{I_n}{I}. \quad (5)$$

$\Delta\Omega_n$ is the initial jump in Ω_n , and I is the total stellar moment of inertia.

The form of the 'post-glitch function' given in (3) is in rough accord with the Vela observations and also, but with different Q and τ , with reported observations of the Crab glitches. A key test of the model is whether or not all post-glitch functions in a given pulsar have the same Q and τ . Period noise intrinsic to the pulsar introduces some ambiguity into the separation between post-glitch function and noise fluctuations in the reduction of the data. At present there is no reported inconsistency with the post-glitch function hypothesis. From a quantitative fit based on (3), to the initial speedups for both pulsars, one obtains the glitch parameters given in Table I.

The observed magnitudes of Q and τ inform us about certain properties of the neutron star interior. Q furnishes a direct measure of the extent to which the glitch reduces the total stellar moment of inertia, since the relative change in I is

$$\frac{\Delta I}{I} = -\frac{(\Delta\Omega)_\infty}{\Omega} = -(1-Q) \frac{(\Delta\Omega)_0}{\Omega}, \quad (6)$$

where $(\Delta\Omega)_\infty$ is the glitch-induced long term increase in the crustal rotation frequency. Q likewise gives an indication of the stellar abundance of neutron superfluid, since as

long as $I_c \ll I_n$, and/or $\Delta I_n \ll \Delta I_c$ (where ΔI_n and ΔI_c are the changes in the moments of inertia of the two components at the glitch), one has

$$Q \simeq I_n/I. \quad (7)$$

In a lighter neutron star ($M \lesssim 0.5 M_\odot$) almost all of the matter beneath the crust is expected to be protons, electrons, and neutron superfluid. The theoretical Q is then ~ 0.90 – 0.95 , just in the range of that inferred from the Crab pulsar glitches. This is an encouraging numerical agreement. In a heavier neutron star (corresponding to a central density, say, of greater than $10^{15} \text{ g cm}^{-3}$), one expects Q to be smaller, since in such stars the easily spun-up charged hyperon core and/or solid neutron core will contribute to I_c , producing a corresponding decrease in the fraction of neutron superfluid. However, no matter how heavy the star, one expects a non-vanishing value of

TABLE I
Speed-up observations for the Vela and Crab pulsars

	Vela	Crab
Ω (rad/sec)	70.5	190
T (yr)	2.4×10^4	2.4×10^8
$\Delta\Omega/\Omega$	2.34×10^{-6}	$(6.9 \pm 0.7) \times 10^{-9}$
$\Delta\dot{\Omega}/\dot{\Omega}$	6.8×10^{-3}	$(8.5 \pm 3.5) \times 10^{-4}$
τ	1.2 yr	(7.7 ± 3) days
Q	0.145	0.96 ± 0.08

Q , since there will always be some neutron superfluid present; an approximate minimum value for Q is ~ 0.05 . For the Vela pulsar, the inferred $Q \sim 0.15$ suggests that it has a mass $\gtrsim 0.7 M_\odot$ (Pines *et al.*, 1972).

The crust-superfluid coupling is characterized by τ , essentially the time for the I_c components to come to rest if the core were suddenly to stop spinning. That this is a macroscopic time strongly suggests the presence of superfluid neutrons in the core of the star, as well as the superfluidity of those protons which interpenetrate the neutrons (Baym *et al.*, 1969a, 1969b); it is incompatible with the interior neutrons forming a 'normal' degenerate quantum liquid. Assuming the core protons to be superfluid, the dominant coupling to the neutrons comes through electron-neutron interaction; this interaction will spin up a normal Fermi liquid of neutrons in a time $\sim 10^{-11}$ s. On the other hand, if the neutron liquid is superfluid, only that part of the neutron fluid within vortex cores can interact in a normal way with the core electrons. Even for these, the interaction is suppressed by a factor $\exp(-\pi\Delta^2/4E_F kT)$ where Δ is the superfluid energy gap and E_F is the neutron Fermi energy (Feibelman, 1971). The net electron-superfluid neutron interaction is proportional to the total length of vortex line in the superfluid. The minimum length of vortex line in a sphere with radius R of uniformly rotating superfluid with angular frequency Ω is $\sim \Omega m_n R^3/\hbar$. The radius of a vortex core $\sim 10^{-12}$ cm. The minimum fraction of the rotating superfluid contained in

the quasi-normal vortex cores of the Crab pulsar is thus 10^{-18} . With this fraction of superfluid neutrons,

$$\frac{1}{\tau_c} \sim \frac{\Omega}{40} \left(\frac{\Delta}{1 \text{ MeV}} \right) \left(\frac{kT}{E_F} \right) \exp \left(- \frac{\pi \Delta^2}{4 E_F k T} \right) s. \quad (8)$$

The estimated magnitude for τ_c is quite reasonable; for a temperature of 10^8 K and typical pulsar densities, τ_c is of the order of days for $\Delta = 1.7$ MeV and of the order of a year for $\Delta = 2.4$ MeV; since, moreover, τ is very sensitive to small changes in temperature, one can easily understand the different τ 's observed for the Vela and Crab pulsars.

