TESTING THE MODELS AGAINST OBSERVATIONS

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Abstract: Tests of the model atmospheres of red giant stars by photometric observations and angular diameter measurements (including limb-darkening) revealed that the present models reasonably represent the thermal structure of the continuum-forming region of red giant atmospheres. On the other hand, tests by line spectra (including line-blanketed fluxes) revealed that classical model atmospheres of red giant stars cannot yet satisfactorily represent the line-forming region. Recent progress in observations also provide some tests of improved models incorporating non-LTE or sphericity effects. More importantly, recent observations offer the means to test some new approaches such as: hydrodynamical simulation of photospheric convection, modeling atmospheres with thermal bifurcation, and incorporating the effects of outer atmosphere having inhomogeneous structure consisting of warm chromosphere and cool molecular dissociation zone.

1 INTRODUCTION

It is now more than 20 years since initial attempts to study atmospheric structure of red giant stars by means of the model-atmosphere method (e.g., Auman 1967; Tsuji 1967). In these 20 years, computations of model atmospheres for red giant stars have made considerable progress with increasing sophistication in treating numerous molecular lines both in oxygen-rich giants (e.g., Gustafsson et al. 1975; Tsuji 1978a; Johnson et al. 1980) as well as in carbon-rich stars (e.g., Querci et al. 1974; Johnson 1982; Eriksson et al. 1984; Johnson & Yorka 1986). Further, some attempts to relax the assumption of LTE (Auman & Woodrow 1975) or of plane-parallel geometry (e.g., Watanabe & Kodaira 1978; Schmid-Burgk et al. 1981; Kipper 1982; Scholz 1985) have been undertaken. As for details of these developments, see the reviews on model atmospheres of cool stars (Carbon 1979; Johnson 1985,1986).

Now, the more important problem should be how to assess the validity of theoretical model atmospheres by observations. However, what happened actually was not so simple as constructing the models and then making the observational tests. For example, initial attempts to construct model atmospheres of cool stars were largely motivated by the progress of infrared astronomy in the early 1960's. Also, in later developments of our studies of red giant stars, what actually happened more often was that observations preceded and perhaps motivated the models. Thus, testing the models against observations is not a simple one-way process; rather, it is a complicated process of successive

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iteration between observations and models, by which our understanding of red giant stars could be successively improved.

Major observational tools for testing theoretical model atmospheres of red giant stars are (1) photometry, especially in the infrared, and (2) angular diameter measurements. Initial tests of model atmospheres on these points provided a rather optimistic view of the validity of the classical model atmospheres of red giant stars (Sects. 2 & 3). On the other hand, the situation appeared to be not so optimistic once line spectra were examined, either by line-blanketed fluxes (Sect. 4) or by high resolution spectra (Sect.5). Thus, it is still difficult to have a unified understanding of the line-forming region on the basis of the present models that were so successful in understanding the continuum-forming region. Inspection on such tests, however, reveals that observations and theories consistently indicate new pictures of atmospheres of red giant stars (Sect.6).

The subject of testing models has already been discussed as a part of more general reviews on model atmospheres of red giant stars (e.g., Carbon 1979; Johnson 1985, 1986), or in connection with problems of determining fundamental stellar parameters (Gustafsson & Jorgensen 1985) or stellar abundances (Lambert et al. 1986; Gustafsson 1989). Thus, we emphasize more recent results in this review. Also, we concentrate on models of photospheres of cool giant stars (M, S, and C types), except when problems related to chromospheres, circumstellar envelopes, or G-K giants strongly impact our main subject. Also, problems related to Mira variables may be discussed by other reviews in this conference.

2 SPECTRAL ENERGY DISTRIBUTION AND ANGULAR DIAMETER

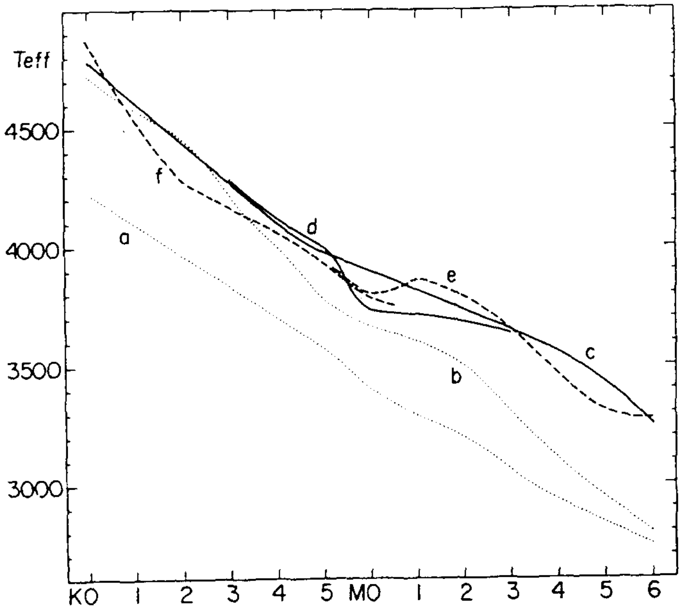
A classical model atmosphere is characterized by 4 parameters; effective temperature (T_{eff}), surface gravity (g), turbulent velocity (v_{tur}), and chemical composition. If the effect of spherical geometry is important, T_{eff} and g should be replaced by more fundamental parameters - mass (M), radius (R), and luminosity (L). For testing a model by observations, however, it is still convenient to define T_{eff} for spherical models as well. As the thermal structure of a stellar atmosphere is primarily determined by T_{eff}, it is natural that the correct assignment of T_{eff} is the first step in testing a model atmosphere by observations. Also, a comparison between T_{eff} determined by direct methods based on measured angular diameters and that determined by indirect methods based on analysis of photometric observations can be regarded as a test of a model atmosphere, since indirect methods depend largely, but indirect methods depend little, on model atmospheres.

