Part 4
Reduced Models

> "I've just caught you in a contradiction. Don't you see." He proudly lettered "Contradiction" on his pad with his thick black pencil.
>
> J. Heller, *Catch-22*

The large-N reduction was first discovered in 1982 by Eguchi and Kawai [EK82], who showed that the $SU(N)$ Yang–Mills theory on a d-dimensional space-time is equivalent at $N = \infty$ to the one at a point. This construction is based on an extra symmetry of the reduced model which should not be broken spontaneously.

Soon after that it was recognized that this symmetry is, in fact, broken for $d > 2$. Two ways were proposed to cure the construction: the quenching prescription [BHN82] and the twisting prescription [GO83a]. Each of these two prescriptions results in a reduced model which recovers multicolor QCD both on the lattice and in the continuum.

While the reduced models look like a great simplification, since the space-time is reduced to a point, they still involve an integration over d infinite matrices which is, in fact, a continual path integral. For some years it was not clear whether or not this is a real simplification of the original theory which can make it solvable, so the point of view on the reduced models was that they are just an elegant representation at large N.

The recent interest in reduced models has arisen from the matrix-model formulation [BFS97, IKK97] of M-theory combining all types of superstring theories. The novel point of view on the reduced models is that they are equivalent [CDS98] to gauge theories on noncommutative space. The gauge field is no longer matrix-valued but rather noncommutativity of matrices in the reduced models is transformed into noncommutativity of coordinates in the noncommutative gauge theory, which in the limit of large noncommutativity reproduces ordinary Yang–Mills theory at large N.

We shall start this part by describing the original Eguchi–Kawai model and its quenched version. Then we discuss twisted reduced models and their equivalence to noncommutative gauge theories. Finally, we concentrate on the properties of noncommutative gauge theories as such.