A longer post-glitch healing time would be expected for the Vela pulsar even if the mutual friction torque were the same as the Crab pulsar, since the healing time is roughly proportional to $IQ(1-Q)$. However, the torques are likely not identical, since Vela is older (and hence colder) and rotating more slowly (and hence possesses a smaller number of vortex lines): both these latter effects are in the right direction.

We note that if the protons, which interpenetrate the neutron superfluid, form a normal Fermi liquid, rather than a superfluid, the above times would be appreciably decreased, since the proton-neutron coupling strength is some 10^3 times that of the magnetic electron-neutron interaction and the proton Fermi energy is much smaller than that of the electrons. Given the above parameters, the resulting τ 's would then be microscopic in contradiction with observation.

We have thus far assumed the neutron superfluid is not turbulent. If, as suggested by Greenstein (1970), there exists any appreciable degree of turbulence, the total length of vortex line is greatly extended beyond its normal length. For the theorists this is literally opening a can of worms, and the questions he must answer are: how many 'worms' (squirming vortex lines) are there, and what is their total length? Any vortex line extension by a factor f would decrease the τ by $1/f$. A huge f (complete turbulence on a microscopic scale would make $f \sim 10^{18}$) would therefore predict a τ_c much too small to be compatible with observation. We conclude there cannot be any very large turbulence present in the rotating neutron superfluid.

3. Origin of Macrogitches

Some fifty-odd papers which seek to explain the macrogitches observed for the Vela and Crab pulsars have by now appeared in print, and yet another fifty may well appear where the problem is regarded as satisfactorily resolved by the astrophysical community. In preparing this talk it seemed to us that a critical review of macrogitch theories might be useful in sorting out whether certain classes of theories could be rejected at this time and whether any theory or combination of theories could be regarded as providing a plausible or even possible explanation of the glitch origin; by relating theories in the latter group to observation on the one hand, and to stellar structure on the other, one might hope to gain further perspective on the nature of these two neutron stars, and on neutron stars in general.

Let us define a macroglitch as a frequency jump such that

$$\frac{\Delta\Omega}{\Omega} \gtrsim 10^{-9}.$$

The present observational facts which a macroglitch theory must explain include the following:

(i) Macroglitch size vs. pulsar period. Why have macroglitches been seen only in the Crab and Vela pulsars, and why are those observed in the Vela pulsar ($\Delta\Omega/\Omega \sim 2 \times 10^{-6}$) some two or more orders of magnitude larger than those seen in the Crab ($10^{-9} \lesssim \Delta\Omega/\Omega \lesssim 10^{-8}$)? In other words, why does one have

$$\left(\frac{\Delta\Omega}{\Omega}\right)_{\text{Vela}} \gg \left(\frac{\Delta\Omega}{\Omega}\right)_{\text{Crab}} \gg \left(\frac{\Delta\Omega}{\Omega}\right)_{\text{other}} ?$$

(ii) Macroglitch sign. Why do all macroglitches thus far observed have the same sign?

(iii) Macroglitch repetition frequency. The time, τ_g , between Vela macroglitches is of the order of a few years, while that between Crab macroglitches is months to years.

(iv) $\tau_g \neq \tau$. The time between glitches is certainly not the same as the healing time, τ , in the case of the Crab pulsar, and most likely is not the same either for the Vela pulsar, since $(\tau_g)_{\text{Vela}} \sim 2$ yr, $(\tau)_{\text{Vela}} \sim 1.2$ yr.

(v) Macroglitches are 'sudden' events. For the Crab pulsar (Lohsen, 1972) one sees that a macroglitch takes place in no more than a few hours; for Vela the corresponding limit is at present a few days.

(vi) $(Q)_{\text{Vela}} \ll 1 \neq (Q)_{\text{Crab}} \simeq 1$. The fraction of the glitch which relaxes is appreciably less than unity in the case of the Vela pulsar.

(vii) No appreciable change in pulse shape is seen following the speedup of either the Crab or Vela pulsars.

The classes of macroglitch theories may be conveniently specified in terms of the hypothesized physical location of the macroglitch; from way out to far in, we have:

- Planets
- Magnetospheric instabilities
- Accretion
- Crustquakes
- Hydrodynamic instabilities associated with the superfluid neutron core
- Corequakes

and it is in this order we shall consider the theories.

3.1. PLANETARY PERTURBATIONS (Michel, 1970; Rees *et al.*, 1970)

It seems rather difficult to assume that the macroglitches and microglitches are a result of a linear motion of the pulsar caused by a system of planets surrounding it.

Even if one accepts their existence in the immediate vicinity of a supernova remnant, there are considerable difficulties in fitting such a model to the observations. For one, it seems impossible to fit the sharp rise of a glitch together with its much slower subsequent slowing down to a near passage of a planet: indeed, no reasonably smooth function can fit the observation. In the Crab pulsar, an imitation of the restless behavior by planets presents even a more formidable project. The Princeton group (Groth, 1971) at one stage obtained a reasonable fit for their 1969–1970 observing season of the Crab pulsar, by postulating the existence of three orbiting planets. Together with their four cubic polynomial parameters, this was a 19 parameter fit. However, they could not then predict correctly their 1970–1971 observations; it is likely that one needs an ever increasing number of planetary companions as the fit period is increased.