2.1 Oxygen-Rich Giants

A test of model atmospheres by the method noted above can best be illustrated by a well-studied bright giant star α Boo. While recent analyses of flux distribution by model atmospheres (FDM) provided rather high temperatures of 4410 ±88 K (Blackwell & Shallis 1977), 4462± 33 K (Augason et al. 1980), or 4375 ± 50 K (Frisk et al. 1982), some angular diameter measurements suggested very low values near 4000 K (see e.g., Trimble & Bell 1981). This confused situation seems to be well resolved by two recent papers. First, Di Benedetto & Foy(1986), after careful analysis of the bias affecting the visibility curve obtained by Michelson interferometry with two telescopes (I2T) in the near infrared, showed that direct determination of effective temperature with accuracy better than 1% is now feasible and that T_{eff} = 4294 $\pm$ 30 K for α Boo. Second, Blackwell et al. (1986) showed that an indirect method based on the infrared flux method (IFM) also provides a value of the effective temperature with an accuracy of about 2% after careful evaluations of photometric data, infrared calibration, and line-blocking effect; the result is T_{eff} = 4230 $\pm$ 80 K or 4307 $\pm$ 80 K, depending on whether absolute flux calibration of the standard star Vega is by direct comparison with a standard furnace or by a model atmosphere, respectively.

Similar tests of model atmospheres by comparisons of direct and indirect effective temperatures have also been done for cooler stars extending to late M giant stars. An initial attempt in this direction revealed that T_{eff} by FDM only showed reasonable agreement with the upper estimates of T_{eff} based on angular diameters then available, and this fact was interpreted as due to some biases not well recognized in the angular diameter measurements (Tsuji 1978a). This fact also suggested a possibility that the effective temperature scale of M giants should be revised upward against the one used before (e.g., Johnson 1965). Extensive analysis on angular diameters from lunar occultation by Ridgway et al. (1980) provided conclusive evidence of the general increase in effective temperatures of M giant stars between M0 and M6. Meanwhile, T_{eff} by indirect method based on FDM (Manduca et al. 1981) as well as on

Fig.1. Effective temperature scale for K and M giants. Dotted lines recall two historical scales: a (Kuiper 1938) and b (Johnson 1965). Solid lines are those based on direct measurements of angular diameters: c (Ridgway et al. 1980) and d (Di Benedetto & Rabbia 1987). Dashed lines are those based on model-atmosphere analysis (the infrared flux method): e (Tsuji 1981a) and f (Leggett et al. 1986).



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IFM (Tsuji 1981a) showed agreement within 100 K with the scale of Ridgway et al. (Fig.1). More recently, the direct method based on I2T in the near infrared provided effective temperatures with internal accuracy of 61 K for a large number of red giant stars cooler than K0 (Di Benedetto & Rabbia 1987), and this demonstrated a new possibility of the interferometric technique for measurements of stellar angular diameters; with such a high accuracy in angular diameter measurement, previous determinations of T_{eff} are confirmed in general (Fig.1), and, further, any possible bias in model atmospheres can now be investigated. For example, systematic deviation of the results by FDM or by IFM from those by interferometric measurements suggests that the cross-section of H^- opacity should still be improved. For such a purpose, however, higher accuracy in photometric data would be required.

2.2 Carbon Stars

For cool carbon stars, T_{eff}'s by the direct method (Ridgway et al. 1977) and by IFM (Tsuji 1981b) show reasonable agreement, and carbon stars turn out to be cooler than M giant stars. Such a recent temperature scale of cool carbon stars has been shown to be fairly consistent with various temperature indicators including infrared colours, band strengths of polyatomic molecules, and molecular excitation temperatures by Lambert et al. (1986), and it is expected that a more detailed analysis of the temperature scale based on their new models will be carried out. Especially, the effective temperature scale of cool carbon stars showed inverse correlation with the temperature class of the C-classification by Keenan & Morgan (1941), which is essentially related to excitation temperature. Such a contradiction between the effective temperature scale and the excitation temperature scale may reflect some serious model effect due to heavy line-blanketing by molecular bands (Tsuji 1985). To resolve the issue, empirical determination of excitation temperatures in cool carbon stars may also be useful.

3 STELLAR SURFACE BRIGHTNESS DISTRIBUTION

For testing of stellar structure, observations of limb-darkening or, more generally, brightness distribution over the stellar disk should be more informative than the single angular diameter measurement. In fact, recent progress in observations with high spatial resolution provides some interesting possibilities.

3.1 K-M Giant Stars

For the K5 giant star α Tau, Ridgway et al. (1982b) have obtained multiple wavelength measurements between 0.4 and 4.0 μm by the lunar occultation technique, and showed that the apparent diameter which is determined by a uniform-disk-model fit to the observed occultation diffraction pattern increases toward longer wavelength. This result showed reasonable agreement with the angular diameter variation based on predicted limb darkening by Manduca et al. (1977) for models of red giant stars by Gustafsson et al. (1975), although the observed variation with wavelength may be stronger than the predicted one. Thus, the limb-darkening effect as predicted by model atmospheres, or possibly somewhat greater, is confirmed for a red giant star. Also, angular diameters

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measured in the optical region by lunar occultations for α Tau (White & Kreidl 1984) and for μ Gem (Ridgway et al. 1977) show excellent agreements with those by Michelson interferometry in the near infrared, if limb-darkening effects are corrected (Di Benedetto & Rabbia 1987). This fact in turn confirmed the limb-darkening effect predicted by model atmospheres.

For other M giant stars, multichannel measurements by the lunar occultation technique also suggested wavelength dependence of angular diameters: an 20% decrease from 0.9 to 1.7 μm, followed by a slight rise at 2.2 μm (Ridgway et al. 1982a). This initial result was confirmed by more extensive observations, but the ratio of angular diameters at 2.2 μm to that at 1.7 μm is not a monotonically increasing function of spectral type (Schmidtke et al. 1986). Such an observation is difficult to understand by an analysis based on spherically extended models (Scholz & Takeda 1987), which showed that the angular diameter at 1.7 μm is a good measure of the photospheric radius defined by the continuum but that interpretation of monochromatic angular diameters are difficult when molecular bands appear in the higher level of photosphere so that the brightness distribution shows an extended halo in the outer disk.

