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Explaining the variation of the meridional circulation with the solar cycle

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Abstract. The meridional circulation of the Sun is observationally found to vary with the solar cycle, becoming slower during the solar maxima. We explain this by constructing a theoretical model in which the equation of the meridional circulation (the ϕ component of the vorticity equation) is coupled with the equations of the flux transport dynamo model. We find that the Lorentz force of the dynamo-generated magnetic fields can slow down the meridional circulation during the solar maxima in broad conformity with the observations.

Keywords. Sun: interior, Sun: magnetic fields, activity

1. Introduction

The meridional circulation is one of the most important large-scale coherent flow patterns within the solar convection zone. It has been found from helioseismic measurements that the meridional circulation varies with the solar cycle (Chou & Dai 2001; Zhao & Kosovichev 2004; Komm *et al.* 2015). These results are consistent with the surface measurements of Hathaway & Rightmire (2010), who found that the meridional circulation at the surface becomes weaker during the sunspot maximum by an amount of order 5 m s^{-1}. We explore whether this variation of the meridional circulation can be due to the Lorentz force of the dynamo-generated magnetic field.

The meridional circulation plays a very critical role in the flux transport dynamo model, which started being developed from the mid-1990s (Choudhuri, Schüssler & Dikpati 1995; Durney 1995). Most of the papers on the flux transport dynamo are of kinematic nature, which do not include the back-reactions of the magnetic field on the large-scale flows. In order to include such back-reactions, we have to couple the mean field dynamo equations with the dynamical equation governing the flow (i.e. essentially the Navier–Stokes equation). This is what we do in our recent paper (Hazra & Choudhuri 2017). The next two Sections summarize the methodology and the results presented by Hazra & Choudhuri (2017).

There is some observational evidence for random fluctuations in the meridional circulation having a time scale longer than the solar cycle. These fluctuations seem crucial in producing such effects as the Waldmeier effect (Karak & Choudhuri 2011), as well as the observed correlation of the decay rate of a cycle with the strength of the next cycle (Hazra *et al.* 2015). These fluctuations also make a contribution in producing grand minima (Choudhuri & Karak 2012). In this study, we do not include such fluctuations and consider only dynamo-induced periodic variations. Although there are indications that the spatial structure of the meridional circulation is more complicated than the single-cell pattern assumed in many dynamo models (see Hazra, Karak & Choudhuri 2014), it is still a debated issue and we assume a single-cell structure.

2. Methodology

To incorporate the effect of the Lorentz force on the meridional circulation, we need to consider the ϕ component of the vorticity equation, which is

$$\frac{\partial \omega_\phi}{\partial t} + s\nabla.\left(\mathbf{v}_m \frac{\omega_\phi}{s}\right) = s\frac{\partial \Omega^2}{\partial z} + \frac{1}{\rho^2}(\nabla\rho \times \nabla p)_\phi + [\nabla \times \mathbf{F}_L]_\phi + [\nabla \times \mathbf{F}_\nu(\mathbf{v}_m)]_\phi. \tag{2.1}$$

where ω_ϕ is the ϕ component of vorticity which comes from the meridional circulation $\mathbf{v}_m$ only, $s = r\sin\theta$, $\mathbf{F}_L$ is the Lorentz force term and $\mathbf{F}_\nu(\mathbf{v}_m)$ is the turbulent viscosity term corresponding to the velocity field $\mathbf{v}_m$. We now break up the the meridional velocity into two parts:

$$\mathbf{v}_m = \mathbf{v}_0 + \mathbf{v}_1, \tag{2.2}$$

where $\mathbf{v}_0$ is the regular meridional circulation the Sun would have in the absence of magnetic fields and $\mathbf{v}_1$ is its modification due to the Lorentz force of the dynamo-generated magnetic field. The azimuthal vorticity ω_ϕ can also be broken into two parts corresponding to these two parts of the meridional circulation:

