BULL. AUSTRAL. MATH. SOC. 65R20, 45G10
VOL. 38 (1988) [151–152]

A NEW COLLOCATION-TYPE METHOD FOR THE NUMERICAL SOLUTION OF HAMMERSTEIN EQUATIONS

SUNIL KUMAR

The new method approximates the solution of the Hammerstein equation

$$y(t) = f(t) + \int_a^b k(t,s)g(s,y(s))ds, \quad t \in [a,b], \tag{1}$$

by first applying the standard collocation method to an equivalent equation for $z(t) := g(t,y(t))$. It then obtains the approximation to y by using the equation

$$y(t) = f(t) + \int_a^b k(t,s)z(s)ds, \quad t \in [a,b]. \tag{2}$$

Let z_n and y_n be the approximations to z and y respectively. It is proved that, under suitable conditions, y_n converges to y at a rate at least equal to that of the best approximation to z from the space in which z_n is sought. In particular, it is shown that if z_n is sought in certain piecewise polynomial function spaces, then y_n may exhibit (global) superconvergence, that is, it may converge to y at a faster rate than z_n does to z.

The discrete version of the method, which arises when numerical quadrature is used to approximate required integrals, is analysed for interpolatory quadrature rules and piecewise polynomial approximations to z. The analysis shows that the discrete approximation to z has the same order of convergence as its 'exact' counterpart if k is sufficiently smooth, and if the quadrature rule used is of sufficient precision.

The new method is used in the computation of a simple turning point $(y = y^c, \lambda = \lambda^c)$ of the parameter-dependent equation

$$y(t) = f(t) + \lambda \int_a^b k(t,s)g(s,y(s))ds, \quad t \in [a,b],$$

where $\lambda \in \mathbf{R}$. In the spirit of the method, the simple turning point $(z = z^c, \lambda = \lambda^c)$ of an equivalent equation for $z(t) := \lambda g(t,y(t))$ is computed first. This involves the

Received 9 December 1987. Thesis submitted to the University of New South Wales, April 1987. Degree approved November, 1987. Supervisor: Professor I.H. Sloan.

application of the standard piecewise polynomial collocation method to a certain system of equations which has (z^c, λ^c) as part of an isolated solution.

Let (z_n, λ_n) denote the approximation to (z^c, λ^c). Then y_n, the approximation to y^c, is obtained as before by the use of (2). It is proved that, under suitable conditions, the approximations to y^c and λ^c are both superconvergent, that is, they both converge to their respective exact values at a faster rate than z_n does to z^c.

Finally, (1) is considered again, and uniform convergence of y_n to y is established for the case where z_n is a polynomial of degree $\leqslant n-1$, with coefficients determined via collocation at the zeros of the nth degree Chebyshev polynomial of the first kind.

Computer Centre
The University of the South Pacific
P.O. Box 1168
Suva
Fiji