Based on lunar occultation data, Bogdanov & Cherepashchuk (1984) tried to derive brightness distribution over the disk of μ Gem. It is shown that the effect of brightness distribution on the lunar occultation diffraction pattern is rather subtle for this star, and requirements for resolution and S/N ratio are quite severe. The conclusions so far reached by them are: the plane-parallel model with limb-darkening coefficient $u > 0.7$ cannot be excluded, and atmospheric extension of this star is rather small, if any, consistent with the extended spherical model atmospheres such as by Watanabe & Kodaira(1979).

3.2 Carbon Stars

Radial brightness distribution over the disk of the carbon star TX Psc has been estimated from the lunar occulation data by the same method as used for μ Gem (Bogdanov,1979): TX Psc may consist of central stellar disk and an extended halo which is more prominent than in μGem.

3.3 M Supergiant Stars

The M supergiant star α Ori has been well observed by interferometric techniques since its stellar angular diameter was first measured by Michelson & Pease (1921). However, the increase of observational data at first increased the discrepancy with model-atmosphere predictions; the observed angular diameters tended to be larger than the photospheric angular diameter based on model analysis of photometric data and also to be larger at shorter wavelength, in contradiction to the expectation from limb-darkening. Also, angular diameters measured at TiO bands tended to be larger than those at continuum bands, and the lower effective temperature was consistent not only with a larger diameter but also with the TiO-related variations of the measured diameters (Belega et al. 1982). Further, a possibility of time variation in the photospheric radius has been suggested (White 1980).

More recent observations based on speckle and other interferometric techniques finally overcame these contradictions by resolving the central stellar disk against the extended halo in α Ori. For example, Ricort et al. (1981) have proposed a model consisting of core (diameter 35 ± 10 mas) and halo (diameter 100 ± 40 mas) for α Ori from their speckle data. Also, Roddier & Roddier (1983, 1985) showed evidence, from their two-dimensional interferometric image reconstruction, for a relatively small stellar disk with angular size of 37 mas and a highly asymmetric circumstellar envelope. Further, Cheng et al. (1986) showed that multi-wavelength speckle data can be interpreted as indicating a central stellar disk of 42.1 ±1.1 mas, if a bias possibly induced by the extended halo, between 1 and 5 R_*, is accounted for. Also, these authors showed that wavelength-dependent limb-darkening, whose coefficients turned out to range from u= 0.9 in strongly line-blanketed spectral regions to u=0.4 in less blanketed regions, are consistent with model prediction of plane-parallel atmospheres (Tsuji 1976). The same speckle data have been reanalyzed by a refined algorithm of high spatial resolution imaging by Christou et al. (1988), who showed that the recovered images of α Ori can be resolved into two Gaussian components which represent the disk and extended components. A mean value for the widths of the Gaussian fits to the disk component at 6 spectral band-passes was found to be 19.8 ± 0.7 mas. Observed apparent radii, however, decreased towards longer wavelengths for the 4 measurements of continuous fluxes, and this is again opposite to what can be expected from the limb-darkening effect. On the other hand, the remaining 2 apparent diameters measured at chromospheric lines around 6563 and 8542 A turned out to be substantially larger than the continuum diameters. Another component, the extended low power halo, could be found at Hα light, as is already known from the detailed analyses of the extended chromosphere by Hα imaging (Hebden et al. 1986, 1987). Furthermore, the extended halo has been found at all the continuum wavelengths out to a distance of 4 R_* in the recovered images.

The anaylses reviewed above consistently showed that the angular diameter of the stellar disk of α Ori is relatively small - in the range between 35 and 41 mas - and this confirms the rather high effective temperature of about 3900 K based on model atmosphere analyses (Tsuji 1976; Vieira 1986). At the same time, the presence of an extended halo around α Ori is now well established, and the problem is to decide what kind of model could explain this finding. A possible model that relies upon dust to provide the scattering to explain the rather large apparent angular diameter of α Ori has been proposed before (Tsuji 1978b; White 1980). Recently some observational evidences for dust condensation near the stellar photosphere have been found (e.g., Roddier & Roddier 1985; Roddier et al. 1986), and the possibility of dust formation in the stellar chromosphere by the so-called condensation instability has been proposed (Stencel et al. 1986). On the other hand, the main objection to such a model is recent observations on the possible presence of a dust-free region out to 10 R_* in α Ori by infrared interferometry or by infrared imaging (e.g., Bloemhof et al. 1984). If the dust model for extended halo cannot be applied, the only possibility remaining appears to be gaseous particle that may scatter or re-emit stellar radiation. Generally, models of the outer atmosphere of α Ori assume a rather low density of $n_H \simeq 3\ 10^{+9} cm^{-3}$ at $r \simeq 1.1 R_*$ (e.g.,

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Hartmann & Avrett 1984; Skinner & Whitmore 1987), and the hydrogen column density in the extended halo is roughly $N(H) \simeq 10^{+23} cm^{-2}$ if $R_* \simeq 600 R_\odot$. Then, with a cross-section of $\sigma \simeq 7.4\ 10^{-29} \lambda(\mu m)^{-4}$ for Rayleigh scattering by hydrogen atoms, the scattering radial optical depth is roughly, $\tau = \sigma N(H) \simeq 10^{-4}$ at $\lambda = 0.5\ \mu m$. This is far short of explaining the observed halo. Also, if H^- b-f emission is considered instead of scattering, the radial optical depth of the halo is still smaller. Note that H^- b-f emission may have some effect on the integrated flux if a rather high density is assumed in the thin transition layer very close to the stellar surface (e.g., Skinner & Whitmore 1987), but this cannot explain the spatially resolved extended halo. Thus, if a small amount of dust does not exist in the inner halo of the outer atmosphere, a larger amount of gas must exist in the inner halo than that assumed in the present models for the outer atmosphere of α Ori.