On the other hand, a planetary passage hypothesis is consistent with the Vela observations, in the sense that it can explain both the magnitude of the apparent speedup, its duration, and its repetition frequency, provided one assumes a highly eccentric orbit with a near passage at $\sim 4.5 \times 10^{12}$ cm (Michel, 1970). (This distance is sufficiently far from the pulsar that tidal effects will not cause a major perturbation.) However, since the two Vela glitches were not of identical magnitude, one needs at least two planets in orbit about the Vela pulsar to fit the data. Clearly more observational data and a more detailed theoretical analysis is required before one can accept or reject this proposal for Vela. If the Vela macroglitches are nothing but a linear motion-induced Doppler shift, then no conclusions concerning the superfluidity of the neutron core can be drawn from analysis of post-glitch behavior. However one can still measure a $Q \equiv 1 - (\Delta\Omega)_\infty / (\Delta\Omega)_0$; so defined, Q should be equal to unity, since any spinup is necessarily followed by a spindown. However, only the 'rapid' spin up is easily detected above the intrinsic pulsar noise, so that the apparent measured Q could turn out to be less than unity. (Whether it could be as small as 0.15, however, is not clear.)

3.2. MAGNETOSPHERIC INSTABILITIES (Pacini and Scargle, 1971; Sturrock, 1971)

The two chief arguments in favor of a magnetospheric instability as an origin of macroglitches – the apparent observation of wisp motion or flaring plus a change in the pulsar dispersion measure following the Crab macroglitch of September, 1969 – have become the two principle observational arguments against this explanation; no similar correlations have been found with other Crab macroglitches. However, those still interested in pursuing this possibility (unstable-magnetospheric-theorists?) must explain:

(i) Why only these two pulsars choose to have unstable magnetospheres and why, in these cases, the instability is such that in the glitch an appreciable fraction of the magnetosphere is blown away? Indeed the hypothesized sudden change in the plasma moment of inertia required for the Vela pulsar is of the same order as the maximally allowable $I_{\text{plasma}} \lesssim B_s^2 R^3 / 6\Omega^2$ (Rosenbluth, 1972), where B_s is the surface magnetic field, R the pulsar radius.

(ii) The characteristic time to fill the pulsar magnetosphere is of the order of seconds; why then should it take months, or years, for an instability to develop?

(iii) Why, if the entire magnetosphere is involved in the instability, is there no change in pulse shape following a macroglitch?

(iv) The behavior of terrestrial plasmas near an instability is generally to avoid a gigantic instability; such plasmas tend rather to fluctuate about some 'minimal' instability. Why should pulsar plasmas be different?

3.3. ACCRETION

There are likewise a number of problems with attributing macroglitches to accretion (see the paper by Börner and Cohen in this volume).

(i) Where does the accreting matter come from? One needs $\sim 10^{-10} M_{\odot}$ per macroglitch for the Crab pulsar, and a thousand times more for Vela. One knows, for example, that the accretion rate for a neutron star which forms a compact X-ray source is $\sim 10^{-9} M_{\odot}$ per year, (Lamb *et al.*, 1973), and that to get this much accreting matter easily, one needs to postulate an accompanying close companion star. The constancy of the pulsars' pulse periods enables one to readily discard any such companions for Crab and Vela. Moreover, the accreting matter could not represent a 'fallback' of matter from the supernova explosion which created the pulsar; any such fallback would take place within the first year of the supernova (Colgate, 1972).

(ii) Assuming that one invents a source of accreting matter, could $10^{-9} M_{\odot}$ reach the stellar surface sufficiently rapidly to produce a macroglitch? Consider a large piece of stellar (or rather planetary) matter incident on the pulsar. It will first of all be ripped apart by tidal forces. To see this, assume the matter to be in the form of an infalling homogeneous sphere of radius R_s , density ρ ($\sim 5 \text{ g cm}^{-3}$) and rigidity μ ($\sim 10^{12} \text{ dyne cm}^{-2}$); the shear angle ϕ of the sphere at distance R from the pulsar will be

$$\phi \sim \frac{kM_p}{4\rho R^3}, \quad (9)$$

where M_p is the pulsar's mass ($2 \times 10^{33} \text{ g}$). The Love number k is roughly given by $G\rho^2 R_s^2/2.5\mu$ for $R_s \ll (1/\rho)\sqrt{\mu/G} \sim 8 \times 10^8 \text{ cm}$, and is ~ 1 for $R_s \gg (1/\rho)\sqrt{\mu/G}$. For a body with $R_s \gg 8 \times 10^8 \text{ cm}$, we must therefore have

$$\phi \sim \frac{10^{32}}{R^3} \lesssim \phi_c, \quad (10)$$

where ϕ_c is the critical angle for break-up.

