BULL. AUSTRAL. MATH. SOC. 46G99, 60B05, 28A25
VOL. 16 (1977), 159-160. (28A30, 60B10, 60J99)

Topological measure theory, with applications to probability

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Several aspects of the theory of measures on topological spaces are investigated in this thesis. For the most part they relate to problems which have arisen in the study of weak convergence of probability measures in settings more general than the usual (complete, separable) metric case.

Throughout, the treatment is based on the correspondence which exists between certain measures and linear functionals on function spaces. This enables simple functional analytic techniques to be applied to problems of measure theory. This correspondence, which is sometimes referred to as an *integral* (or *Riesz*) representation, is studied in some detail in Chapter 2. The results obtained are of sufficient generality to give a unified treatment of a number of well-known representations - Pollard and Topsøe [*1*].

The next chapter continues with the linear functional interpretation of measures. Simple derivations of necessary and sufficient conditions for compactness in spaces of measures under the topology of weak convergence are given, thus generalising a famous theorem of Prohorov.

Chapter 4 consists of a collection of examples which demonstrate some of the advantages of working with τ-additive measures for more general topological spaces. [A measure is τ-additive if $\mu\left(\bigcap_\alpha F_\alpha\right) = \inf_\alpha \mu\left(F_\alpha\right)$ for every downward filtering family of closed sets.] It is argued that the combination of τ-additive measures and uniform spaces provides the natural way of generalising the theory of measures on separable metric spaces.

Received 9 July 1976. Thesis submitted to the Australian National University, June 1976. Degree approved, November 1976. Supervisors: Dr C.C. Heyde (CSIRO, Division of Mathematics and Statistics), Professor P.A. P. Moran.

The final chapters contain some examples of the way in which the techniques of topological measure theory can be applied to probability. The existence and weak convergence of random measures (and point processes) on locally compact spaces is discussed in Chapter 5, while the concluding chapter treats some more specialised problems relating to the R-theory of Markov chains on general topological spaces.

Reference

[1] David Pollard and Flemming Topsøe, "A unified approach to Riesz type representation theorems", *Studia Math.* 54 (1975/1976), 173-190.

https://doi.org/10.1017/S0004972700023133 Published online by Cambridge University Press