4 LINE-BLANKETED FLUXES AND COLOURS

Once effective temperature - one of the major parameters that specify a model atmosphere - is well settled (Sects. 2 & 3), and global thermal structure of the photosphere is confirmed, one may expect that line-blanketed fluxes and colours could provide further tests of model atmospheres. While such tests provide positive results for models of G-K giant stars, many problems remain for those of cooler giants.

4.1 Oxygen-Rich Giants

For G-K giant stars, line-blanketed colours predicted from model atmospheres have been tested againt observed colours by various photometric systems for stars with well defined fundamental parameters, and the results showed fine success in general, except for minor discrepancies in the UV or in strong molecular bands (Gustafsson & Bell 1979). Also, the predicted surface brightness-colour relations such as F_V - (V-R) agree very well with those observed for $6000 > T_{eff} > 4000$ K (Bell & Gustafsson 1980).

Some attempts in the same direction have been made for cooler giant stars, including late K and M giants. Piccirillo et al. (1981) showed that the empirical relation between colour temperature and effective temperature can be well reproduced by considering the effect of weak TiO bands in predicting the near-infrared colours defined by Wing (1971). Recently, Steiman-Cameron & Johnson(1986) showed that agreements between predicted and observed colours are qualitatively good in general but not quantitatively, partly because of the difficulty of defining zero-points of photometric systems. Especially, these authors showed that the predicted TiO bands are systematically too weak relative to observations for models with T_{eff} below 3500K, and suggested it might be due to sphericity effect not accounted for in their models. In fact, it is known that the TiO absorption is stronger in models with larger atmospheric extension (Watanabe & Kodaira 1979) and, further, this fact can be used to measure the geometrical extension as a new classificaton parameter besides T_{eff} and g (Scholz & Wehrse 1982). Such a proposition is still to be tested by observations, for which it is essential to find stars with known fundamental parameters such as mass, radius, and luminosity, since TiO is already known to be sensitive to temperature. Also,

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cool giants having weak TiO bands for their color tend also to show weaker Mg II fluxes than stars of similar color (Steiman-Cameron et al. 1985), and this may indicate that other factors not yet considered above could be important in testing models by TiO.

Other examples of molecular bands that remains difficult to understand are the CO vibration-rotation bands in the spectrophotometric scans observed by Strecker et al. (1979); predicted strengths of the fundamental bands appear to be too weak compared to observed ones while predicted overtone bands appear to be stronger than those observed (Manduca et al. 1981). It is noted that this difficulty appears already in late K and early M giants. It is not clear if the same problem appears in cooler giants in the analysis of similar observational data by Scargle & Strecker(1979), since observed data are rather forced to fit predicted ones by adjusting several parameters that are not necessarily checked by other observations. However, we will see the same difficulty of understanding CO line strengths in an analysis of the high resolution spectra (Sect.5.3).

4.2 Carbon-Rich Stars

So far as diatomic opacity sources are concerned, predicted line-blanketed fluxes (Querci & Querci 1974) and observed infrared fluxes (Goebel et al. 1980) show relatively good agreement. However, the nature of opacity sources, especially of polyatomic molecules, in cool carbon stars is not yet fully understood. For example, atmospheric structure of cool carbon stars is drastically changed by polyatomic opacities, especially if the veil opacity due to many combination bands of HCN are included (Jorgensen et al. 1985), and a large reduction of gas pressure in the surface layers suggested a possibility of large extension of the outer layers (Eriksson et al. 1984). On the other hand, a preliminary computation of spherical models, including HCN opacity based on transitions confirmed in the laboratory and by observation (Ridgway et al. 1978) revealed that the atmospheric extension in carbon stars may be rather modest as compared with that in M giant stars (Scholz & Tsuji 1984). Detailed observational tests of model atmospheres based on polyatomic molecular opacity including the veil opacity were done by Lambert et al. (1986): the predicted depression due to HCN absorption as a whole is too strong compared with that observed, while predicted strengths of the individual lines of the fundamental bands of HCN identified in the spectrum of TX Psc show reasonable agreement with those observed. This fact may suggest that the veil opacity was over-estimated, but other possibilities suggested are: departure from LTE, heating of the surface layers by mechanical energy flux or by dust formation, errors in adopted effective temperature scale, etc. (see also Jorgensen, this conference).

Another problem associated with opacity is the origin of the violet depression in cool carbon stars, a classical interpretation of which was to assume a pseudo-continuum due to C_3 or grain opacity due to SiC. Recently, this problem has been reexamined in detail by new photometric data, including IUE data, and the two previously suggested opacity sources, C_3 and SiC, are both shown to be inadequate (Faulkner et al. 1988). Also, recent measurements of the Balmer decrement in Mira type carbon stars revealed that the the source of the violet opacity may be located deep in the photosphere (Orlati 1987). Finally, it is shown that the violet opacity in cool carbon stars is due to the cumulative effect of several sources, primarily b-b transitions (including the Mg I resonance line) and b-f opacities from low lying states of neutral metals, with partial contribution of CH photo-dissociation and C_3 pseudo-continuum (Johnson et al. 1988). These authors also noted that the violet flux deficiency in cool carbon stars could have been over-estimated and may largely be a temperature effect, if effective temperatures of carbon stars are lower than those of M giant stars (Sect.2).

5 LINE SPECTRA

Some of the difficulties encountered in testing model atmospheres of red giant stars by line-blanketed fluxes (or colours) may suggest the necessity of detailed analyses of line spectra. However, detailed analyses of line spectra for testing atmospheric structure of cool giant stars is rather meager, because most of these analyses have so far been done in connection with chromospheric and circumstellar problems. This may largely be due to the intrinsic difficulty of measuring line profiles accurately in the spectra of cool stars because of the intrinsic blending and of the uncertain continuum level. Such difficulties can partly be removed by the recent progress in high resolution and high quality spectroscopy, especially in the infrared.