If $\phi_c \sim 10^{-4}$, a typical value for such objects, than at a distance of $R \sim 10^{12} \text{ cm}$, the infalling chunk will start breaking up. As the pieces continue to fall in, we must eventually have

$$\frac{G\rho R_s^2 M_p}{10 R^3 \mu} \simeq 7 \times 10^{13} \frac{R_s^2}{R^3} \lesssim \phi_c \quad (11)$$

so that, when the pieces finally reach the pulsar's surface, at $R \sim 10^6$ cm, they have a radius of roughly 1 cm!

We may also get a rough idea as to when the torn pieces will start to move independently in the pulsar's gravitational field. For the pulsar's gravitational pull to be more important than that of a neighboring piece, we must have

$$\frac{GM_p}{R^2} \gg \frac{(4\pi/3)G\rho R_s^3}{R_s^2}$$

or

$$R \ll \frac{10^{16}}{\sqrt{R_s}}. \quad (12)$$

Therefore, at about $R \sim 3 \times 10^{10}$ cm one may expect that the chunk will not only be torn to bits, but that the bits will also start to move independently of one another. Also, an incoming chunk of material is likely to carry a large amount of angular momentum with it. This means that the large infalling chunk goes first into orbit, and, because of the tidal tearing, forms a disc of particles around the pulsar. The disc will indeed fall in gradually, because of friction in the pulsar's 'atmosphere' and the tidal forces, but it is very difficult to see why this will not be a continuous long process rather than a sharp 'glitch.'

We also note in passing, that with the required rate of infalling material, there should be substantial X-radiation produced (which is not observed), and, indeed, the neutron star may not function as a pulsar at all (Shvartsman, 1971; Lamb *et al.*, 1972).

3.4. CRUSTQUAKES

Crustquakes – the sudden release of elastic energy in the solid outer crust of a neutron star – were one of the first mechanisms suggested to explain the Vela macrogitch (Ruderman, 1969). What is appealing about crustquakes is that one is dealing with a physical process which has a clear terrestrial analogue, for which the time between macrogitches, all of which will have the same spin, varies from one to another (and is not correlated with τ), that crustquakes are expected to be a common phenomenon only in comparatively young pulsars, and that there exists a wide variety of mechanisms, many of them plausible, for inducing critical strains in the stellar crust. While, as shall see, they continue to be a likely mechanism for the Crab-pulsar macrogitches, it is no longer likely that they provide a mechanism for those observed in the Vela pulsar.

The mechanism which has been considered in most detail is rotational-induced strain arising from the gradual spin-down of the star as a result of the emission of electromagnetic radiation and charged particles. The crust of the star, formed when the star is spinning comparatively fast, is subject to increasing gravitationally induced stresses as the star slows down; when these stresses exceed the yield point, the crust will crack (a starquake). In the process, some stress is suddenly relieved, the crustal

moment of inertia is suddenly reduced, and, by conservation of angular momentum, its rotation rate is suddenly increased, hence a speedup.

A simple description of such crustquakes may be given in terms of a quadrupolar stellar deformation described by a single time-dependent distortion parameter, the crustal oblateness (Baym and Pines, 1971). In this description, if appreciable plastic flow does not take place, the time to the next quake is proportional to the stress relieved in the preceding quake, and may be estimated for a given model of neutron star. To make quantitative estimates of the relevant parameters, we assume the star is axially symmetric and define the oblateness ϵ according to $I = I_0(1 + \epsilon)$ where I_0 is the moment of inertia for a non-rotating spherical star. The time varying portion of the mechanical energy may be written as

$$E = -\frac{L^2}{2I_0} \epsilon + A\epsilon^2 + B(\epsilon_0 - \epsilon)^2, \tag{13}$$

where ϵ_0 is a reference oblateness (which changes only as a result of plastic flow or crustquakes), and the coefficients A and B measure the gravitational and elastic energy stored in the star as a result of rotation. An order of magnitude estimate of A and B may be obtained from the expressions appropriate to a self-gravitating incompressible homogeneous sphere of radius R and crustal volume V_{cr} , $A = (3/25) GM^2/R$, $B = (57/50)\mu V_{cr}$, where μ is the shear modulus of the crustal material. On minimizing the energy, (13), at fixed L and ϵ_0 , we have

$$\epsilon = \frac{I_0\Omega^2}{4(A + B)} + \frac{B}{A + B} \epsilon_0 \tag{14}$$

and, since $B \ll A$ for all stable neutron stars of this type (Baym and Pines, 1971), $\epsilon \simeq I_0\Omega^2/4A$, its perfect fluid value. In this one parameter description, which may be appropriate to ‘astrologically young’ pulsars, a quake takes place when the mean stress in the crust, $\sigma = (1/V_{cr})(\partial E_{el}/\partial \epsilon) = \mu(\epsilon_0 - \epsilon)$ exceeds some critical value, σ_c . In the quake, both the oblateness and reference oblateness decrease according to

$$\Delta \epsilon = [B/(A + B)] \Delta \epsilon_0 \simeq (B/A) \Delta \epsilon_0, \tag{15}$$

where $\Delta \epsilon$ is directly observable, since

$$\Delta \epsilon = \Delta I/I = -(\Delta \Omega)_\infty/\Omega = -(1 - Q)(\Delta \Omega)_0/\Omega. \tag{16}$$