$$\omega_\phi = \omega_0 + \omega_1. \tag{2.3}$$

We substitute Eqs. 2.2 and 2.3 in Eq. 2.1. Then we subtract from it the version of Eq. 2.1 for $\mathbf{v}_0$ alone ($\mathbf{F}_L$ will be absent in this case). Neglecting quadratic terms in perturbed quantities, we get

$$\frac{\partial \omega_1}{\partial t} + s\nabla.\left(\mathbf{v}_0 \frac{\omega_1}{s}\right) + s\nabla.\left(\mathbf{v}_1 \frac{\omega_0}{s}\right) = [\nabla \times \mathbf{F}_L]_\phi + [\nabla \times \mathbf{F}_\nu(\mathbf{v}_1)]_\phi \tag{2.4}$$

This equation is solved along with the mean field equations of the flux transport dynamo, which give the magnetic field from which the Lorentz force appearing in Eq. 2.4 is calculated. A solution of Eq. 2.4 first yields the perturbed vorticity ω_1 at different steps. We can then compute the perturbed velocity $\mathbf{v}_1$ using streamfunction-vorticity formalism. For more details of the methodology, we refer the reader to Hazra & Choudhuri (2017). We have assumed that the dynamo-generated magnetic fields do not affect the thermodynamics significantly, making the thermal wind term ($\frac{1}{\rho^2}(\nabla\rho \times \nabla p)_\phi$) to drop out of Eq. 2.4. This is what makes the theory of the modification of the meridional circulation decoupled from the thermodynamics of the Sun and simpler to handle than the theory of the unperturbed meridional circulation.

3. Results

Our discussions in this Section will refer to the northern hemisphere of the Sun, some quantities having the opposite sign in the southern hemisphere. The unperturbed meridional circulation there is anti-clockwise, implying a negative vorticity ω_0. We need to generate a positive perturbed vorticity ω_1 at the time of the sunspot maximum, if the meridional circulation is to be weakened by the Lorentz force at that time. Figure 1 reproduced from Hazra & Choudhuri (2017) illustrates this. The three rows of this figure plot three quantities in the $r-\theta$ plane: the toroidal field B_ϕ, the ϕ-component of the curl of the Lorentz force $[\nabla \times \mathbf{F}_L]_\phi$ and the perturbed vorticity ω_1 along with the associated streamlines. The five vertical columns correspond to five time intervals during a solar cycle. The second column corresponds to a time close to the sunspot maximum, whereas the the fourth column corresponds to the sunspot minimum.