5.1 Atomic Lines

One problem in using atomic lines as probes of atmospheric structure is that atomic line spectra may not always be interpreted under the assumption of LTE. For example, considerable departures from LTE in ionization equilibrium have been predicted from model analyses by Auman & Woodrow (1975). The predicted over-ionization may be consistent with observations for red supergiant stars (Ramsey 1981), but may be over-estimated for red giant stars if the result of Ramsey (1977) is reinterpreted by a more recent effective temperature scale (Sect.2) which is revised upward compared with that used by Ramsey (1977).

Observational evidence for pronounced departures from LTE in line formation was shown for β Gem (KOIII) by Ruland et al. (1980). Abundances of iron-peak elements determined by LTE analysis of ionized and high-excitation neutral lines appeared to be normal while those by similar analysis of low-excitation neutral lines appeared to be deficient by about 0.3 dex. A similar effect, indicating substantial departure from LTE, was noted in the Zr/Ti abundance ratio for a larger sample of G-K giant stars (Brown et al. 1983). A physical explanation of such observations was offered by Steenbock (1985), who has solved detailed statistical equilibrium equations of Fe I/Fe II in β Gem and showed that non-LTE abundance corrections can be as large as 0.2 dex. The observed effect in β Gem is consistent with such a model, even though the observed effect is somewhat larger than predicted.

Similar analyses, both theoretical as well as observational, on the possible departures from LTE would be highly desirable for cooler giant stars. At present, quantitative analyses of atomic lines (e.g., for abundance determination) in cooler giants are usually done differentially with respect to some standard stars such as α Tau to minimize the non-LTE effects (e.g., Smith & Lambert 1986). In this connection, a de-

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tailed quantitative analysis of the high resolution infrared spectrum of α Ori based on solar f-values by Vieira (1986) is quite interesting. The analysis is self-consistent within the framework of quantitative stellar spectroscopy based on classical model atmospheres. Yet the result shows a surprisingly low metal abundance for this typical population I supergiant star. Whether this is due to a real abundance anomaly or whether this is due to some serious model effect deserves further study.

5.2 H_2 Molecule

A possible importance of the quadrupole transitions of H_2 molecule for testing atmospheric structure of cool stars was well recognized at the infancy of infrared spectroscopy (Spinrad 1966; Lambert et al. 1973). So far, however, clear identification of H_2 quadrupole lines due to fundamental VR transition near 2.4 μm is limited to S type stars (Hall & Ridgway 1977) and cool carbon stars (Johnson et al. 1983). The H_2 1-0 S(0) line in a large sample of cool carbon stars has been used to test model atmospheres of carbon stars (Lambert et al. 1986). Models with polyatomic molecular opacity (including veil opacity) and with a certain range of the C/O ratio reproduce well the observed H_2 intensities for the entire range of effective temperatures of cool carbon stars, while models without polyatomic molecular opacities predict H_2 intensities much stronger than observations. The major reason for this difference is that the high opacity due to HCN lowers the gas pressure in the upper layers where H_2 lines are formed. However, one remaining problem is that the HCN veil opacity, which makes the gas pressure so low, has not yet been confirmed by observation (Sect.4.2). Although this problem should still be settled, the analysis of Lambert et al. (1986) showed a possibility of explaining the observed H_2 intensity without an assumption of hydrogen deficiency in cool carbon stars - a problem that was examined in detail by Johnson et al. (1985).

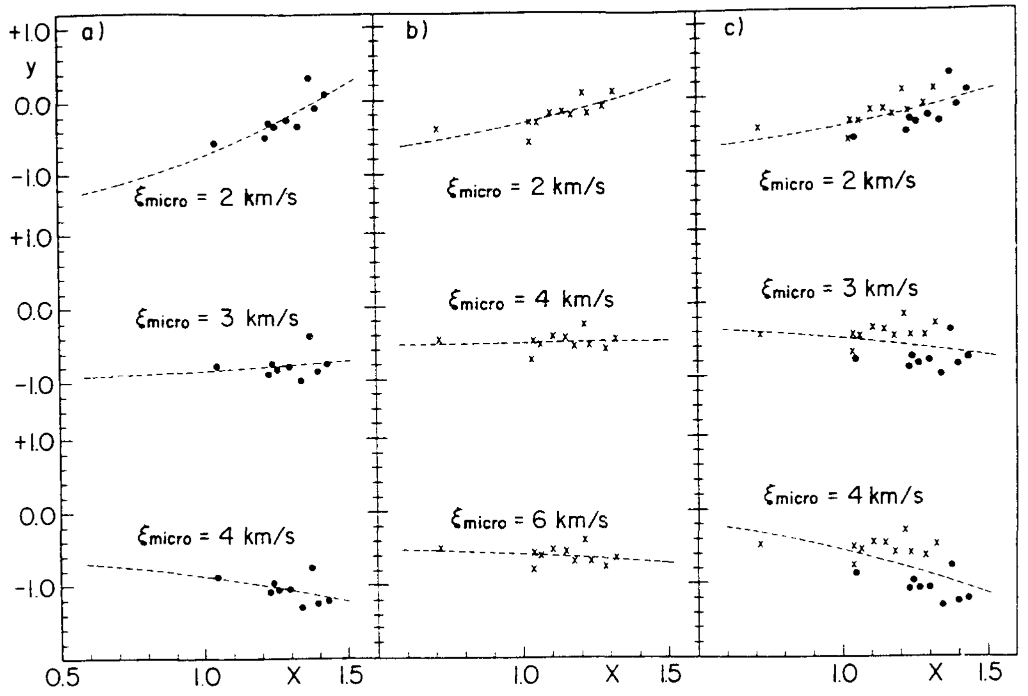
For M giant stars, the H_2 1-0 S(1) line is not clearly identified and the observed strengths are anyhow appreciably less than those predicted by the present model atmospheres (Tsuji 1983). This may suggest that temperature may be hotter (including the possibility of a chromosphere or inhomogeneity; see Sect.5.3) or gas pressure may be lower in the surface layers of actual stars than in present model atmospheres.