After the quake, the stress will start to build up once more, and the time to the next quake is given by

$$\tau_q^\epsilon = T(\omega_q^2/\Omega^2) |\Delta \Omega_\infty/\Omega|, \tag{17}$$

where T is the slowing-down time of the pulsar ($T = |\Omega/\dot{\Omega}|$), and effects of stellar structure are described through the parameter,

$$\omega_q^2 = 2A^2/B I. \tag{18}$$

Results of microscopic stellar model calculations of ω_q^2 are given in Table II, together with the predicted time between quakes, for a speedup involving a relative

jump in the moment of inertia of one part in 10^9 [recall that for the Crab pulsar, if $Q \simeq 0.9$, this corresponds to an initial speedup of one part in 10^8 , since $\Delta I/I = -(\Delta\Omega_\infty)/\Omega = -(1-Q)(\Delta\Omega)_0/\Omega$ according to (6)]. We see that for a model star of mass $\sim 0.3 M_\odot$, the predicted interval between macroglitches of initial relative magnitude 10^{-9} to 10^{-8} , is of the order of months and years as observed, and further note that the corresponding critical strain angle, $\phi_c \sim \sigma_c/\mu \sim 10^{-4}$, a reasonable *ad hoc* value. For the Vela pulsar, however, the corresponding calculated interval between macroglitches of relative magnitude $(\Delta\Omega)_0/\Omega \sim 10^{-6}$, while depending on the assumed mass, is at least a few centuries, and more probably many millenia. This is because the released strain is much larger than in the Crab, while both the slowing down rate, T , and the rate at which strain is replenished ($\sim \Omega^{-2}$) are an order of magnitude smaller. The observed two-year interval between macroglitches, if typical, is therefore not compatible with the treatment of Vela as an 'astrologically young' pulsar in which the strain energy released in one quake is replenished before the next.

TABLE II
Dynamic stellar parameters for the Crab pulsar

Mass (M_\odot)	$\omega_q^2(\text{s}^{-2})$	$\tau_q^\varepsilon(\text{yr})$	$2\pi/\Omega_w(\text{days})$	$\varepsilon/10^{-4}$
0.10	2.7×10^8	0.018	3.8×10^{-3}	14
0.15	1.7×10^9	0.11	2.4×10^{-2}	5
0.20	6.3×10^9	0.42	8.8×10^{-2}	3
0.25	2.4×10^{10}	1.6	3.3×10^{-1}	2
0.30	4.7×10^{10}	3.1	6.5×10^{-1}	2
0.46	5.2×10^{11}	35	7.3	1
0.80	7.0×10^{12}	470	9.8×10^1	0.9
1.08	3.7×10^{13}	2400	5.0×10^2	0.7
1.41	4.0×10^{14}	27000	5.5×10^3	0.5

A decrease in ε is, of course, not the only way to build up strain energy in the stellar crust. Pines and Shaham (1972a) have considered the buildup of strain energy as a result of the misalignment of the rotation axis and the elastic reference axis in a star subject to a radiation torque nearly perpendicular to the axis of rotation. The elastic energy then contains an angular contribution which can play an important role (indeed, as we shall see, it is likely responsible for the macroglitches observed in the Crab pulsar); however, although this angular term modifies the above estimate of τ_q^ε , it does not seem capable of doing so sufficiently to explain in plausible fashion the two-year interval between Vela macroglitches.

Still another mechanism for inducing crustquakes has been proposed by Dyson (1969, 1970) who has considered the possible existence of volcanoes on the pulsar surface, through which matter pours out until sufficient material has built up that one gets a starquake. The mountain building process can be a comparatively slow one (\sim months to years) and the mountains are not especially high. (Indeed, one can show that the maximum height, h , of a mountain on the stellar surface is $h/R \simeq 3\phi_c(B/A)$; since $\phi_c \sim 10^{-4}$ and $B/A \sim 3 \times 10^{-3}$, one concludes that mountains on the pulsar

surface will not greatly exceed 1 cm in elevation.) The volcano mechanism, while appealing, suffers from a certain lack of plausibility in that to get volcanoes one needs substantial internal temperature gradients, and these are not likely found in the extraordinarily highly (thermally) conducting superdense interior matter. Moreover, in common with the above glitch theories, it cannot easily explain the *size* of the Vela macroglitches. Vela, assuming it has a liquid interior, has a fluid oblateness $\varepsilon \sim I_0 \Omega^2 / 4 A \sim 10^{-5}$; hence, in macroglitches the fractional oblateness change approaches the disconcertingly large value of $\sim 10\%$.

3.5. HYDRODYNAMIC INSTABILITIES ASSOCIATED WITH THE FLUID NEUTRON CORE

Cameron and Greenstein (1969) have suggested that the neutron core fluid may become classically unstable, as the slowing down torques set up a rotational flow in which the angular momentum per unit mass decreases with distance from the rotation axis. This model has the great virtue that it is based upon an instability that is known to exist. However, the evidence against such a glitch model includes:

(i) In such models the interval between glitches should be approximately the relaxation time for the fluid to return to its preglitch state. But the observed τ in the case of the Crab is two orders of magnitude less than the glitch interval while in the case of Vela the pulsar did not nearly finish its relaxation when the second glitch occurred.