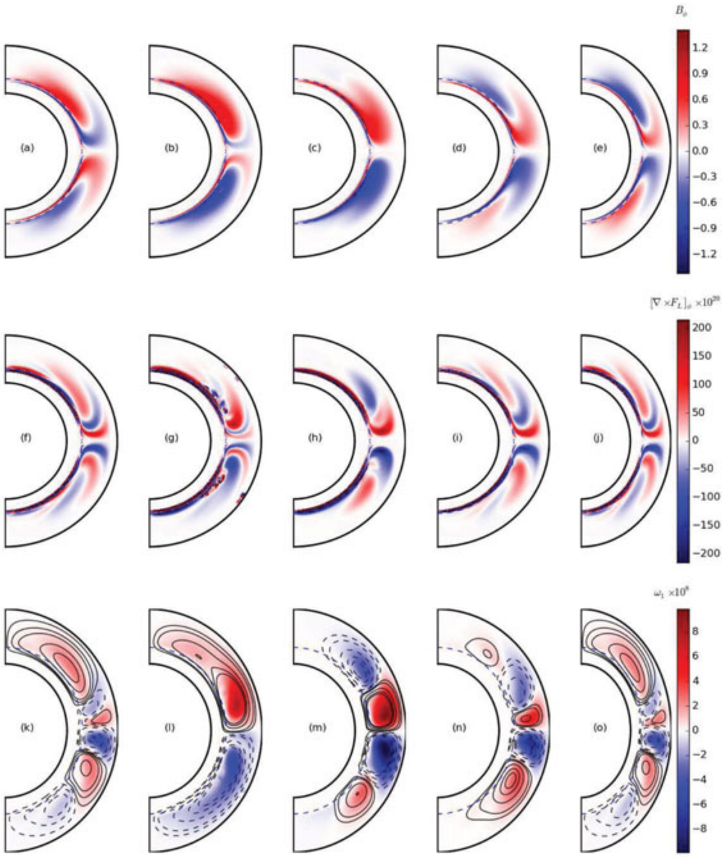

Figure 1. Snapshots in the $r-\theta$ plane of different quantities spanning an entire solar cycle: the toroidal field B_ϕ, the ϕ component of the curl of the Lorentz force $[\nabla \times \mathbf{F}_L]_\phi$ and the perturbed vorticity ω_1 with streamlines are shown in the first row (a)-(e), the second row (f)-(j), and the third row (k)-(o) respectively. The five columns represent time instants during the midst of the rising phase, the solar maximum, the midst of the decay phase and the solar minimum, followed by the midst of the rising phase again. In the third row (k)-(o), the solid black contours represent clockwise flows and the dashed black contours represent anti-clockwise flows. Taken from Hazra & Choudhuri (2017).

It is clear from the second column of Figure 1 (corresponding to the solar maximum) that $[\nabla \times \mathbf{F}_L]_\phi$ is negative in the northern hemisphere at that time, giving rise to a negative perturbed vorticity. This will clearly make the meridional circulation weaker at that time. When the flow velocity is calculated from the total vorticity as given by Eq. 2.3, we find that it varies in a periodic fashion. Figure 2 shows the time evolution of the total v_θ at 25° latitude just below the surface along with the sunspot number. We see that the meridional circulation reaches its minimum a little after the sunspot maximum. This figure can be compared with observed variation of the meridional flow at the surface (Fig. 4 of Hathaway & Rightmire 2010).

It may be noted that, while solving the dynamo equations in this exploratory study, we have used only the unperturbed meridional circulation given by $\mathbf{v}_0$. If we want to include the full time-varying meridional circulation in the dynamo equations, then we have to solve the streamfunction-vorticity equation at every time step to find the perturbed velocity from the perturbed vorticity. Although this is computationally expensive, we

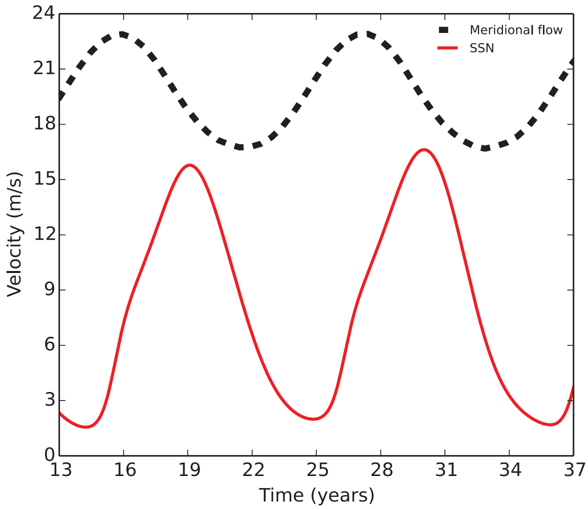

Figure 2. The black dashed line shows the time variation of the total meridional circulation just below the surface at 25° latitude. The red solid line shows the yearly averaged sunspot number. Taken from Hazra & Choudhuri (2017).

plan to do this in future for a typical illustrative case. While a varying meridional circulation would affect the magnetic field generated by the dynamo (Karak & Choudhuri 2011; Choudhuri & Karak 2012), a meridional circulation varying periodically with the solar cycle is not expected to change the behavior of the system qualitatively (Karak & Choudhuri 2012).

4. Conclusion

We conclude that the back-reaction due to the Lorentz force of the dynamo-generated magnetic field on the meridional circulation is the most likely reason for its variation with the solar cycle and our results are in good agreement with the observed variation of the meridional circulation.

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