5.3 CO Molecule

Observed strengths of molecules other than H_2 depend not only on atmospheric structure but also on chemical abundance. Thus, a single line cannot be used for the testing of model atmospheres, but quantitative analysis of a large number of lines covering the range from weak to strong lines could provide a useful test of atmospheric structure from deep to outer layers. For this purpose, CO vibration-rotation bands, which consist of many measurable transitions, may be the best candidate, since the assumption of LTE is well warranted for the first and second overtones (Carbon et al. 1976), and molecular data such as oscillator strengths are relatively well known (e.g., Chackerian & Tipping 1983). Actually, abundance analysis based on CO VR lines can also be regarded as such a test, since abundance determination is based on the assumption that equivalent widths of different transitions with different strengths can consistently be understood by a model atmosphere with well defined parameters such as T_{eff}, g, v_{tur}, and chemical composition.

Such an analysis of CO has been done for oxygen-rich (Smith & Lambert 1985, 1986; Tsuji 1986) as well as for carbon-rich (Lambert et al. 1986) giants. Each of these results appear to be internally consistent, if low excitaton strong lines are excluded. For example, medium strong lines of the first overtone bands at 2.3μ m can consistently be understood by a unique set of parameters to be interpreted as carbon abundance and turbulent velocity; an example of such an analysis is shown in Fig.2a, from which we obtain log A_C =7.68 (by standard notation with log A_H = 12.00) and ξ_{micro} = 3.20 km sec^{-1}(Tsuji 1986). However, for several M giant stars in common with the analysis by Smith & Lambert (1985,1986), the differences in the resulting carbon abundance are as large as 0.6 dex (e.g., 30 g Her). To trace the origin of this discrepancy, the second overtone bands are analyzed by the same method (Fig. 2b), and also the first and second overtone bands are analyzed together (Fig.2c), as was done by Smith & Lambert (1985). The results from the second overtone alone are log A_C = 8.01 and ξ_{micro} = 4.28 km sec^{-1} while

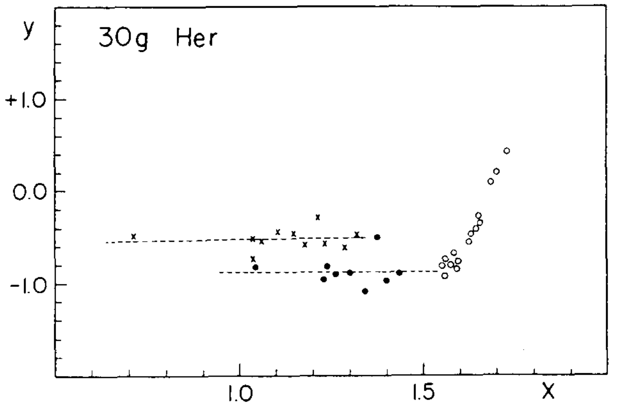
Fig.2. Abundance parameter y = log(C/H)+3.45 obtained from CO lines with reduced equivalent width of x = log(W/ν)+6.0 for several assumed values of the micro-turbulent parameter ξ_{micro}, by using a model atmosphere with T_{eff} = 3200K, log g=0.0 and v_{micro} = 3.0km sec^{-1} for 30 g Her (M6III). The analysis is based on high resolution infrared spectra by 4m FTS of Kitt Peak National Observatory (NOAO). (a) CO first overtone band (excluding low excitation strong lines); (b) CO second overtone band; and (c) CO first and second overtone bands together.



those by the first and second overtones are log A_C = 8.04 and ξ_{micro} = 2.46 km sec^{-1}. The resulting abundance parameters agree rather well with each other and with the result by Smith & Lambert (1985), but differ appreciably from the result by the first overtone alone (Fig.2a). Probably, micro-turbulent velocity may be more difficult to determine from weak lines of the second overtone alone than from saturated lines of the first overtone, and more detailed analysis should be needed before the difference of the turbulent velocities due to different bands can be confirmed. This fact also reveals that the nature of turbulence in M giant stars cannot yet be regarded as well known, whereas the distribution of turbulent velocities in G-K giant stars is rather well established (e.g., Gray 1982). The turbulent velocity determined from the combined analysis of the first and second overtone bands now shows good agreement with that by Smith & Lambert (1985, 1986), but this result is also open to question, since this is biased by the systematic effect that is revealed by the separate analyses of the first and second overtone bands.

Now, it is evident that the first and second overtone bands show drastically different results in abundance and turbulent velocity or, in other words, the first and second overtone bands of CO cannot be understood consistently by a unified model within the framework of classical atmosphere. More or less similar differences in the derived parameters from the first and second overtone bands have been found in some 20 M giant and supergiant stars while such an anomaly did not appear in K giant stars. This later result implies that the difficulty just found for M giant stars cannot be due to systematic error in the absolute scale of f-values of CO, which is accurate to within 10% (Tipping 1988). Other possible sources of such difference can be systematic differences in line blending, in continuum location, or in unrecognized opaci-

Fig.3. The result of the analysis by Fig.2 (notations have the same meanings as in Fig.2): CO second overtone (cross) gives log A_C =8.01 and ξ_{micro} =4.28 km sec^{-1} while CO first overtone (filled circle) gives log A_C =7.68 and ξ_{micro} = 3.20 km sec^{-1}. The low excitation strong lines of the CO first overtone (open circle) give an unreasonably large abundance parameter, indicating the presence of excess absorption that cannot be accounted for by the photospheric model.