(ii) It may not be possible to set up the unstable flow in a non-turbulent superfluid. Wherever the external torque causes the angular momentum per unit mass to become independent of radius – the limit of stability – the vortex density, and thus the push of the external torque, have to vanish.

(iii) Why are such glitches not seen in the somewhat slower pulsars?

Packard (1972) has proposed that glitches may be manifestations of crust-pinned vortex lines tearing loose. Such a phenomenon is known for individual vortex lines in the laboratory. Arguments against this model include:

(i) As in the Cameron-Greenstein model, the observed relationship between relaxation time and glitch interval is then paradoxical.

(ii) The tension from vortex line bundles and even vortex lines considered as if isolated (a great underestimate of the tension) is so great that for the estimated parameters for pulsar superfluid and crust nuclei no pinning should occur.

(iii) Why, when pinning is easier in slower pulsars, are no glitches observed for them?

3.6. COREQUAKES

Starting with the recent plausible suggestion that the heavier neutron stars may possess a solid inner neutron core, we have attempted to explain the Vela macroglitches as arising from corequakes which represent the sudden release of elastic energy stored in the solid *inner* neutron lattice (Pines *et al.*, 1972). This neutron solid likely occurs at stellar densities $\geq 1.5 \times 10^{15} \text{ g cm}^{-3}$ (Canuto and Chitre, 1972); because its shear modulus will be some five orders of magnitude larger than that of the crustal material ($\mu \sim 10^{35} \text{ dyne cm}^{-2}$ instead of $\sim 10^{30} \text{ dyne cm}^{-2}$), the core possesses a substantial reservoir of elastic and gravitational energy which can be released in starquakes. Thus

one has $B \sim 10 A$, and the possibility of a quite brittle crustal material, while $\varepsilon \simeq \varepsilon_0$, so that as the star slows down, ε , following ε_0 , will change only discontinuously in starquakes. There is thus sufficient elastic energy present that the crust does not have to build up the strain energy released in the previous quake, before quaking anew [and in this respect a solid neutron core resembles the earth (Pines and Shaham, 1972b)], so that there is no difficulty in principle in understanding the appearance of macroglitches every few years in a star rotating as slowly as the Vela pulsar. Moreover the size of the glitches is no longer a major problem because an initial core oblateness of order 10^{-2} would have been reduced by little less than an order of magnitude by macroglitches of magnitude $\Delta\varepsilon \sim 10^{-6}$ occurring over a period of 10^4 yr. Moreover, such macroglitches can be regarded as involving only a minute fraction of the equatorial bulge, rather than the appreciable fraction required for models which yield a current oblateness of $\varepsilon \sim 10^{-5}$.

We further note that the presence of a solid core reduces appreciably the structure factor, $Q \simeq I_n/I$, since the solid neutron core will corotate rigidly with the crust and interior charged particles within microscopic times. The fraction of neutron superfluid can easily be 0.15; indeed, an I_n/I of that magnitude is characteristic of neutron stars with a mass $\sim 0.7 M_\odot$.

4. Microglitches

In addition to the overall slowing down – polynomial behavior – and the various macroglitches, there is observational evidence that the Crab pulsar – and, possibly, the Vela pulsar as well – display a ‘restless’ or noisy behaviour, which manifests itself in erratic small variations in arrival times. A detailed analysis of this restless behavior by the Princeton group (Boynton *et al.*, 1972), shows that no reasonably smooth function can fit the corresponding phase residuals over a large observation period. Rather, these behave like shot noise, corresponding to many minute microglitches with possibly both spin downs as well as spin-ups present. From a Fourier analysis of that shot noise, the Princeton group could determine the average value of the rate of jumps (r) times their magnitude squared, $\langle r(\Delta\Omega)^2 \rangle$. It can also be concluded from their analysis, that the noise before the September 1969 macroglitch was larger than the noise after that event; recently Lohsen (1972) found further evidence of that in connection with the October 1971 macroglitch. A shot noise interpretation of the restless behavior explains as well the earlier reported periodicities of the order of months, since shot noise produces apparent periodicities of the order of the time span of the data.

Nelson *et al.* (1970) have interpreted the restless Crab behavior in terms of larger, less frequent, frequency jumps, which are manifestly of both signs; the average value of the small jumps cannot be obtained from the Princeton analysis, since it is lost in the overall slowing down of the pulsar. Further, a grouping of very frequent, small, frequency jumps can occur, to provide the transition from the Princeton interpretation to that of Nelson *et al.* (1970). Clearly, longer periods of observation as well as higher temporal resolution are required to determine definitely the microscopic structure of this restless behavior.