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ty sources. Such a difference in CO abundances from the first and second overtones has also been noted in cool carbon stars with $T_{eff} < 2900K$ (Lambert et al. 1986) and has been attributed to poorly determined continua in cooler stars. In M giant stars, such a difference already appears in early M giants, and a similar inconsistency also appears in photometrically calibrated line-blanketed fluxes which can be analyzed independently of continuum level (Sect.4.1). Thus, there is a possibility that the contradictory results from the first and second overtone bands may imply a serious difficulty for the classical model atmospheres in the line forming layers. Probably, the weak lines of the second overtone are fairly well represented by classical models and may give the best estimate of the carbon abundance. The apparent decrease of carbon abundance, as revealed by the high excitation lines of the first overtone, reminds us of the case of the Sun in which CO lines gave carbon abundance about 0.3 dex less than that implied by other indicators (Tsuji 1977; Lambert 1978). This result was interpreted as due to inhomogeneity in the surface layers of the Sun, and recent observations of CO with high spatial resolution confirmed such a possibility (Ayres et al. 1986). Now, it is interesting to consider a similar possibility for M giant stars.

Further, strong low excitation lines of the CO first overtone bands, which have not been used in the analyses in Figs.2a–c, show excess absorption that cannot be explained by the photospheric models. As shown in Fig.3, carbon abundances derived from low excitation CO lines of the first overtone (open circle) turn out to be much larger than those from high excitation CO lines of the first overtone (filled circle; from Fig.2a) and those from CO lines of the second overtone bands (cross; from Fig.2b). Also, the low excitation CO lines show shifts and asymmetries that indicate excess absorption in the blue wing in some stars and in the red wing in other stars. Thus, it may be reasonable to assume that the excess absorption originates from an extra molecular layer in the outer atmosphere distinct from the stellar atmosphere, and the possible presence of a quasi-static molecular formation zone in the outer atmosphere of red giants has been suggested (Tsuji 1988a). Such a possibility can also be inferred for cool carbon stars from the presence of excess absorption in low excitation CO lines (Dominy et al. 1978) and from the distinct radial velocity difference of low excitation CO lines relative to high excitation lines (Lambert et al. 1986). Also, such an excess absorption cannot be the effect of such photospheric structure as a sphericity effect because of the different kinematical behaviours of the excess absoption relative to the photospheric spectrum. The effect of geometrical extension is not very clear in the predicted spectrum of CO lines based on a spherical model for α Her (Höflich et al. 1986) since other effects, such as those due to fundamental parameters, are not well separated.

At present there is no actual model to be tested by observations outlined above, but some interesting theoretical ideas have already been proposed. A possible origin of inhomogeneity or bifurcation in the solar upper atmosphere through efficient CO cooling in the presence of mechanical heating was suggested by Ayres (1981). Further, Kneer (1983) suggested that the atmospheres of cool stars might be destabilized by the radiative instability due to the high temperature

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sensitivity of CO and other molecular formation. Under such CO cooling, radiative equilibrium models are still obtained by a time-dependent numerical simulaton, but the resulting models have a bistable character in that they are distinctly divided into hot or cool models depending on effective temperature (Muchmore & Ulmschneider 1985), or they have an even more distinctively bistable character in that two different temperature structure can be possible in the same model for limited effective temperature range (Muchmore 1986). Further, it was suggested that the similar cooling instability due to SiO formation may induce autocatalytic molecular formation in cool envelope of red giant stars (Muchmore et al. 1987). These works provide a theoretical basis for the possible presence of inhomogeneities in stellar surface layers and a molecular formation zone in the outer atmospheres of red giant stars.

Now, an interesting possibility may be to assume that the high excitation lines of the CO first overtone in Fig.3 originate from upper layers with bifurcated structure. More recently, actual application of such an idea to Arcturus has been made by Cuntz & Muchmore(1988), who showed that an atmosphere with a hot chromospheric component and a cool molecular component is generated, depending on the strength of the shock due to acoustic waves. This result lent support to the suggestion based on high resolution spectra of the CO fundamental band that the upper atmosphere of Arcturus should be composed of hot and cool components (Heasley et al. 1978). Although CO first overtone bands revealed no marked anomaly in Arcturus, possibly because they are not yet as strong as in M giant stars, the first overtone bands in cooler M giants may already be strong enough to show some effects of a bifurcated atmosphere that appear only in CO fundamental bands in Arcturus.

Further, the identification of the quasi-static molecular dissociation zone in the outer atmosphere of red (super)giant stars, as revealed by the low excitation lines of CO first overtone bands, can be regarded as an observational support to the theory of molecular formation by a thermal instability (Kneer 1983; Muchmore et al. 1987) since otherwise it is difficult to explain the presence of a rather large abundance of CO in the outer atmospheres of red (super)giant stars. As the excitation temperatures of the molecular dissocation zone are relatively high - between 1000 and 2000 K (Tsuji 1988a), it may be located near the photosphere. Also, the fact that the molecular dissociation zone produces distinct absorption indicates that it may not be so extended as to produce emission. However, as emissivity at 2.3μm decreases rapidly at about 1300 K, it is possible that the cooler part of the molecular dissociation zone is significantly extended. For Betelgeuse, the CO column density of the molecular dissociation zone is 10^{+20}cm^{-2} and, with C/H ratio of 2.5 10^{-4} (Lambert et al. 1984), the hydrogen column density is 4 10^{+23}cm^{-2} or the mass column density of the molecular component of the outer atmosphere is 0.4 g cm^{-2}. This is probably a lower-limit estimate, since possible emission from the extended part of the CO dissociation zone may have filled in the absorption. For comparison, the mass column density of the hot component of the chromosphere deduced from Ca II emission is 4.3 g cm^{-2} (Basri et al. 1981).

Finally, the development of inhomogeneities in surface layers and in outer atmosphere may have its root in the convective zone, and granular convective cells themselves will appear as temperature inhomogeneities in red (super)giant stars (e.g., Schwarzschild 1975). An attempt to model the granular convection in the solar atmosphere has already been done (Nordlund 1985) and tested by detailed analysis of line asymmetries and shifts (Dravins et al. 1981). While detailed theoretical models are not yet available for red giant stars, empirical approaches to such a model have already been done by an analysis of infrared CO lines whose bisectors showed a systematic red shift in the line wings, in consistency with a granular convective model (Ridgway & Friel 1980), or by measuring differential shifts of CO lines relative to some Fe I lines, which show a dependence on excitation as expected for convective model (Nadeau & Maillard 1988).

5.4 Some Other Molecules

OH: Strong lines of VR bands show much larger strengths than those expected from model atmospheres in a curve-of-growth analysis for Betelgeuse, and this result was interpreted as due to anomalous atmospheric structure or of depth dependent turbulent velocity (Lambert et al. 1984). More recently, pure rotation lines of OH in the mid-infrared have been observed in Betelgeuse (Jennings et al. 1986); the measured equivalent widths show better agreement with predictions based on a semi-empirical model (Basri et al. 1981) than those based on a classical model atmosphere (Johnson et al. 1980).

TiO: This molecule is highly sensitive to temperature and could be used to test the surface layers of model atmospheres, once basic parameters such as T_{eff} are well established. Such a test on models of Arcturus and Aldebaran showed that the best agreement with observations can be obtained for models having boundary temperatures close to the empirical values (Hänni & Sitska 1986). To resolve the problems that appeared in colours blanketed by TiO (Sect.4.1), similar analyses by high resolution spectra are desirable.

MgH: Generally, hydrides are known as good luminosity criteria and hence can be indicators of surface gravity. This idea was applied to MgH in Arcturus (Bell et al. 1985); the surface gravity estimated from MgH showed good agreement with those by pressure broadening of strong metal lines and by FeI/FeII ionization equilibrium. Other problems related to spectroscopic determination of stellar mass have been reviewed by Trimble & Bell (1981) with special reference to Arcturus. Unfortunately, determination of surface gravity is more difficult in cooler giants than in Arcturus.

6 CONCLUDING REMARKS

A brief survey of major observational tests revealed that red giant atmospheres are far more complicated than those described by available models. In fact, positive confirmation of present model atmospheres of red giant stars is limited to the global thermal structure in the continuum-forming region, as can be tested by the flux distribution and angular diameter measurements(Sect.2) together with limited information on limb-darkening (Sect.3). On the other hand, it is difficult to find positive confirmation of the atmospheric model in the line-forming region in general, as is evidenced by the fact that many observations related to line spectra cannot be satisfactorily explained

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by classical model atmospheres (Sects.4 & 5). There is no doubt that model atmospheres of the first generation (classical model atmospheres) played a major role in interpretations and analyses of new observations in the 1960's and 1970's and served to provide basic knowledges on red giant stars. However, it is also clear that the classical models are now challenged by new observations of higher quality.

One problem in line spectra of cool stars is to obtain measurements of line intensity or profile of sufficient accuracy under the inherent difficulties in locating the continuum and in finding spectral regions free from intrinsic blending. Even with the recent progress in observational techniques, such difficulties could not entirely be resolved, and some of the difficulties encountered in testing models by line spectra may be due to difficulty in defining the true continuum, for example. In this regard, in addition to high resolution spectroscopy, well calibrated spectrophotometry in line-blanketed regions is very important. In fact, limited information by line-blanketed fluxes also reveals serious difficulties of classical models (Sect.4) and thus this problem cannot be attributed to the difficulty of continuum location alone. Some of these difficulties can be resolved by further improvements of the classical models (e.g., opacities) or in extending the available sophistications (e.g., non-LTE, spherical geometry). At the same time, observational tests already suggest that more radical improvements may be needed.

One major problem revealed by the recent analyses of line spectra is that inhomogeneity may prevail throughout the subphotospheric convective zone, through the surface layers, and on to the outer atmospheres. Theoretical modeling of the granular convection in red giant stars will certainly be very important in the future, and observational tests of such a model are already possible with available observational techniques (Sect.5.3). Also, an attempt of two-dimensional imaging of the stellar disk by temperature sensitive molecular bands, using the technique of speckle interferometry, will provide more direct tests of such inhomogeneities (Lynds et al. 1976). The temperature inhomogeneity due to the granular convection and the mechanical energy flux generated in the convective zone may further induce bifurcated structure in the surface layers of cool stars because of the characteristic molecular cooling function (Ayres 1981; Muchmore & Ulmschneider 1985). Observational tests of such a model are rather difficult by spatially unresolved spectroscopic observations, but some evidences for such a model can be found through systematic effects in temperature sensitive lines. Further, such a bistable structure in the upper atmosphere may develop into thermal instability due to molecular cooling (Kneer 1983) and may induce autocatalytic molecular (Muchmore et al. 1987) or dust (Stencel et al. 1986) formation in the outer atmosphere. Observational identification of a molecular dissociation zone in the outer atmosphere by low excitation CO lines can be regarded as the observational manifestation of such a possibility (Tsuji 1988a). Thus the presence of a cool molecular component, in addition to the warm chromosphere, in the outer atmospheres of red giant stars appears to be confirmed, and such a component may play an important role in determining the structure of the outer atmosphere and, eventually, in producing molecular outflow and mass-loss (Tsuji 1988b).

Now, the problem of model atmosphere of red giant stars may be closely related to an effort to have a unified model of the photosphere (including subphotospheric convective zone) and outer atmosphere (including a chromosphere with cool and warm components, dust envelope, and cool wind). For this purpose, observational as well as theoretical backgrounds are being developed, and there is no doubt that further efforts towards understanding the physical structure of the photosphere, together with the outer atmosphere, of red giant stars will be rewarding in promoting our basic understanding of red giant stars, including their evolution.

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