Let us define a microglitch as a frequency jump of either sign of relative magnitude, $\Delta\Omega/\Omega \lesssim 10^{-10}$. In principle most of the macroglitch mechanisms discussed earlier can be scaled down to explain microglitches; to some extent the various anti-macroglitch arguments we have used are buried in the observational noise, in that one can no longer observe postglitch behavior if one has microglitches which take place hourly, or even daily. We consider briefly three ‘new’ microglitch mechanisms: microquakes induced by angular strains in the crust, relaxation of magnetic field stresses, and superfluid ‘vacillation.’

4.1. MICROQUAKES

As shown by Pines and Shaham (1972a), one can expect in general that there is an angular contribution to the elastic energy which arises from a misalignment of the rotation axis and the elastic reference axis. However, a crustquake induced by purely angular strains is, as a rule, smaller than an ‘e’ quake, since it involves a smaller area of the stellar surface (Pines and Shaham, 1972b). Also, it is a spin-down, since it involves a crustal motion which tends to orient the crustal axis towards the direction of the instantaneous axis of rotation. The coexistence of oblateness strains has the effect of producing either spin-ups or spin-downs, depending on the specific geometry of the quake and the relative importance of the two kinds of strains. When misalignment has a much faster characteristic time than that of the slowing down, then most of the time strain is relieved by microquakes; eventually, however, these are ineffective in relieving oblateness strains and a ‘macroquake’ must occur. Detailed considerations show that as a result pulsars will be noisier before a macroglitch than after it.

4.1.1. *Relaxation of Pulsar Magnetic Fields*

If a pulsar is born in a violent event, it is possible that its crust begins to solidify before all of its conducting fluid components have finished moving to maximally relax the stellar magnetic field stresses (Ruderman, 1972). If so, the present crust may sustain considerable magnetic stresses up to $B^2/8\pi \sim 10^{23}$ dyne cm^{-2} . Such stresses might cause a continual crumbling in weaker parts of the crust which could manifest itself in ‘noise’ in the pulsar spin frequency. This model would not account for a reported increase in timing noise before the last major glitch in the Crab.

4.1.2. *Superfluid Vacillation*

After a glitch, angular momentum is transferred to the neutron superfluid in the (observed) relaxation time τ . Because the torque which communicates this to the superfluid core is not spatially uniform, a differential angular velocity is induced in the superfluid in addition to any differential rotation the core fluid may have in its usual spinning down state. The slight increase in angular momentum induced locally by the crust is spread throughout the superfluid by vortex-vortex interactions even though there is no viscosity. (Typical estimates for this so-called Tykachenko-wave angular momentum redistribution suggest a wave velocity of order 0.1 cm s^{-1} .) The added angular momentum is shared among a large number of incommensurate normal

modes of the superfluid which cause it to oscillate back and forth about its average motion as the angular momentum continually redistributes itself (Ruderman, 1972). There is no viscosity and, in the low temperature limit, no way of damping this added differential rotation 'vacillation,' except by rubbing back on the crust whose sudden 'glitch' initially stirred the superfluid. An estimate of the pseudorandom motions of the crust caused by this underlying superfluid vacillation gives amplitudes comparable to those observed in the Crab pulsar. Again, however, this model would not account for an increase in timing noise *before* a macroglitch.

4.1.3. Other Ways of Determining Stellar Structure

We discuss three other possible observational handles on neutron star structure. First, as we have mentioned, energy balance considerations, together with the assumption that the rotational energy of the Crab pulsar is the sole source of power for the Crab nebula, in principle provides a determination of the stellar moment of inertia (and hence the mass). There are two problems (see Ruderman, 1972):

(i) One cannot be certain of the absolute luminosity of the Crab nebula because its distance is not known to within a factor of two or so.

(ii) One is not sure what fraction of the nebular luminosity must be supplied by the pulsar – is it only the energy radiated by the highest energy and the shortest lived electrons in the nebula ($\sim 5 \times 10^{37}$ erg s⁻¹ on the basis of current distance estimates) or does one have to supply as well nebular radiation and the kinetic energy of expansion (which could require as much as 4×10^{38} erg s⁻¹). The best one can therefore conclude is that $I = (3.6 \pm 2.8) \times 10^{44}$ gm cm², which translates into pulsar masses as (Baym *et al.*, 1971)

$$M_{\text{crab pulsar}} \sim (0.8 \pm 0.6) M_{\odot}.$$

One may note that this covers almost the whole range of stable neutron stars!

Obviously, a direct measurement of pulsar mass would be highly desirable. The observation of a wobble of the star (analogous to the Chandler wobble of the Earth), which results from the misalignment of the rotational axis and principle reference inertial axis discussed earlier, provides such a determination under certain circumstances. The wobble frequency is given by (Pines and Shaham, 1972b)

$$\Omega_w = \frac{3}{2} \left(\frac{B}{A+B} \right) \varepsilon_0 \Omega. \quad (19)$$

For a star with a liquid interior (the Crab pulsar?), $\varepsilon_0 \approx \varepsilon \approx I_0 \Omega^2 / 4A$, and one has (Pines and Shaham, 1972a)

$$\Omega_w \simeq \frac{3}{4} \frac{\Omega^3}{\omega_q^2} \quad (20)$$

so that measurement of the wobble frequency provides a direct measurement of ω_q^2 , from which the mass can be inferred from theoretical calculations of B and A . Indeed, it is at least as likely that in this fashion one will be able to determine the distance to

the Crab pulsar (since I is 'directly' determined) as it is that better limits on I can be obtained through better distance determinations of the conventional sort. For a star with a solid neutron core (the Vela pulsar?) on the other hand, observation of Ω_w provides a direct measure of ε_0 , but gives no information on the stellar mass.

Further information on stellar structure is in principle contained in the fine structure of a macroglitch (Lohsen, 1972). For example, if one interprets a macroglitch as a crustquake, the fine structure provides information on a possible sequence of related crustquakes, one acting to trigger the next, in a fashion which may be sensitive to the pulsar mass.

4.1.4. *Observational Tests*

We should like to emphasize once more the importance of carrying out a more detailed analysis of existing timing data on both the Crab and Vela pulsars, in order to obtain the following information:

- (i) The magnitude and timing of successive macroglitches.
- (ii) Post macroglitch behavior (does the two-component theory provide an adequate description?)
- (iii) Nature of the residual 'restless' behavior (frequency noise, larger but less frequent microglitches of both signs, or?).

For the Crab pulsar, all three aspects of the data are interrelated, so that, as we have mentioned, it is non-trivial to make an analysis which distinguishes in unambiguous fashion between the above phenomena. (Indeed, the distinction we have made between macroglitches ($\Delta\Omega/\Omega \gtrsim 10^{-9}$) and microglitches ($\Delta\Omega/\Omega \lesssim 10^{-10}$) is itself an arbitrary one.) For the Crab pulsar, it is in principle possible to combine the results obtained by different groups of observers in order to obtain this information and indeed a start has been made in that direction; however, the present 'state of the art' is such that one cannot yet give a 'Glitch table' which lists the magnitude and time of occurrence of the macroglitches which have taken place since September, 1969 (and it is for this reason that one is not included here). For the Vela pulsar, essentially all the relevant observations have been made by Reichley and Downs with the Goldstone array, and we can only encourage them in the difficult task of data analysis, and hope that an early answer will be provided to the questions we have posed.

Discovery of pulsar wobble would likewise represent a significant forward step. To the extent that one can decide on the existence of a liquid core, through theory or observation, observation of pulsar wobble will provide a direct determination of either the pulsar mass or current oblateness, both quantities of fundamental interest. [It is, we suppose, a measure of the difference between astronomy and particle physics that one does not have six competing teams (and proposals) currently attempting this fundamental observation.]

4.1.5. *Concluding Remarks*

At first sight, two pulsars provide a singularly slender observational base on which to construct an elaborate theoretical superstructure. However, we are fortunate in that

TABLE III
Present status of macroglitch theories^a

Observational Property	Magnitude	Rapid rise time	Sign.	$(\Delta\Omega/\Omega)_{\text{Vela}} \gg (\Delta\Omega/\Omega)_{\text{Crab}} \gg (\Delta\Omega/\Omega)$ and other	Macro-glitch function	Q values	Frequency	$t_g \neq \tau$	Persistence of pulse shape	Explain microglitches as well?	Physically plausible for neutron stars
Theory											
Planets:											
Crab	1	3	2	1	3	2	2	2	1	3	1
Vela	1	3	2	1	2	2	1	2	1	2	1
Magnetospheric instability	2	2	1	2	2	1	1	1	3	1	2
Accretion	2	3	2	2	2	1	2	1	1	2	3
Crab	1	1	1	1	1	1	1	1	1	1	1
Crust quakes:											
Vela	2	1	1	2	1	1	3	1	1	1	1
C. G. Hydrodynamic instability	1	1	2	3	3	3	1	3	1	1	1
Crab	1	1	1	1	1	3	2	1	1	1	1
Core-quakes:											
Vela	1	1	1	1	1	1	1	1	1	1	1

^a 1 ≡ Not inconsistent with observation
 2 ≡ May be inconsistent with observation
 3 ≡ Inconsistent with observation

there is now good reason to believe that the Crab and Vela pulsars correspond to different classes of neutron stars, in that they display different internal structure. We have discussed at some length the various theories for the origin of macroglitches; our discussion may be summarized in tabular form, and such a summary is presented in Table III. At this stage it would seem that crustquakes offer a plausible explanation for the Crab macroglitches, while corequakes can explain the Vela macroglitches; it may be that suitable modification of many of the other theories would render them equally plausible, with the exception of the accretion mechanism which seems out of the question for either the Crab or Vela pulsars.

On the basis of the evidence in at this time (both observational and theoretical) we conclude that the Crab pulsar has a mass $\lesssim 0.5 M_{\odot}$, and is $\sim 90\%$ superfluid neutrons, while the Vela pulsar may well possess a solid neutron core, have a mass of $\sim 0.7 M_{\odot}$, with a stellar superfluid neutron abundance of $\sim 15\%$. It will be illuminating to see whether future observations and theoretical developments confirm this preliminary identification.

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